MATH 43 First Exam October 12, 2007

NAME: ______________________________
(Last, First)

1. (15 *points*) (a) Find the projection of $\vec{b} = [1, 3, -2, 1]$ on $\vec{a} = [2, -1, 3, 4]$.

(b) Find the length $||\vec{v}||$ of the vector $\vec{v} = [3, 2, -2]$.

(c) Find the unit vector in the direction of the vector $\vec{v} = [3, 2, -2]$.

2. (10 *points*) Suppose that a homogeneous system of linear equations in five variables has

(i) $x_3 = s$ and $x_5 = t$ as free variables; and that

(ii) $x_1 = 5x_3 - 2x_5 = 5s - 2t$, $x_2 = 7x_3 = 7s$ and $x_4 = -4x_5 = -4t$.

Write the solution of the system as a vector $[x_1, x_2, x_3, x_4, x_5]$ and find two vectors that span the solutions.

3. (20 *points*) Find all solutions of the following systems of equations. For each system state whether the system is consistent or inconsistent. If the system is consistent, state whether the solution is unique or not.

$$\begin{aligned} x_1 - x_2 \qquad &= 1 \\ x_2 + x_3 &= 2 \\ x_1 + x_2 + x_3 &= 1 \end{aligned}$$

$$\begin{aligned} x_1 - x_2 \qquad &= 3 \\ x_2 + x_3 &= 2 \\ x_1 \qquad + x_3 &= 5 \end{aligned}$$

4. (15 *points*) Use the Gauss-Jordan method to solve the following system. Start with the augmented matrix of the system. In your reduction of the augmented matrix include a clearly identified reduced row echelon matrix for the coefficient matrix. Identify the free variables and give your answer as a vector $[w, x, y, z]$.

$$\begin{aligned} w + 2x + y + 2z &= 0 \\ 2w + 4x - y - 5z &= 3 \end{aligned}$$

[continued on back of page]

5. (10 *points*) If the augmented matrix of a system of equations has the form given below, find all k for which the system has a unique solution. Find all k (if any) for which the system is inconsistent. Find all k (if any) for which the system has infinitely many solutions.

$$[A \mid \vec{c}] = \left[\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 3+k & 2k+2 \end{array}\right]$$

6. (15 *points*) (a) Find the general equation of the plane through $P = (-5, 4, 3)$ with normal vector $\vec{n} = [1, 3, -1]$.

(b) Find the vector and parametric equations of the line through $P = (4, -1)$ and $Q = (2, 3)$.

7. (15 *points*) (a) Determine whether $\vec{v} = \begin{bmatrix} -1 \\ 2 \\ -5 \end{bmatrix}$ is a linear combination of the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 4 \\ -6 \\ 10 \end{bmatrix}$. Explain your work and your conclusion.

(b) Determine whether the vectors $\vec{v}_1 = \begin{bmatrix} -2 \\ 3 \\ -5 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 4 \\ -6 \\ 10 \\ 3 \end{bmatrix}$ and $\vec{v}_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 3 \end{bmatrix}$ are linearly independent. Explain your work and your conclusion.