TEAM ROUND

1. In quadrilateral $ABCD$, $AB = 2\sqrt{6}, BC = 7 - 2\sqrt{3}, CD = 5$, $\measuredangle B = 135°$, and $\measuredangle C = 120°$. Compute AD.

2. When expanded as a decimal, the fraction 1/97 has a repetend (the repeating part of the decimal) that begins right after the decimal point and is 96 digits long. If the last three digits of the repetend are A67, compute the digit A.

Problem 3. For real numbers α, B, and C, the zeros of $T(x) = x^3 + x^2 + Bx + C$ are $\sin^2\alpha$, $\cos^2\alpha$, and $-\csc^2\alpha$. Compute $T(5)$.

4. For an olympiad geometry problem, Tina wants to draw an acute triangle whose angles each measure a multiple of $10°$. She doesn't want her triangle to have any special properties, so none of the angles can measure $30°$ or $60°$, and the triangle should definitely not be isosceles.

How many different triangles can Tina draw? (Similar triangles are considered the same.)

5. A geometric progression of positive integers has n terms; the first term is 10^{2015} and the last term is an odd positive integer. How many possible values of n are there?

6. Let a, b, m, n be positive integers with $am = bn = 120$ and $a \neq b$. In the coordinate plane, let $A = (a, m)$, $B = (b, n)$, and $O = (0, 0)$. If X is a point in the plane such that $AOBX$ is a parallelogram, compute the minimum area of $AOBX$.

7. If

$$\tan(120° - x) = \frac{\sin 120° - \sin x}{\cos 120° - \cos x},$$

where $0° < x < 180°$, compute x.

8. In square $ABCD$ with diagonal 1, E is on $\overline{AB}$ and F is on $\overline{BC}$ with $m\angle BCE = m\angle BAF = 30°$. If $\overline{CE}$ and $\overline{AF}$ intersect at G, compute the distance between the incenters of triangles AGE and CGF.

Problem 9. Let S be the set of integers from 0 to 9999 inclusive whose base-2 and base-5 representations end in the same four digits. (Leading zeros are allowed, so $1 = 0001_2 = 0001_5$ is one such number.) Compute the remainder when the sum of the elements of S is divided by 10,000.

10. Let S be the set of integer triplets (a, b, c) with $1 \le a \le b \le c$ that satisfy $a + b + c = 77$ and:

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{5}.$$

What is the value of the sum $\sum_{(a,b,c)\in S} a \cdot b \cdot c$?