Math 205, Spring 2012

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Text - Goode-Annin, 3rd edt.

Office hours: Mon, Thurs, Fri: 12:20-1:30

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1. Course Info

2. Homework 1:

Chapter 1, Sections 1, 2, 3,

Due Thursday, Jan 19th

3. Homework 2:

Chapter 1, Sections 4, 5, 6

Due Tuesday, Jan 24th

Chapter 1: Intro, Slope Fields, verify solution
(Tuesday, 1/17)

1.4, 1.5: Separable DE, Population/logistic growth

1.6 Linear Equations
(Thursday, 1/19)

Problem 1:

Verify that the function $y = c_1\sqrt{x}$ is a

solution of $y\prime = \dfrac{y}{2x}$

Solution:

Compute $y\prime$ and check.

$$y\prime = c_1(\tfrac{1}{2})x^{-\frac{1}{2}}.$$

$$\frac{y}{2x} = \frac{c_1\sqrt{x}}{2x}$$

$$= c_1(\tfrac{1}{2})\frac{\sqrt{x}}{(\sqrt{x})^2}$$

$$= c_1(\tfrac{1}{2})\frac{1}{\sqrt{x}}$$

$$= y\prime.$$

Problem 2:

Determine all values r so $y = e^{rx}$ is a solution to $y'' - 4y' + 3y = 0$.

Linear DE, Main Step: to solve linear DE

$$\frac{dy}{dx} + p(x)y = q(x)$$

multiply by the integral factor $f = e^{\int p\,dx}$

and use

$$\frac{d}{dx}(fy) = f\left(y' + py\right)$$

on the left to get $\frac{d}{dx}(fy) = fq$.

Problem:

Solve $\frac{dy}{dx} + \frac{2x}{(1-x^2)}y = 4x, \ -1 \le x \le 1$.

Solution:

Find integral factor, inside integral first:

$$\int \frac{2x}{(1-x^2)}\,dx$$

$$= -\ln(1-x^2) = \ln\left((1-x^2)^{-1}\right) \quad \text{(simplify!)}.$$

So $e^{\int \frac{2x}{(1-x^2)}\,dx} = e^{\ln\left((1-x^2)^{-1}\right)}$

$$= \left(1-x^2\right)^{-1} = \frac{1}{1-x^2} = f.$$

Multiply by $f = \dfrac{1}{1-x^2}$ and use Main Property:

$$\frac{1}{1-x^2}\left(\frac{dy}{dx} + \frac{2x}{(1-x^2)}y\right) = \frac{4x}{1-x^2},$$

$$\frac{d}{dx}\left(\frac{y}{1-x^2}\right) = \frac{4x}{1-x^2}.$$

Now integration gives $\dfrac{y}{1-x^2} = (-2\ln(1-x^2)) + c,$

so $y = (1-x^2)\left(-\ln((1-x^2)^2) + c\right).$

Notice that we can check this instance of the Main Property directly (using the product rule).