Math is Beautiful

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What this talk is not about

- Mathematics that makes beautiful things
- Mathematics in art

  Painting is a science and all sciences are based on mathematics. No human enquiry can be a science unless it pursues its path through mathematical exposition and demonstration.

  Leonardo Da Vinci
- The nature of beauty and how it relates to math (mostly)

What this talk is about

- Explain how one can find beauty inside mathematics.
- Show how beauty has driven mathematics.

Worst Question Ever

The worst question you can possibly ask a mathematician is:

What is this stuff good for?

“Good” Answers

1. Engineers use it to build bridges.
2. Economists can use it to maximize profit.
3. Planimeters use Green’s Theorem.
4. Computer graphics use linear algebra.
5. Physical modelling uses differential equations.

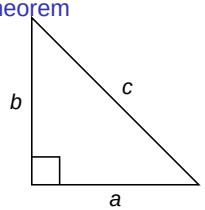
Good Answer

Math is beautiful.

The Pythagorean Theorem

Theorem
In right-angled triangles the square on the side opposite the right angle is equal to the sum of the squares of the sides containing the right angle.

Theorem



$a^2 + b^2 = c^2$

Euclid’s Proof

$\triangle ABF \cong \triangle ACE$ by SAS.

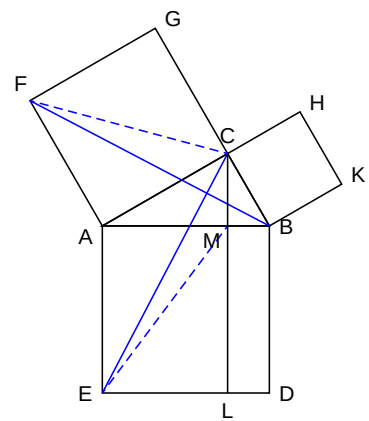
$|\triangle ABF| = |\triangle ACF|$ and $|\triangle ACE| = |\triangle AFM|$ by shearing.

$2|\triangle ACE| = |AELM|$ and $2|\triangle ACE| = |ACGF|$

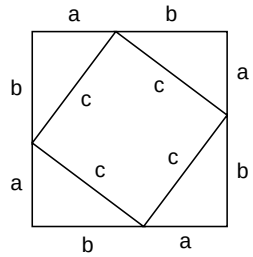
$\therefore |ACGF| = |AELM|$ by conguences

The same argument follows for the other triangles.

$\therefore |\mathbb{A}| + |\mathbb{B}| = |\mathbb{C}|$



Alternate Proof



$$(a+b)^2 = c^2 + 4 \cdot (1/2)(ab)$$
$$a^2 + 2ab + b^2 = c^2 + 2ab$$
$$a^2 + b^2 = c^2$$

How many proofs?

- The Pythagorean Proposition by Elisha Shoomis claims to have 367 distinct proofs.
  - Not all are correct or that distinct
  - Estimates of the number of correct and distinct proofs range from 256 to 365
- http://www.cut-the-knot.org/pythagoras/index.shtml has 72 proofs

Why so many proofs?

- Euclid's proof is completely correct.
- No one proves it out of doubt.
- Many since Euclid have proved it out of enjoyment.
  - President James Garfield came up with a proof in 1876

Elegance in General

- It's not uncommon for theorems to reproved.
- It's usually not about trusting the previous proof, but about finding "beauty".
- This search has pushed the research of many mathematicians.

Argumentum ad Nauseam

Here are other examples:

- Fundamental Theorem of Algebra
- Infinitely many primes
- Four Color Theorem

Argumentum ad Verecundiam

Some proofs command assent. Others woo and charm the intellect. They evoke delight and an overpowering desire to say, “Amen, Amen”.

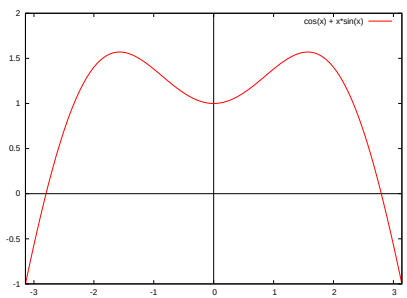
Lord Rayleigh

I have attested it as true in my deepest soul and I contemplate its beauty with incredible and ravishing delight.

Johannes Kepler

Cartesian Coordinate System

$$f(x) = \cos(x) + x * \sin(x)$$



Cartesian Coordinate System

- 1637 - Descartes in *Discourse on Method* and *La Géometrié*
- Created analytic geometry
- Allowed the creation of calculus

What is power?

- Provides a deep insight
- Connects two seemingly different ideas

Argumentum ad Nauseum

- The Fundamental Theorem of Calculus
- Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$
- Algebraic notation

Argumentum ad Verecundiam

[Euler's forumla is] the most remarkable formula in mathematics.

Richard Feynman

Infinite Hotel

A hotel has a billion rooms and all of them are full. A man comes to the hotel and wants a room. Can he get one?

What about for any other number of rooms?

What if there are an infinite number of rooms numbered 1,2,3,. . . ?

What if there an infinite number of rooms and an infinite number of people who want to check in?

What about an infinite number of buses labelled 1,2,3,. . . - each with an infinite number of passengers in seats 1,2,3,. . . ?

Counterintuitive Results

- When abstracted, the results make sense.
- When applied, the results contradict intuition.
- The result uses something subtle about the realm of math that doesn't apply to the real world.

OR

- The intuition is wrong.

Argumentum ad Nauseum

- Monty Hall problem
- Well-ordering theorem
- Banach-Tarski Paradox

Argumentum ad Verecundiam

When reality clashes so violently with intuition, people are shaken.

Marilyn vos Savant

The Axiom of Choice is obviously true, the well-ordering principle obviously false, and who can tell about Zorn's lemma?

Jerry Bona

Conclusions

- Beauty can be found in math.
- The search for beauty in mathematics has driven plenty of research.
- When a mathematician says he studies math because he enjoys it, he's not just pulling your leg.

Argumentum ad Verecundiam

Mathematics, rightly viewed, possesses . . . supreme beauty - a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as in poetry.

Bertrand Russell

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