Week 4 Homework: Pre-view

7.1 - 7.5 as needed on new homework

7.8 improper integrals (rest of cases)

8.2 surface areas (rotation about y-axis)

8.3 centroid (with sketch)

9.1 (no hw), 9.3 separable differential eqn.

8.1 arc length

10.1, 10.2 (start parametric curves)

Homework notes: Until further notice, problems and schedule are from the Fall 2004 list.

Problem 8.2.14:

Find the surface area of the surface obtained by

revolving the curve $y = 1 - x^2, 0 \leq x \leq 1$

about the y-axis.

Solution:

We use $A = \int 2\pi x\sqrt{1 + (y')^2}\, dx$.

We have $y' = -2x$, so

$$A = \int_0^1 2\pi x\sqrt{1 + 4x^2}\, dx.$$

Take $u = 1 + 4x^2$, then $du = 8x\, dx$, so

$$\pi \int 2x\sqrt{1 + 4x^2}\, dx$$

$$= \frac{\pi}{4} \int u^{\frac{1}{2}}\, du$$

$$= \frac{\pi}{4}\left[\frac{2}{3}u^{\frac{3}{2}}\right] + C$$

$$= \frac{\pi}{4}\frac{2}{3}(1+4x^2)^{\frac{3}{2}} + C.$$

So $A = \left[\frac{\pi}{6}(1+4x^2)^{\frac{3}{2}}\right]_0^1$

$$= \frac{\pi}{6}(5^{\frac{3}{2}} - 1).$$