Math 205 Second Exam November 7, 2006

NAME : _____ Section _____
(Last, First)

LECTURE :

Prof. Dodson (MWF 9am, Sect. 10) ___

Prof. Dodson (MWF 11am, Sect. 11) ___

Prof. D'Arcy (MWF 2pm, Sect. 12) ___

Prof. Venkataraman (TuTh 8am, Sect. 13) ___

Question 1: _____ /20

Question 2: _____ /15

Question 3: _____ /25

Question 4: _____ /10

Question 5: _____ /15

Question 6: _____ /15

Total: _____ /100

Instructions : This is a 60 minute exam. You are not to have or use a calculator during the exam. You will be graded on the work shown on your paper; an answer without explanation may receive no credit. Solutions to differential equations are not to include integrals; and must be expressed only in terms of real numbers and real functions.

1. In this problem, solutions to differential equations are not to include integrals.

(a) (10 *points*) Solve the differential equation $x\dfrac{dy}{dx} + y^2 = 0$, and write your solution as an explicit function $y = y(x)$.

(b) (10 *points*) Find the general solution of $\dfrac{dy}{dx} - \dfrac{y}{x} = 3x^3$.

2. (15 *points*) A tank whose volumn is 50 L initially contains 25 g of salt dissolved in 25 L of water. A solution containing 5 g/L of salt is pumped into the tank at a rate of 3 L/min, and the well-stirred mixture flows out at a rate of 1 L/min. Find the amount of salt in the tank after 12 minutes. Include in your solution a clear statement of the differential equation satisfied by the amount of salt $A(t)$ in the tank at time t.

3. (25 *points*) Find the general solution of the following differential equations. Partial credit in parts (b) and (c) will be given for a clearly stated solution to the homogeneous equation. In this problem solutions to differential equations must be expressed only in terms of real numbers and real functions.

(a) $y''(t) + y'(t) - 6y(t) = 0.$

(b) $y''(t) + y'(t) - 2y(t) = -12e^{-2t}.$

(c) $y''(x) + 4y'(x) + 4y(x) = 3\cos(2x).$

4. (10 *points*) Recall that the motion $y(x)$ of a spring-mass system is governed by

$$my''(x) + c\,y'(x) + k\,y(x) = 0,$$

where m is the mass, c is the friction constant and k is the spring constant. If $m = 2, c = 4$ and $k = 10$ find the solution to the Initial Value problem $y(0) = 3, y'(0) = -1$. In this problem solutions to differential equations must be expressed only in terms of real numbers and real functions.

5. (15 *points*) You are given that the characteristic polynomial of the matrix $A = \begin{pmatrix} 2 & 2 & -2 \\ 0 & -2 & 0 \\ -2 & -1 & -1 \end{pmatrix}$ is $p(\lambda) = (\lambda + 2)^2(\lambda - 3)$.

(a) Find an eigenvector with eigenvalue $\lambda_2 = 3$.

(b) Give a diagonal matrix D and a matrix P so that $A = PDP^{-1}$.

6. (15 *points*) You are given that the matrix $B = \begin{pmatrix} 0 & 3 \\ -3 & 6 \end{pmatrix}$ has characteristic polynomial $P(\lambda) = (\lambda - 3)^2$.

(a) Find an eigenvector with eigenvalue $\lambda = 3$.

(b) Find the Jordan Form J of B and a matrix P so that $B = PJP^{-1}$. Show each step in the work needed to find each column of P that is either (i) an eigenvector with eigenvalue $\lambda = 3$ or (ii) a generalized eigenvector with eigenvalue $\lambda = 3$ (and index 2).