

#1 Find the domain of the following functions

a) $f(x) = \frac{3x^2 - 2x + 1}{x^2 - 4x - 21}$

The domains of $3x^2 - 2x + 1$ and $x^2 - 4x - 21$ are all real numbers. So the domain of f is all real numbers except where $x^2 - 4x - 21 = 0$. So

$$\begin{aligned}x^2 - 4x - 21 &= 0 \\(x - 7)(x + 3) &= 0 \\x &= 7, -3\end{aligned}$$

So the domain is all real numbers except $x = -3$ and $x = 7$.

b) $f(x) = \sqrt{x+3} - \sqrt{x-2} + \sqrt[3]{x^2-1}$

You can only take the square root of a positive number, so the domain of $\sqrt{x+3}$ is where $x+3 \geq 0$, or $x \geq -3$.

Similarly, the domain of $\sqrt{x-2}$ is $x-2 \geq 0$ or $x \geq 2$.

You can take the cube root of any number, so the domain of $\sqrt[3]{x^2-1}$ is all real numbers.

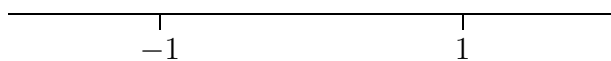
The domain of f is the intersection of all of these. So it's all numbers that are bigger than both -3 and 2. This means the domain is $x \geq 2$ or $[2, \infty)$

c) $f(x) = \frac{1}{\sqrt{x^2-1}}$

The domain of f is the domain of $\sqrt{x^2-1}$ minus the points where it's zero. The domain of $\sqrt{x^2-1}$ is where $x^2-1 \geq 0$. To find where this is positive, first find where it's zero and then test values.

$$\begin{aligned}x^2 - 1 &= 0 \\(x + 1)(x - 1) &= 0 \\x &= -1, 1\end{aligned}$$

So now we look at the number line:

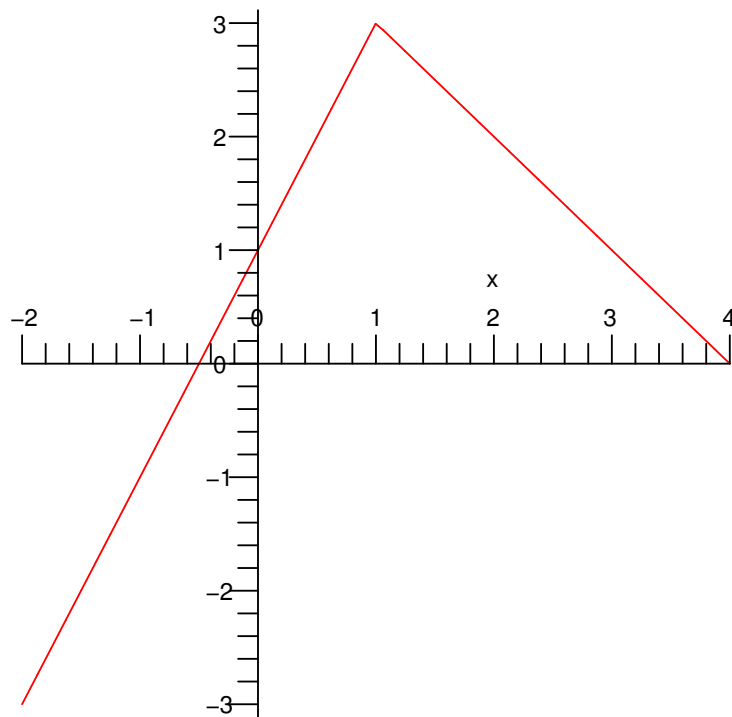


So we want to test values less than -1, between -1 and 1, and greater than 1. Plugging in -2 gives us $(-2)^2 - 1$, which is positive. Plugging in 0 gives us -1, which is negative. Plugging in 2 gives us $2^2 - 1$ which is positive. Thus the domain of $\sqrt{x^2 - 1}$ is $x \leq -1$ and $x \geq 1$. However, at 1 and -1, it's 0 so those points aren't in the domain of f .

