Hypothesis Tests on the Mean

- H_0: $\mu = \mu_0$
- H_1: $\mu \neq \mu_0$

$$Z_0 = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}}$$

Reject H_0 if $Z_0 > z_{\alpha/2}$ *or* $Z_0 < z_{\alpha/2}$

Fail to reject H_0 if $-z_{\alpha/2} \le Z_0 \le z_{\alpha/2}$

Hypothesis Tests (one side)

- H_0: $\mu = \mu_0$
- H_1: $\mu > \mu_0$

Reject H_0 if $Z_0 > z_\alpha$

- H_0: $\mu = \mu_0$
- H_1: $\mu < \mu_0$

Reject H_0 if $Z_0 < -z_\alpha$

Example: two-sided test

- Suppose Guido takes a random sample of *n*=25 and obtains an average burn rate of 51.3 cm/s.
- Specs require that burn rate must be 50 cm/s, and the standard deviation is known to be 2 cm/s.
- He decides to specify a type I error probability (significance level) of 0.05.
- What conclusions can be drawn?

P-value

- The P-value is the smallest level of significance that would lead to rejection of the null hypothesis H0 with the given data.

$$P = \begin{cases} 2[1-\Phi(|z_0|)] & \text{for test} : H_0 : \mu = \mu_0 \quad H_1 : \mu \neq \mu_0 \\ 1-\Phi(z_0) & \text{for test} : H_0 : \mu = \mu_0 \quad H_1 : \mu > \mu_0 \\ \Phi(z_0) & \text{for test} : H_0 : \mu = \mu_0 \quad H_1 : \mu < \mu_0 \end{cases}$$

Probability of Type II Error

- The probability of the type II error is the probability that Z_0 falls between $-z_{\alpha/2}$ and $z_{\alpha/2}$ given that H_1 is true.

$$\beta = \Phi\left(z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma} \right) - \Phi\left(-z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma} \right)$$

β Requirement Sample Size

For two - tailed tests

$$n \approx \frac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2}, \delta = \mu - \mu_0$$

For one - tailes tests

$$n \approx \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2}, \delta = \mu - \mu_0$$

Example (cont.)

- Suppose Guido wants to design the burn rate test so that if the true mean burn rate differs from 50 cm/s by as much as 1 cm/s, the test will detect this (i.e. reject the null hypothesis) w.p. 0.90. Determine the sample size required to detect this departure.

Confidence Intervals

If $\bar{x}$ is the sample mean of a random sample of size n from a population with variance σ^2, a $100(1-\alpha)\%$ CI on μ is given by

$$\left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

Example

- Construct a 95% confidence interval for the burn rate. 25 samples were taken. The sample mean is 51.3 and the standard deviation is 2.
- What is the relationship between hypothesis testing and confidence intervals?

Error Bound Sample Size

If $\bar{x}$ is an estimate of μ, we can be $100(1-\alpha)\%$ confident that the error $|\bar{x}-\mu|$ will not exceed a specified amount E when the sample size is:

$$n = \left(\frac{z_{\alpha/2}\sigma}{E} \right)^2$$

Example

- Suppose that we wanted the error in estimating the mean burn rate of the rocket propellant to less than 1.5 cm/s, with 95% confidence. What is the required sample size?

One-Sided CIs

The $100(1-\alpha)\%$ upper-CI for μ is

$$\mu \le \overline{x} + z_{\alpha}\frac{\sigma}{\sqrt{n}}$$

The $100(1-\alpha)\%$ lower-CI for μ is

$$\mu \ge \overline{x} - z_{\alpha}\frac{\sigma}{\sqrt{n}}$$

General CIs

Let $\hat{\theta}$ be an estimator for θ.

The $100(1-\alpha)\%$ CI, (L, U), is given by:

$$\mathrm{P}\left(L \le \hat{\theta} \le U\right) = 1 - \alpha$$