Standard error

- Standard error $\sigma_{\hat{\Theta}} = \sqrt{V(\hat{\Theta})}$
- Estimated standard error $\hat{\sigma}_{\hat{\Theta}}$,$s_{\hat{\Theta}}$, $se(\hat{\Theta})$

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} \qquad \hat{\sigma}_{\overline{X}} = \frac{S}{\sqrt{n}}$$

Example 1

- While measuring the thermal conductivity of Armco iron, using a temperature of 100F and a power of 550W, the following 10 measurement of thermal conductivity (in Btr/hr-ft-F) were obtained:

41.60	41.48	42.34	41.95	41.86
42.18	41.72	42.26	41.81	42.04

- Find point estimate of the mean conductivity and the estimated standard error.

Mean square error

- Definition: $MSE(\hat{\Theta}) = E(\hat{\Theta} - \theta)^2$

$$MSE(\hat{\Theta}) = E[\hat{\Theta} - E(\hat{\Theta})]^2 + [\theta - E(\hat{\Theta})]^2$$
$$= V(\hat{\Theta}) + (bias)^2$$

- Relative efficiency $\frac{MSE(\hat{\Theta}_1)}{MSE(\hat{\Theta}_2)}$

Maximum Likelihood Estimation

- The Maximum Likelihood Estimate, $\hat{\theta}$ of a parameter θ is the numerical value of θ for which the observed results in the data at hand would have the highest probability of arising.

Example: Defect Rate

- Suppose that we want to estimate the defect rate f in a production process. We learn that, in a random sample of 20 items that completed the process, 3 were found to be defective. Given only this outcome, we must estimate f.

MLE Definition

- The Maximum Likelihood Estimator, $\hat{\Theta}$, is the value of θ that maximizes

  $$L(\theta) = f(x_1; \theta).f(x_2; \theta). \ldots .f(x_n; \theta)$$

- Example: Bernoulli
- Example: Exponential
- Other examples in text: Normal

Properties of MLE

- In general, when the sample size *n* is large, and if $\hat{\Theta}$ is the MLE of θ:
  - $\hat{\Theta}$ is an approx unbiased estimator for $\theta [E(\hat{\Theta}) \approx \theta]$
  - The variance of $\hat{\Theta}$ is almost as small as that obtained by any other estimator
  - $\hat{\Theta}$ has an approx normal distribution

The Invariance Property

- Let $\hat{\Theta}_1, \hat{\Theta}_2, \ldots, \hat{\Theta}_k$ be the MLE of θ_1, θ_2 ,..., θ_k . Then the MLE of any function $h(\theta_1, \theta_2 ,..., \theta_k)$ of these parameters is the same function $h(\hat{\Theta}_1, \hat{\Theta}_2, \ldots, \hat{\Theta}_k)$ of the estimators.

MLE vs. unbiased estimation

- What is the difference between MLE and unbiased estimation?
- Can a procedure meet one standard and not the other?

- Example: uniform distribution on [0,a].