IE 121 – Spring 2002

Midterm exam 2

1. (15 points) In this problem, you need to select the correct answer.

   1. The null hypothesis is H_0: $\mu = \mu_0$, and the variance of the population is known. Which test statistic has to be used for the hypothesis testing?
      a) z_0
      b) t_0
      c) χ_0^2

   2. Let L_{95} be the length of the 95% confidence interval for a certain mean. Let L_{90} be the length of the 90% confidence interval for the same mean. Which of the following statements is true?
      a) $L_{95} > L_{90}$
      b) $L_{95} < L_{90}$
      c) $L_{95} = L_{90}$

   3. Suppose we want to test the null hypothesis on the population proportion H_0: $p = p_0$ against the alternative one H_1: $p > p_0$ at the significance level α. Which of the following is the correct rejection criterion?
      a) $|z_0| > z_{\alpha/2}$
      b) $z_0 > z_\alpha$
      c) $z_0 < -z_\alpha$
      d) $z_0 > z_{\alpha/2}$

   4. When the number of degrees of freedom of the t distribution is very large, it becomes:
      a) a normal distribution with mean 0 and variance 2
      b) a χ^2 distribution
      c) the standard normal distribution
      d) an exponential distribution

   5. Which of the following distributions is symmetric?
      a) t distribution
      b) χ^2 distribution

   Answers: 1. a) 2. a) 3. b) 4. c) 5. a)

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2. (25 points) A random sample of 200 semiconductor devices were tested and 4 of them were found to be defective.

a) Find a 95% confidence interval on the true proportion of defective devices.

b) Using the point estimate of p from the above sample, how many devices must be tested to be 90% confident that the error in estimating the true value of p is no more than 0.02?

c) What the sample size should be if we want to be at least 90% confident that the error in estimating p does not exceed 0.02 regardless of the value of p?

Solution:

a)

$$0.02 - 1.96\sqrt{\frac{0.02 \cdot 0.98}{200}} \leq p \leq 0.02 + 1.96\sqrt{\frac{0.02 \cdot 0.98}{200}}$$

$$0.010 \leq p \leq 0.030$$

b) $n = \left(\frac{1.64}{0.02}\right)^2 0.02 \cdot 0.98 = 131.8 \simeq 132$

c) $n = \left(\frac{1.64}{0.02}\right)^2 0.25 = 1681$

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3. (30 points) A chocolate factory wants to make sure that the mean of sugar content μ of one of their products does not exceed 30%. To test this hypothesis H_0: $\mu = 30$ against the alternate one H_1: $\mu > 30$ they choose a random sample of size 20. The sample mean is found to be $\bar{x} = 31.1\%$, and the sample standard deviation turns out to be $s = 2.5\%$ (they have no independent knowledge of the variance at this point). They want to use $\alpha = 0.1$.

a) Will they reject the null hypothesis given this data?

b) Now suppose they know that the true standard deviation is $\sigma = 2.4$. What sample size should they use in order to be able to detect the true mean sugar content of 31.5% with probability 0.95?

Solution:

a) $t_0 = \frac{31.1-30}{2.5/\sqrt{20}} = 2.050 > t_{0.1,19} = 1.328$

So they will reject the null hypothesis.

b) Since the standard deviation is known, we can use the corresponding methods (formula 8-25 from the old book and 9-20 from the new book):

$n = \frac{(1.28+1.64)^2 \cdot 2.4^2}{1.5^2} = 21.8 \simeq 22$

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4. (30 points) We would like to test the hypothesis that whether a student likes coffee is independent on the major. For that purpose we poll a 100 Engineering and a 100 Arts and Sciences majors. The results are shown in the table below.

a) Would we reject the hypothesis at $\alpha = 0.1$ significance level?

b) What is the range on the corresponding P-value?

	Like coffee	Do not like coffee
Engineering	45	55
Arts and Sciences	30	70

Solution: a) The table of expected frequencies if the hypothesis of independence is true reads:

	Like coffee	Do not like coffee
Engineering	37.5	62.5
Arts and Sciences	37.5	62.5

So the test statistic is

$$\chi_0^2 = \frac{(45-37.5)^2}{37.5} + \frac{(30-37.5)^2}{37.5} + \frac{(55-62.5)^2}{62.5} + \frac{(70-62.5)^2}{62.5} = 4.80 > 2.71 = \chi^2_{0.1,1}$$

so we reject the hypothesis at $\alpha = 0.1$.

b) From the χ^2 table: $0.025 \leq P \leq 0.05$