The domain of f is the set of numbers such that $x < -1$ and $x > 1$ or $(-\infty, -1), (1, \infty)$

#2 Graph the following piecewise function

$$f(x) = \begin{cases} 2x + 1 & , x \leq 1 \\ 4 - x & , x > 1 \end{cases}$$



#3 §1.2 # 2

#4 §1.2 # 12

#5 $f(x) = \sqrt{3 - 2x}$ and $g(x) = \sqrt{x + 4}$. Find $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $(f/g)(x)$. Find the domain of each as well.

The domain of f is $3 - 2x \geq 0$ or $x \leq \frac{3}{2}$. The domain of g is $x + 4 \geq 0$ or $x \geq -4$.

$(f + g)(x) = \sqrt{3 - 2x} + \sqrt{x + 4}$ and the domain is all numbers that are less than $\frac{3}{2}$ and greater than -4 . This is $-4 \leq x \leq \frac{3}{2}$ or $[-4, \frac{3}{2}]$.

$(f - g)(x) = \sqrt{3 - 2x} - \sqrt{x + 4}$ and the domain is $[-4, \frac{3}{2}]$ again.

$(fg)(x) = \sqrt{3 - 2x}\sqrt{x + 4}$ and the domain is $[-4, \frac{3}{2}]$.

$(f/g)(x) = \frac{\sqrt{3-2x}}{\sqrt{x+4}}$. The domain is the same as before but without the points where $g(x) = 0$. So the domain for this one is $(-4, \frac{3}{2}]$ or $-4 < x \leq \frac{3}{2}$.

#6 Find $(f \circ g)(x)$ and $(g \circ f)(x)$ and the domain of each for:

a) $f(x) = \sqrt{x-15}$, $g(x) = x^2 + 2x$

$$\begin{aligned}(f \circ g)(x) &= \sqrt{(x^2 + 2x) - 15} \\ &= \sqrt{(x^2 + 2x - 15)}\end{aligned}$$

The domain of g is all real numbers, but the domain of $f \circ g$ needs what's under the radical to be positive. So the domain is where $x^2 + 2x - 15 \geq 0$. Again, we need to use the number line

$$\begin{aligned}x^2 + 2x - 15 &= 0 \\ (x + 5)(x - 3) &= 0 \\ x &= -5, 3\end{aligned}$$

So we need to look at $x < -5$, $-5 < x < 3$, and $x > 3$. Plugging in -6 gives a positive number. Plugging in 0 gives a negative number. Plugging in 5 gives a positive number again. Thus the domain is all numbers such that $x < -5$ or $x > 3$. Alternatively, the domain is $(-\infty, -5], [3, \infty)$.

$$\begin{aligned}(g \circ f)(x) &= (\sqrt{x-15})^2 + 2\sqrt{x-15} \\ &= x - 15 + 2\sqrt{x-15}\end{aligned}$$

Since g is a polynomial, any number will work with it. All that matters here is the domain of f , which is $x - 15 \geq 0$ or $x \geq 15$. So the domain of $g \circ f$ is $x \geq 15$ or $[15, \infty)$.

b) $f(x) = \frac{x}{x-2}$, $g(x) = \frac{3}{x}$

$$\begin{aligned}(f \circ g)(x) &= \frac{\frac{3}{x}}{\frac{3}{x} - 2} \\ &= \frac{3}{3 - 2x}\end{aligned}$$

This is a fraction, so the denominator can't be zero. That means that $3 - 2x = 0$ or $x = \frac{3}{2}$ is not in the domain. However, remember that any number first has to go through

$g(x)$, so we're only looking at the numbers in the domain of g . That means that $x = 0$ also isn't in the domain. The domain for $f \circ g$ is all real numbers except $x = 0$ and $x = \frac{3}{2}$

