Week 4a: 4.3 Undetermined Coef [Tables]

TABLE of trial y_p: Usual trial y_p

$F(x)$	Usual	Modified
ce^{ax}	$y_p = A_0 e^{ax}$	
$ce^{ax}\cos(bx)$ or $ce^{ax}\sin(bx)$	$y_p = e^{ax}(A_0\cos(bx) + B_0\sin(bx))$	
cx^k	$y_p = A_0 + A_1 x + \cdots + A_k x^k$	

When: The root associated with $F(x)$ is NOT a root of the characteristic polynomial $P(r)$ giving y_H in the general solution $y = y_H + y_p$.

In the 3 cases, $P(a) \neq 0$; $P(a+bi) \neq 0$; and $P(0) \neq 0$.

Applications often feature the "pure imaginary" complex case, where $a = 0$, for which we get the usual solution when $P(ib) \neq 0$.

TABLE of trial y_p: Simple modified solution

$F(x)$	Usual	Modified
ce^{ax}	$y_p = A_0 e^{ax}$	$y_p = A_0 x e^{ax}$
$ce^{ax}\cos(bx)$ or $ce^{ax}\sin(bx)$	$y_p = e^{ax}(A_0\cos(bx) + B_0\sin(bx))$	$y_p = xe^{ax}(\ \ldots\ )$
cx^k	$y_p = A_0 + A_1 x + \cdots + A_k x^k$	$y_p = x(\ \ldots\ )$

When: The root associated with $F(x)$ is a simple root of the characteristic polynomial $P(r)$. If the DE was $y'' + a_1 y' + a_2 y = F$, so $P(r) = r^2 + a_1 r + a_2$, the cases are (1) $P(r) = 0$ has distinct real roots, one of which is $r_1 = a$; (2) $P(r) = 0$ has complex roots $a \pm bi$, the SAME complex roots as the root associated with $F(x)$. (We include the possibility that the "real part" $a = 0$.) And (3) $P(0) = 0$, so $a_2 = 0$; but $a_1 \neq 0$.

TABLE of trial y_p: Modified solution(s)

$F(x)$	Usual	Modified
ce^{ax}	$y_p = \dots$	$y_p = A_0 x^m e^{ax}$
$ce^{ax}\cos(bx)$ or $ce^{ax}\sin(bx)$	$y_p = \dots$	$y_p = xe^{ax}(A_0\cos(bx) + B_0\sin(bx))$
cx^k	$y_p = \dots$	$y_p = x^m(A_0 + A_1 x + \cdots + A_k x^k)$

When & Where: Usually $m = 1$ (simple root, previous table!). The only other case is $m = 2$, which occurs when $P(r) = 0$ has a repeated root that coincides with the root associated with $F(x)$. For cases, $P(r)$ NEVER has repeated complex roots (why?). In the polynomial case, the DE is $y'' = F(x) = cx^k$, since to get $r = 0$ as a repeated root of $P(r) = 0$ we'd need both $a_2 = 0$ and $a_1 = 0$. In the exponential case, we'd have $P(r) = (r - a)^2 = r^2 - 2ar + a^2$, where $F(x)$ also involves the exponential function e^{ax}, with the same a.

The single **crucial step** is to take the trial function y_p from the correct table: we may have to multiply by x or even by x^2, but only when needed: due to a repetition as above.

Finally, in the complex case, the modified function with both x and e^{ax} gives a tedious product rule exercise, so in Math 205, we only include the modified case $a = 0$.