Math 205, Summer I 2016

B. Dodson

Text - Goode-Annin, 3rd edt.

email: bad0@lehigh.edu

1. Course Info

2. Homework 1:

Chapter 1, Sections 1, 2, 3, 4

3. Homework 2:

Chapter 1, Sections 5, 6

Chapter 1: Intro, Slope Fields, verify solution

1.4, 1.5: Separable DE, Population/logistic growth

1.6 Linear Equations

Problem 1:

Verify that the function $y = c_1\sqrt{x}$ is a

solution of $y\prime = \dfrac{y}{2x}$

Solution:

Compute $y\prime$ and check.

$y\prime = c_1(\frac{1}{2})x^{-\frac{1}{2}}$.

$\dfrac{y}{2x} = \dfrac{c_1\sqrt{x}}{2x}$

$= c_1(\frac{1}{2})\dfrac{\sqrt{x}}{(\sqrt{x})^2}$

$= c_1(\frac{1}{2})\dfrac{1}{\sqrt{x}}$

$= y\prime$.

Problem 2:

Determine all values r so $y = e^{rx}$ is a solution to $y'' - 4y' + 3y = 0$.

Linear DE, Main Step: to solve linear DE

$$\frac{dy}{dx} + p(x)y = q(x)$$

multiply by the integral factor $f = e^{\int p\,dx}$

and use

$$\frac{d}{dx}(fy) = f(y' + py)$$

on the left to get $\dfrac{d}{dx}(fy) = fq$.

Problem:

Solve $\dfrac{dy}{dx} + \dfrac{2x}{(1-x^2)}y = 4x,\ -1 \leq x \leq 1.$

Solution:

Find integral factor, inside integral first:

$$\int \frac{2x}{(1-x^2)}\,dx$$

$$= -\ln(1-x^2) = \ln\left((1-x^2)^{-1}\right) \text{ (simplify!).}$$

So $e^{\int \frac{2x}{(1-x^2)}\,dx} = e^{\ln\left((1-x^2)^{-1}\right)}$

$= \left(1-x^2\right)^{-1} = \dfrac{1}{1-x^2} = f.$

Multiply by $f = \dfrac{1}{1-x^2}$ and use Main Property:

$$\frac{1}{1-x^2}\left(\frac{dy}{dx} + \frac{2x}{(1-x^2)}y\right) = \frac{4x}{1-x^2},$$

$$\frac{d}{dx}\left(\frac{y}{1-x^2}\right) = \frac{4x}{1-x^2}.$$

Now integration gives $\dfrac{y}{1-x^2} = (-2\ln(1-x^2)) + c,$

so $y = (1-x^2)\left(-\ln((1-x^2)^2) + c\right).$

Notice that we can check this instance of the Main Property directly (using the product rule).