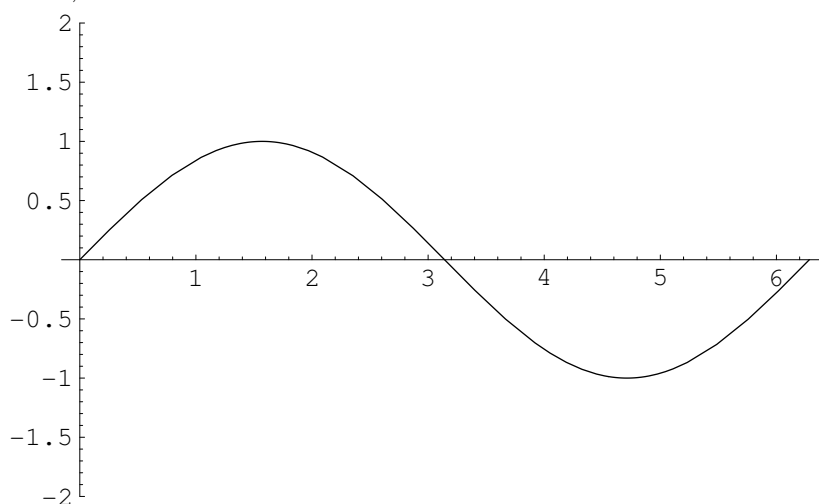
$$\begin{aligned}(g \circ f)(x) &= \frac{3}{\frac{x}{x-2}} \\ &= \frac{3(x-2)}{x}\end{aligned}$$

Again, the denominator can't be zero, so $x = 0$ is not in the domain. But similar to $f \circ g$, we only care about numbers that are in the domain of f , so $x = 2$ is also not in the domain of $g \circ f$. The final domain for $g \circ f$ is all real numbers except $x = 0$ and $x = 2$.

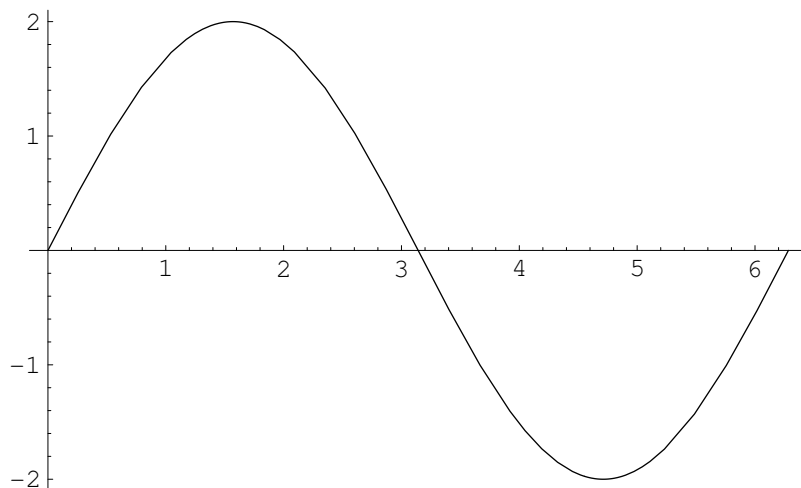
#7 Graph the following functions by starting with the graph of a standard function and applying the appropriate transformations.

