Math 22 Hand-in Homework problems

1. Find the area of the region bounded by the line $y = \frac{1}{2}x$ and the parabola $y^2 = 8 - x$ in two ways. First do it by integrating with respect to x, and then do it by integrating with respect to y.
2. Find the volume of the solid obtained by revolving the region between $y = x^2$ and $y = 2x$ about the y-axis. Use the method of washers.
3. Find the volume of the annular ring obtained by revolving the region between the curves $y^2 = x$ and $y = x^3$ around the line $x = -1$. Use the method of washers.
4. Find the volume of the annular ring in #3 using the method of cylindrical shells.
5. For a body falling from rest, its position at time t is given by $s = \frac{1}{2}gt^2$. Write its velocity as a function of time, and then write its velocity as a function of s. Suppose it falls for T seconds, and let v_T denote its final velocity. Write the average velocity during these T seconds as a certain number times v_T, first averaging with respect to t and then averaging with respect to s.
6. Evaluate $\int x^3\sqrt{1-x^2}\,dx$.
7. Evaluate $\int x^7 \cos(x^4)\,dx$.
8. Evaluate $\int_0^{\pi/8} \cos^2(4x)\,dx$.
9. Evaluate $\int \frac{\sin^3 x}{\sqrt{\cos x}}\,dx$.
10. Evaluate $\int \tan^3 x \sec^3 x\,dx$.
11. Evaluate $\int \frac{\sqrt{x^2-16}}{x}\,dx$, where $x > 4$.
12. Evaluate $\int x \sin^{-1} x\,dx$ by first integrating by parts and then using a trigonometric substitution.
13. Evaluate $\int_0^2 \frac{x}{x^2+5x+6}\,dx$.
14. Evaluate $\int \frac{dx}{x(x^2+x+1)}$.
15. Evaluate $\int \frac{\tan^{-1} x}{(x-1)^3}\,dx$.
16. Use the Trapezoid Rule with $n = 5$ to find an approximate value of $\frac{\pi}{4} = \int_0^1 \frac{1}{1+x^2}dx$. Compare the error given by the error estimate with the actual error.
17. Modification of exercise 24 on page 506. Replace $\sqrt{4-x^3}$ by $\sqrt{4-x^5}$.
18. Tell whether the improper integrals $\int_1^\infty \frac{1}{x+x^2}\,dx$, $\int_0^1 \frac{1}{x+x^2}\,dx$, and $\int_0^\infty \frac{1}{x+x^2}\,dx$ converge or diverge. Explain your reasoning.
19. Find the length of the curve $x = \frac{1}{6}y^3 + \frac{1}{2y}$, $1 \le y \le 2$.
20. Note that the surface obtained by rotating the curve $y = \sqrt{1-x^2}$, $0 \le x \le 1$, around the x-axis is a hemisphere of radius 1. Compute the surface area of the portion between $x = 0$ and $x = x_0$, and show this is proportional to x_0.

21. Find the centroid of the region in the first quadrant which lies above the curve $y = x^2$ and below the line $y = 1$.

22. Use the Theorem of Pappus and the known values of the area of a circle of radius r and the volume of a sphere of radius r to find the centroid of the portion of the first quadrant which lies inside the circle $x^2 + y^2 = r^2$. Use the fact that when this region is revolved about the y-axis, one obtains a hemisphere.

23. Which of the following is the general solution of the DE $y' = xe^{-\sin x} - y\cos x$?: $y = \frac{1}{2}x^2 e^{-\sin x} + C$ or $y = (\frac{1}{2}x^2 + C)e^{-\sin x}$. Then find the solution which satisfies the initial condition $y(\pi) = 0$.

24. Find the solution of the separable DE $y' = (xy - y)/(y + 1)$.

25. A chemical manufacturing company has a large tank which it uses to control the release of pollutants into a sewage system. Initially the tank contains 360 gallons of water containing 2 pounds of pollutant per gallon. Water containing 3 pounds of pollutant per gallon enters the tank at the rate of 60 gallons per hour and is uniformly mixed with the water already in the tank. Simultaneously, water is released from the tank at the rate of 60 gallons per hour. How many pounds of pollutant are in the tank after t hours?

26. Work a modified version of number 4 on page 598. In this version, add 100 to the number of yeast cells at each time. Thus, for example, at time 0 there are 118 and at time 10 there are 609. You need not work part (a); instead, use 780 as the carrying capacity.

27. Work number 6 on page 598, using 2 billion (= 2000 millions) as the carrying capacity. Let $P(t)$ denote the population in millions t years after the year 1990.

