MATH 205 Quiz 6: Take-home due 10am August 10, 2015

**NAME:** ______________________________
(Last, First)

1. Consider the coefficient matrix $B = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -3 & 2 \\ 1 & -2 & 2 \end{pmatrix}$. You are given that $B$ has characteristic polynomial

$$p(\lambda) = (\lambda + 2)(\lambda - 1)^2.$$

Further, an eigenvector with eigenvalue $\lambda_1 = -2$ is $\vec{v}_1 = (0, 2, 1)$.

(a) Find all eigenvectors of $B$ with eigenvalue $\lambda_2 = 1$.

(b) Use the given information and your answer in part (a) to give the general solution to $\mathbf{x}' = B\mathbf{x}$, where $\mathbf{x} = \mathbf{x}(t)$ is the vector function with entries $x_1(t), x_2(t), x_3(t)$.

2. You are GIVEN that the matrix $A = \begin{pmatrix} 3 & 6 \\ 2 & -1 \end{pmatrix}$ has an eigenvector $\vec{v}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ for eigenvalue $\lambda_1 = 5$ and an eigenvector $\vec{v}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ for eigenvalue $\lambda_2 = -3$.

(a) Use the given information to give the general solution to $\mathbf{x}\,' = A\mathbf{x}$, where $\mathbf{x} = \mathbf{x}(t)$ is the vector function with entries $x_1(t), x_2(t)$. [No computation!]

(b) Use the given information and your answer in part (a) to find find a particular solution $\mathbf{x}_p$ to $\mathbf{x}\,'(t) = A\mathbf{x}(t) + \begin{pmatrix} 0 \\ e^{2t} \end{pmatrix}$, where $A$ is the above matrix.