Math 205 Second Exam April 4, 2006

NAME : _____ Section _____
(Last, First)

LECTURE :

Prof. Sun (MWF 8am, Sect. 10) ___

Prof. Johnson (MWF 10am, Sect. 11) ___

Prof. D'Arcy (MWF 3pm, Sect. 12) ___

Prof. Dodson (TuTh 9am, Sect. 13) ___

Prof. Weintraub (TuTh 10:45am, Sect. 14) ___

Prof. Dodson (TuTh 1pm, Sect. 15) ___

Question 1: _____ /20

Question 2: _____ /20

Question 3: _____ /15

Question 4: _____ /10

Question 5: _____ /15

Question 6: _____ /20

Total: _____ /100

Instructions : This is a 60 minute exam. You are not to have or use a calculator during the exam. You will be graded on the work shown on your paper; an answer without explanation may receive no credit. Solutions to differential equations are not to include integrals; and must be expressed only in terms of real numbers and real functions.

1. (20 *points*) Solve the following differential equations. (a) $\left(\frac{x}{y-3}\right)\frac{dy}{dx} - x^3 = 0$

(b) $\frac{dy}{dx} + \left(\frac{1}{1-x}\right)y = 3$

2. (20 *points*) Find the general solution of the following differential equations. Partial credit in part (b) will be given for a clearly stated solution to the homogeneous equation.

(a) $y''(t) + 2y'(t) + 5y(t) = 0.$

(b) $y''(t) - y'(t) - 6y(t) = -12e^{-t}.$

3. (10 *points*) Recall that the motion $y(x)$ of a spring-mass system is governed by

$$my''(x) + c\,y'(x) + k\,y(t) = 0,$$

where m is the mass, k is the spring constant, c is the friction constant. (a) If $m = 2, c = 8$ and $k = 6$ find the solution to the Initial Value problem $y(0) = 1, y'(0) = -5.$

(b) (5 *points*) Which of the following best describes the motion? Give a reason for your answer.

(i) the motion $y(x)$ oscillates above and below the x-axis, since the effect of the friction is small relative to the other values.

OR

(ii) the effect of friction is so strong that the motion never gets below the x-axis.

4. (10 *points*) Find a particular solution of $y''(x) + y(x) = 3\cos(2x)$.

5. (15 *points*) Find all eigenvalues and eigenvectors of the matrix $B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$.

6. (20 *points*) You are given that the characteristic polynomial of the matrix $A = \begin{pmatrix} 2 & 2 & -1 \\ 0 & -2 & 0 \\ 1 & 1 & 4 \end{pmatrix}$ is $p(\lambda) = (\lambda - 3)^2(\lambda + 2)$.

(a) Give a reason why A is not diagonalizable. (b) Give the Jordan form J of A.

(c) Find an eigenvector with eigenvalue $\lambda_2 = -2$.

(d) Find a matrix P so that $A = PJP^{-1}$. Show each step in the work needed to find the columns of P that are either (i) eigenvectors with eigenvalue $\lambda_1 = 3$ or (ii) generalized eigenvectors with eigenvalue $\lambda_1 = 3$ (and length 2).