28. Find the general solution of $y' + 2xy = x$ and find the particular solution that satisfies $y(0) = 2$.

29. Work problem 33 on page 607 with the 100 replaced by 400. (You must explain why the differential equation is as stated, and then solve it.)

30. Show that the parametric curve $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$, $-\infty < t < \infty$, is a circle minus one point. Give the equation of the circle and the coordinates of the missing point. Explain how the circle is traced as the parameter t varies.

31. Find the points on the curve $x = 3 - 4\sin t$, $y = 4 + 3\cos t$, $-\infty < t < \infty$, where there is horizontal tangent and those where there is a vertical tangent. Then find the Cartesian (x-y) equation of the curve and identify it. (Hint: Use $\cos^2 t + \sin^2 t = 1$.)

32. Find the length of the spiral curve $x = t\cos t$, $y = t\sin t$, $0 \le t \le 2\pi$. You may use the integral tables in the back of the text to evaluate the integral.

33. Sketch the polar curve $r = \sin(4\theta)$. Explain how you obtained the graph.

34. Sketch the polar curve $r^2 = \sin(\theta)$. Explain how you obtained the graph.

35. Find the coordinates (x, y) of the points where the curve in number 34 has a vertical tangent.

36. Find the length of the cardioid $r = 1 - \cos\theta$.

37. Find the area that lies within the limacon $r = 1 + 2\cos\theta$ and outside the circle $r = 2$.

38. Evaluate the limit of the sequence with $a_n = (2/n)^{3/n}$.

39. Tell whether each of the following infinite series converges or diverges. If it converges, find its sum.

$$(a)\ \sum_{n=1}^{\infty}\left(\left(\tfrac{1}{2}\right)^n - \left(\tfrac{1}{3}\right)^n\right); \qquad (b)\ \sum_{n=0}^{\infty}\frac{1}{1+(.9)^n}.$$

40. Find the values of x for which the series $\displaystyle\sum_{n=0}^{\infty}\frac{(x+4)^n}{3^n}$ converges, and find the sum of the series for these values of x.

41. Find the sum $\sum_{n=1}^{\infty}\frac{1}{n^6}$ correct to four decimal places. Explain your error analysis.

42. Tell whether each of the following series converges or diverges. Explain your reasoning.

$$\sum_{n=1}^{\infty}\frac{n^{3/2}}{n^2+4} \qquad \sum_{n=1}^{\infty}\frac{\sin^2 n}{n^2}$$

43. Show that $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\left(\frac{\pi}{3}\right)^{2n+1}$ converges. How many terms of the series must we sum in order to know that the absolute value of the error is less than 10^{-5}? (We will learn later that this sum equals $\sin(\pi/3) = \sqrt{3}/2$.)

44-45. For each of the following series, tell whether it converges absolutely, converges conditionally, or diverges. Justify your answer.

$$44.\ \sum_{n=1}^{\infty}(-1)^n\frac{n!(2n)!}{(3n)!} \qquad 45.\ \sum_{n=1}^{\infty}(-1)^n(\sqrt{n+1}-\sqrt{n})^2$$

46. Does the series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^{1+\frac{1}{n}}}$ converge or diverge? Justify your answer.

47. Find the radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty}\frac{(n!)^3}{(3n)!}x^n$. You need not check the endpoints; they require methods beyond the scope of this course (Stirling's approximation of $n!$).

48. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty}\frac{(n+1)(3-2x)^n}{n^2}$.

49. Find the interval of convergence of the power series $f(x) = 1+3x+x^2+3x^3+x^4+\cdots$, whose coefficients are alternately 1 and 3. Then find an explicit formula for $f(x)$ in this interval.

50. Find a power series representation for the function $f(x) = \dfrac{x^2}{3x^3+2}$ and determine the interval of convergence.

51. Evaluate the indefinite integral $\int x \ln(1+x^3)\, dx$ as a power series. Find the radius of convergence.

52. Use a power series to approximate the definite integral $\int_0^{1/2} x^2 \arctan(x^2)\, dx$ to six decimal places. Use a calculator or computer, and explain your accuracy analysis.

53. Find the Taylor series for $f(x) = \cos(x)$ centered at $x = \pi/2$. Find the region of convergence.

54. Use series to evaluate $\lim\limits_{x\to 0} \dfrac{e^x \cos x - \frac{1}{1-x} + \sin(x^2)}{\sin x - x}$.

55. Compute the Maclaurin polynomial $T_3(x)$ for $f(x) = \sqrt{1+x}$. What does it give as the approximate value of $\sqrt{1.1}$? What does Taylor's inequality tell you about the possible error in this estimate?