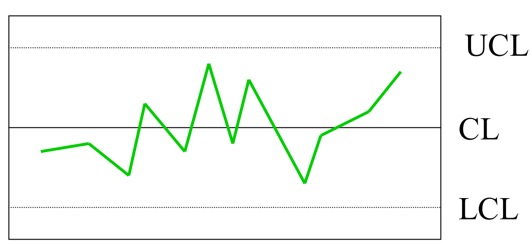
Statistical Process Control

- Is a tool for
  - achieving process stability
  - improving capability by reducing variability
- Variability can be due to
  - **chance causes** (relatively small)
  - **assignable causes** (generally large compared to background noise)

Control Charts



- $UCL = \mu_W + k\sigma_W$
- $CL = \mu_W$
- $LCL = \mu_W - k\sigma_W$

Types of Control Charts:

- Variables Control Charts
- Attributes Control Charts

Purpose of Control Charts

- Improving productivity
- Preventing defects
- Preventing unnecessary process adjustments
- Providing diagnostic information
- Providing information about process capability

Design of a Control Chart

- An $\overline{X}$ control chart:
  - Standard deviation of the sample average.
  - Choosing the control limits.
- Relationship to hypothesis testing.
- Decisions:
  - Sample size to use.
  - Frequency of sampling.

Example of the control chart

- In manufacturing automobile engine piston rings, the inside diameter of the rings is a critical quality characteristic. Given the process average diameter is 74mm, the process standard deviation of diameter is 0.01mm and the sample size is 5.
- How the control limits are determined?

Analysis of Patterns

- Types of patterns:
  - Run (8 or more)
  - Cyclic behavior
- Western Electric Rules (Zone Rules):
  - One point outside 3σ control limits.
  - 2 / 3 consecutive points beyond a 2σ CL.
  - 4 / 5 consecutive points at $\geq 1\sigma$ from CL.
  - 8 consecutive points on one side of the CL.

The $\bar{X}$ Control Chart

- When μ and σ are unknown…
- R = Range of the sample
- W = Relative range = R / σ
- E(W) = d_2; Stdev(W) = d_3
- What would be an estimator for σ?
- What are the UCL and LCL?

Definitions

$$\bar{\bar{X}} = \frac{1}{m}\sum_{i=1}^{m}\bar{X}_i \qquad \bar{R} = \frac{1}{m}\sum_{i=1}^{m}R_i$$

$\bar{X}$ control chart

$$UCL = \bar{\bar{x}} + A_2\bar{r}$$
$$CL = \bar{\bar{x}}$$
$$LCL = \bar{\bar{x}} - A_2\bar{r}$$

R control chart

$$UCL = D_4\bar{r}$$
$$CL = \bar{r}$$
$$LCL = D_3\bar{r}$$

The R Control Chart

- What is a reasonable estimator for σ_R?
- What are the corresponding CL's?

- Example: $n = 5; \bar{\bar{x}} = 33.32, \bar{r} = 5.8$

P Chart

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$CL = \bar{p}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

- where $\bar{p}$ is the observed value of the average fraction defective

U Chart

$$UCL = \overline{u} + 3\sqrt{\frac{\overline{u}}{n}}$$

$$CL = \overline{u}$$

$$LCL = \overline{u} - 3\sqrt{\frac{\overline{u}}{n}}$$

- where $\overline{u}$ is the observed value of the average fraction defective

Control Chart Performance

- ARL = Average Run Length

$$ARL = \frac{1}{p}$$

  - p -- probability that any point exceeds control limits