1. Write as a simple fraction $\frac{1}{2}/(\frac{1}{4}+\frac{1}{5})$.
2. A drink is one-third syrup and the rest water. If six ounces of water were added, it would become 20% syrup. How many ounces of total liquid (water plus syrup) were in the original mixture?
3. Sam can answer each question on a test in 4 minutes, and Dave can answer each question in 1 minute. Dave takes a nap for an hour in the middle of the test. They finish at the same time. How many questions were on the test?
4. A runner runs around a circular track in the same amount of time that a slower runner takes to run around a smaller concentric circular track. The faster runner is twice as fast as the slower runner, and is running on a track of circumference 120π meters. How many meters apart are the two tracks?
5. If a segment of length 64 is divided into three parts whose lengths are proportional to 2, 4, and 6, what is the length of the shortest part?
6. A straight pole which was vertical is broken 36 feet from the ground, and the top part comes down diagonally from the break until it touches the ground 27 feet from the bottom of the pole. How many feet tall was the pole?
7. A sphere of radius 2 is inscribed inside a cube of side length 4. Let v be a vertex of the cube. Let S be the set of points inside the cube and outside the sphere which are closer to v than to any of the other vertices of the cube. What is the volume of S?
8. Determine the product of the roots of the equation

$$|x|(|x|-5)=-6.$$

9. In what base $b>1$ does the equation $24^2=642$ hold? Here both numbers are written in base b.

10. A cubical die with the numbers 1, 2, 3, 4, 5, and 6 on its faces is loaded in such a way that the probability that the number j turns up is proportional to j. What is the probability that an even number turns up?

11. What is the distance between the points of intersection of the curves $y = x^2 + x$ and $y = 3x + 4$?

12. Triangle ABC has $AC = BC$. The point D lies on the segment BC so that $AB = AD = CD$. What is the measure of $\angle ACD$ in degrees?

13. What is the sum of all fractions of the form $\dfrac{3^n + 4^n}{12^n}$ as n ranges over all nonnegative integers? That is, what is the value of $2 + \frac{7}{12} + \frac{25}{144} + \cdots$?

14. In a 3-by-3 grid, all squares except the center square are filled in with integers such that only two distinct integers occur, the sum of the eight integers is 111, and for each of the four sides of the figure, the sum of the three integers along that side is 42. What are the two integers in the grid?

15. For $n \geq 3$, let $f(n) = \log_2(3) \cdot \log_3(4) \cdots \log_{n-1}(n)$. What is the value of $\displaystyle\sum_{k=2}^{99} f(2^k)$?

16. Let $f(x) = |x - a| + |x - 10| + |x - a - 10|$, where a is some number satisfying $0 < a < 10$. What is the minimum value taken by f? (Your answer may be an expression involving a or may be purely numerical, but should be in simplest form.)

17. Let A denote the set of the 26 letters of the alphabet. Let B be a subset of A such that exactly 7/8 of all subsets of A have nonempty intersection with B. How many elements are in B?

18. Evaluate $\displaystyle\sum_{k=1}^{21} \frac{(-2)^k}{k!(21-k)!}$ as a simple fraction. You may use factorial symbols in your answer.

19. There are two circles which pass through the points $(1,9)$ and $(8,8)$ and are tangent to the x-axis. What are their radii?

20. Evaluate $\dfrac{\cot^2\theta - \tan^2\theta}{2+\cot^2\theta+\tan^2\theta}$ when θ equals 15 degrees.

21. A boat travels around an equilateral triangle. Along each side, it has a constant speed. The first 3/4 of the distance requires 3.5 hours, while the last 3/4 of the distance requires 4.5 hours. The middle third takes 10 minutes longer than the first third. How many hours did it take to travel around the triangle?

22. Let $[x]$ denote the greatest integer equal to or less than x. What is the smallest positive value of $[x]$ for which $[x^2]-[x]^2 = 2015$?

23. What is the remainder when 2^{1000} is divided by 13?

24. Evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^{n-1}}$.

25. How many ordered pairs (x, y) of integers (not necessarily positive) satisfy $\frac{1}{x}+\frac{1}{y}=\frac{1}{4}$?

26. The equation $x^3 - 3x^2 + bx + 6 = 0$ has three distinct real solutions which form an arithmetic progression. What is the ordered pair (b, r), where r is the smallest solution of the equation?

27. There is only one common prime divisor of 193499, 180253, and 160921. What is it?

28. Two perpendicular chords of equal length in a circle of radius 1 divide each other into segments of ratio 1:4. What is the length of one of these chords?

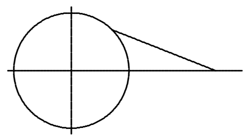
29. If $\displaystyle\sum_{i=1}^{162} \frac{1}{i}$ is written as a reduced fraction $\frac{a}{b}$, what is the largest integer n such that 3^n divides b?

30. The distances of a line from vertices A, B, and C of parallelogram $ABCD$ are 1, 2, and 5, respectively. What are the possible values of the distance from D to that line?

31. If $(x+1)^{2000}$ is expanded, how many of the coefficients are odd?

32. How many paths of exactly six moves may a King follow on a 5-by-5 chessboard in getting from the top left square to the bottom right square without backtracking? The possible moves are 1 space to the right, 1 space down, or 1 space diagonally down and to the right.

33. The left end of a rod of length 3 moves counterclockwise around the circle $x^2 + y^2 = 1$ at 240 revolutions per minute, while the other end is constrained to move along the x-axis. What will be the x-value of the right end of the rod $\frac{91}{3}$ seconds after it is at $x = 4$? (A picture of the circle and the rod at a typical moment appears below.)



34. Determine the set of real values of x that satisfy the inequality
$$\frac{4x^2}{(1-\sqrt{1+2x})^2} < 2x + 9.$$

35. There is only one prime number p which can be written as $a^4 + b^4 + c^4 - 3$, with a, b, and c all prime, not necessarily distinct. What is the value of p?

36. An *interior diagonal* of a convex 3-dimensional polyhedron P is a line segment whose endpoints are two vertices of P and which, except for its endpoints, lies entirely in the interior of P. What is the maximum number of interior diagonals of a convex polyhedron with 13 vertices?

37. What is the largest area of an ellipse that can be inscribed in a triangle with sides 3, 4, and 5?

38. In triangle ABC, points D and E lie on side AB dividing the side in a ratio 1:2:1; i.e., DE is twice as long as the segments AD and EC on either side of it. Let AM be the median to BC, and let G and H denote the intersection points of this with CD and CE, respectively. What is the ratio $AG{:}GH{:}HM$? Write your answer as integers with no factors common to all.

39. Let $P(n)$ denote the probability that in n tosses of a fair coin there will be a string of at least 3 consecutive Heads. What is the smallest n such that $P(n) \geq 1/2$?

40. A piece of paper in the shape of an equilateral triangle ABC has $AB = 12$. When A is folded over to the point D on BC for which $BD = 3$, a crease is formed along a line that joins a point on AB to a point on AC. What is the length of this crease?