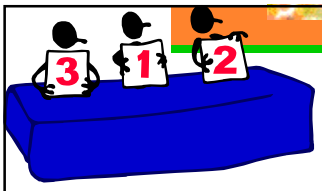
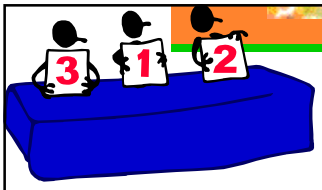
Wilcoxon Signed-Rank Test

- Uses both direction (sign) and magnitude.
- Applies to the case of symmetric continuous distributions:
  - Mean equals median.



Method

- H_0: $\mu = \mu_0$
- Compute differences, $X_i - \mu_0$, i = 1,2,…,n
- Rank the absolute differences $| X_i - \mu_0 |$
- W^+ = sum of positive ranks
- W^- = sum of negative ranks
- From Table X in Appendix: critical w_α*
- What are the rejection criteria for different H_1?

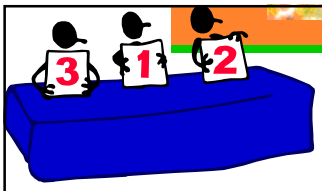


Large Samples

- If sample size is large, n > 20
- W^+ (or W^-) is approximately normal with

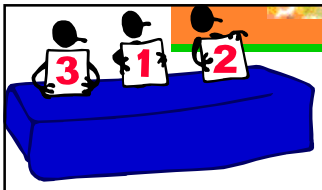
$$\mu_{W+} = \frac{n(n+1)}{4}$$

$$\sigma^2_{W+} = \frac{n(n+1)(2n+1)}{24}$$



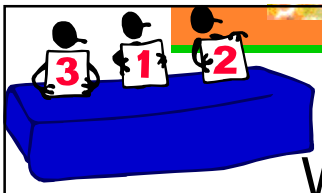
Paired Observations

- Paired data has to be from two continuous distributions that differ only wrt their means.
- Their distributions need not be symmetric.
- This ensures that the distribution of the differences is continuous and symmetric.



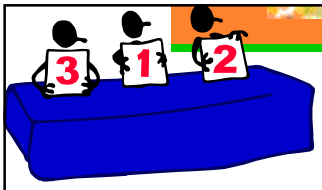
Compare to t-test

- If underlying population is normal, t-test is best (has lowest β).
- The Wilcoxon signed-rank test will never be much worse than the t-test, and in many nonnormal cases it may be superior.
- The Wilcoxon signed-ran test is a useful alternate to the t-test.



Wilcoxon Rank-Sum Test

- Data from two samples with underlying distributions of same shape/spread, $n_1 \leq n_2$
- Rank all n_1+n_2 observations in ascending order
- W_1 = sum of ranks in sample 1
- $W_2 = 0.5(n_1+n_2)(n_1+n_2+1) - W_1$
- Table XI in the Appendix contain the critical value of the rank sums. What are the rejection criteria?

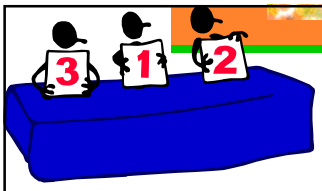


Large Samples

- If sample sizes are large, $n_1, n_2 > 8$
- W_1 is approximately normal with

$$\mu_{W_1} = \frac{n_1(n_1 + n_2 + 1)}{2}$$

$$\sigma^2_{W_1} = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}$$



Compare to t-test

- When underlying distributions are normal, the Wilcoxon signed rank and rank-sum tests are approx 95% as efficient as the t-test in large samples.
- Regardless of the distribution, the Wilcoxon tests are at least 86% as efficient.
- Efficiency of Wilcoxon test relative to t-test is usually high if distributions have heavier tails than the normal.