Nonparametric Tests

- Underlying population distribution is continuous. No other assumptions.
- Data need not be quantitative, but may be categorical or rank data.
- Very quick and easy to perform.

Parametric vs. Nonparametric

- When distribution is normal, parametric test is better. The nonparametric test requires larger sample size to achieve same power.
- When distribution is not close to normal, nonparametric methods are much better.
- Choose the parametric procedure whenever possible.

Sign Test (1)

- Tests hypotheses about the median.
- Null hypothesis $H_0: \tilde{\mu} = \tilde{\mu}_0$
- Find the differences $x_i - \tilde{\mu}_0$
- Test statistic: R^+ is number of differences that are positive.
- What are the P-values for different tests?
- Reject H_0 if the P-value is less than α.

Sign Test (2)

- When n is large (at least 10), and p=0.5, the binomial is approximately normal.
- R^+ has a normal distribution with mean 0.5n and variance 0.25n.
- The test statistic is:

$$Z_0 = \frac{R^+ - 0.5n}{0.5\sqrt{n}}$$

Sign Test for Paired Samples

- $D_j = X_{1j} - X_{2j}$
- The test statistic is:

$$R = \min\{R^+, R^-\}$$

Type II Error

- In determining β, we need:
  - The alternative median
  - AND the *form* of the underlying distribution

Comparison to the t-Test

- If underlying population is normal
  - Either test can be used
  - T-test has smallest β for all one-sided tests
  - T-test has smallest β for all two-sided tests with symmetric critical regions
- If underlying population is nonnormal but symmetric
  - T-test will have smaller β unless tails are heavy
- Sign test is considered test for median, rather than serious competitor for the t-test.