Random Sampling

- Population
- Random sample:
  - Independent variables
  - Same probability distribution
- Statistics
- Point estimate

Examples

- Mean μ of a single population
- Variance σ^2 of a single population
- Difference in means of two population

$$\mu_1 - \mu_2$$

Unbiasedness

- An estimator Θ of some unknown quantity θ is said to be unbiased if the procedure that yields Θ has the property that, were it used repeatedly the long-term average of these estimates would be θ.
- That is to say, $E(\Theta) = \theta$.

Example 1

- Given X = (5.8, 4.4, 8.7, 7.6, 3.1)
- The Sample Mean $\overline{X}$
- The Sample Median $\tilde{X}$
- The 20% trimmed mean $\overline{X}_{tr(20)}$

Example 2

- Sample Variance

$$S^2 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}$$

- Expectation of Sample Variance

$$E(S^2) = \sigma^2$$

Variance of a Point Estimator

- MVUE – Smallest variance estimator
- Theorem

  If $X_1,...X_n$ is a random sample of size n form a normal distribution with mean μ and variance σ^2, then the sample mean $\overline{X}$ is the MVUE for μ

Example 1 (continued)

$$V(\bar{X}) = \sigma^2/n$$

$$V(\tilde{X}) = \sigma^2$$

$$V(\bar{X}_{tr(20)}) = \sigma^2/0.6n$$