a) $y = 2 \sin 2x$

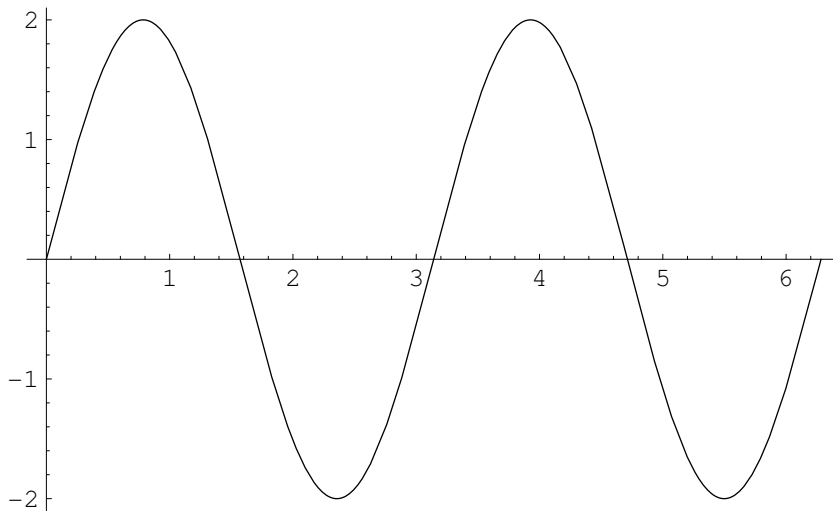
First, we start with $\sin x$



Next we do a vertical stretch of 2 to get $2 \sin x$

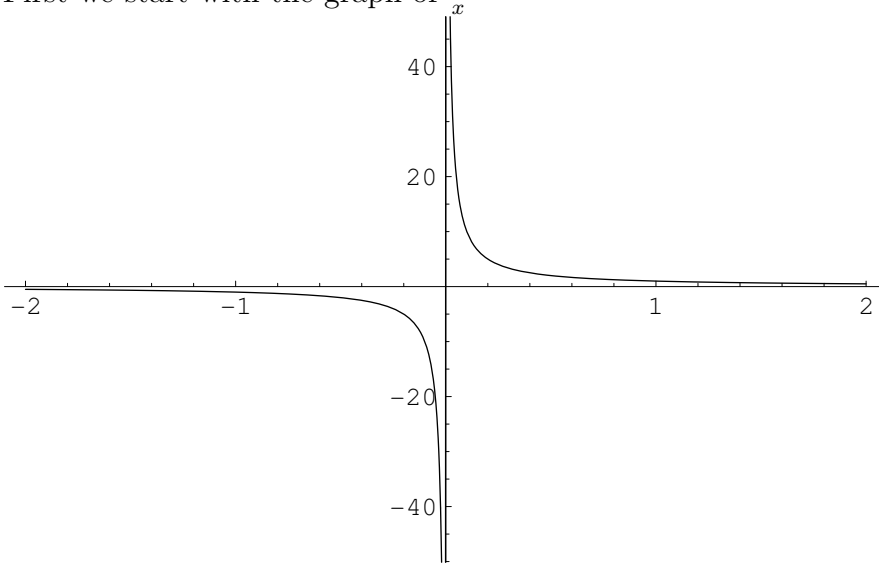


Finally we do a horizontal compression to get $2 \sin 2x$

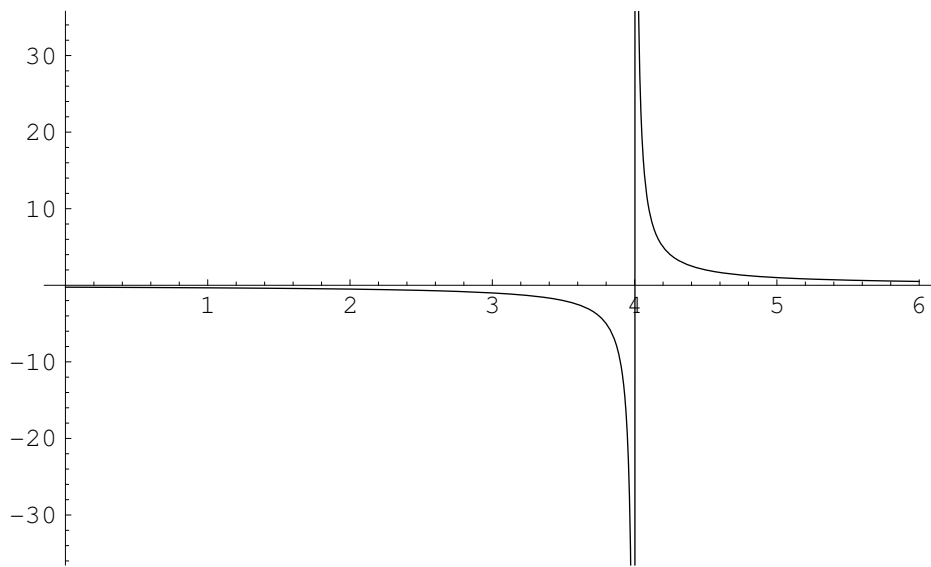


b) $y = \frac{1}{x-4}$

First we start with the graph of $\frac{1}{x}$



Then we do a horizontal shift to the right by 4



#8 Find the exact value of:

a) $\cos\left(\frac{2\pi}{3}\right)$

If we draw this in standard position, we see we're in the second quadrant and that this has an angle with the x-axis of $\pi - 2\pi/3 = \pi/3$. We look at the table and see that $\cos \frac{2\pi}{3} = \sqrt{12}$. However, cos is negative in this quadrant so $\cos \frac{2\pi}{3} = -\frac{1}{2}$.

b) $\sin\left(\frac{-5\pi}{6}\right)$

Again, draw the angle in standard position and see that it makes an angle of $\frac{\pi}{6}$ with the x-axis. $\sin \frac{\pi}{6}$ is $\frac{1}{2}$ and sin is negative in this quadrant, so $\sin \frac{-5\pi}{6} = -\frac{1}{2}$.