Week 5 Homework: Pre-view

10.2 Calculus on parametric curves

10.3 Polar coordinates:

a. translations (x, y) to (r, θ)

b. translations (r, θ) to (x, y)

c. standard curves ($r = a\cos\theta$; $r = a + b\cos\theta$; $r = \cos n\theta$.)

10.4 Area and arc length in polar coordinates

11.1, 11.2 Sequences and Series

If $x = x(t), y = y(t)$ is a parametric curve, we have

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}.$$

This formula is used in Problem 10.2.5 to find the equation of the tangent line to the curve at a point.

To clarify, we still have $x = x(t)$, but a new parametric value $h(t) = \frac{dy}{dx}(t)$. We read the above formula for the first derivative as $x = x(t), y = g(t)$ having $\frac{d}{dx}(g(t)) = \frac{\frac{d}{dt}(g(t))}{\frac{dx}{dt}}$,

and apply this to the function h, and get

$$\frac{d}{dx}(\frac{dy}{dx}) = \frac{\frac{d}{dt}(\frac{dy}{dx})}{\frac{dx}{dt}}$$

as the formula for the 2nd derivative. We use this on the homework.

Problem 10.2.14:

Find the points on the parametric curve

$x(t) = t + \ln t,\ y(t) = t - \ln t$ where

the curve is concave up $(y'' > 0)$.

Solution:

$$\frac{dy}{dx} = \frac{1 - \frac{1}{t}}{1 + \frac{1}{t}} = \frac{t-1}{t+1}.$$

So $$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\dfrac{d}{dt}\left(\dfrac{t-1}{t+1}\right)}{\dfrac{d}{dt}(t + \ln t)},$$

which simplifies to $\dfrac{2t}{(t+1)^3}$.

Now we check signs of the numerator (changes from minus to plus at $t = 0$) and the denominator (changes from minus to plus at $t = -1$) to see that the curve is concave up at the points where $t < -1$ and where $t > 0$.