

Summary of Some Research Findings From Early Fall 2007

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Let $G_{n,d}$ be the n th group in the Magma/GAP database of transitive groups of degree d . Consider the minimal transitive group $G_{24,1489}$ found in the list in [2]. The center of $G_{24,1489}$ is trivial, so $G_{24,1489}$ cannot be the Galois group of a CM-field. However, by looking at the minimal partitions of $G_{24,1489}$, we can determine if it serves as a witness that another group of order $2 \cdot |G_{24,1489}| = 2 \cdot 576 = 1152$ is ρ -minimal. (See [1] for a definition and discussion of ρ -minimality). Magma verifies that $G_{24,1489}$ has three minimal partitions, each consisting of 12 sets of imprimitivity of size 2. Call them P_1, P_2 , and P_3 .

Magma can find the image and kernel of the action $G_{24,1489}$ on each of these block systems. Let K_i and I_i be the kernel and image, respectively of each of these actions, for $i = 1, 2, 3$. Magma calculates that $K_1 \cong K_3$ with $|K_1| = |K_3| = 1$ and $I_1 \cong I_3$ with $|I_1| = |I_3| = 576$. When we compare these isomorphic images to the two minimal transitive groups of degree 12 and order 576, found in [2], we find that $I_1 \cong I_3 \cong G_{12,162}$. Thus, P_1 and P_3 are equivalent block systems (in some sense).

The situation with respect to P_1 and P_2 is different. $|K_2| = 16$ and $|I_2| = 36$, so they are not isomorphic to K_1 and I_1 . Magma verifies that $I_2 \cong G_{12,17}$, which is the unique minimal transitive group of degree 12 and order 36 found in [2].

To determine if $G_{24,1489}$ serves as a witness that another larger group is ρ -minimal, we search Magma's database of transitive groups to find a group G of degree 24 and order 1152 that has a center of size 2. Then we examine the kernel and image of the action of an orbit \mathcal{O} of $Z(G)$ on G . We would like to find two such groups, say H and K . We want the image of the action of \mathcal{O} on H to be isomorphic to $G_{24,1489}$. Similarly, we would like to find K so that the image of the action of \mathcal{O} on K is isomorphic to $G_{12,17}$. When we do this, we find that $H = G_{24,2777}$ and $K = G_{24,2798}$, respectively, fit these criteria. Thus, $G_{24,2777}$ and $G_{24,2798}$ are both ρ -minimal, with $G_{24,1489}$ as a witness.

References

- [1] Bruce Dodson, *Galois Groups of CM-Fields*, Trans. of the AMS **283** (1984), 1-32.
- [2] Alexander Hulpke, *Constructing transitive permutation groups*