Math 23 Quiz 3 June 18, 2008

NAME: ________________________________ (please print clearly)
(Last, First)

1. (10 *points*) If $f(x,y) = xy - x^2 - 2y^2 + 7y + 1$, find the solutions of $f_x = 0, f_y = 0$ (these are the *critical points* of f).

Soln: $f_x = y - 2x = 0 \rightarrow \ y = 2x$

$f_y = x - 4y + 7 = 0 \rightarrow x - 4(2x) + 7 = -7x + 7 = 0 \rightarrow x = 1$; so $y = 2$,

and $(1,2)$ is the only critical point.

(b) For each solution use the second derivative test to determine whether the point is a local max, local min or neither.

Soln: $D(x,y) = f_{xx}f_{yy} - f_{xy}^2 = (-2)(-4) - 0 = 8$,

so $D(1,2) = 8 > 0$, and $(1,2)$ is NOT a saddle point. Next, check

$A = f_x x(1,2) = -2$, so a curve through the critical point is *concave down,*

and the critical point is a local maximum.

(Note: if asked, the *maximum value* is $f(1,2) = 2 - 1 - 8 + 14 + 1 = 8$.)

2. (10 *points*) Show each step as you evaluate the iterated integral
$\int_0^1 \left(\int_{\sqrt{x}}^2 x + y^2 \ dy \right) dx$.

Soln: For the inside integral, $\int x + y^2 dy = xy + \frac{1}{3}y^3$,

so $\int_{\sqrt{x}}^2 x + y^2 dy =$

$$[xy + \tfrac{1}{3}y^3]_{y=\sqrt{x}}^{y=2} = (x)(x) + \tfrac{1}{3}(x^{\frac{3}{2}}).$$

Then the value of the integral is given by the outside integral,

$$\int_0^1 \left(x^2 + \tfrac{1}{3}x^{\frac{3}{2}} \right) dx = \frac{1}{3} + (\frac{1}{3})(\frac{2}{5}) = \frac{1}{5}.$$