Math 205, Fall 2009
B. Dodson

Week 8:

4.1 Higher Order DE

4.2 Constant Coef, Homogeneous DE

General Solution: (1) to solve the 2nd order linear DE $y'' + a_1 y' + a_2 y = 0$
find two linearly independent solutions y_1 and y_2, then the general solution is $y_H = c_1 y_1 + c_2 y_2$.

(2) to solve the non-homogeneous 2nd order linear DE $y'' + a_1 y' + a_2 y = F$, find a particular solution y_p, then the general solution is $y = y_H + y_p$, where y_H is the solution of (1).

Problem 1:

Determine all values r so $y = e^{rx}$ is a solution to $y'' - 4y' + 3y = 0$.
Find the general solution.

Solution:

For $y = e^{rx}$, we have $y' = re^{rx}$ and $y'' = r^2 e^{rx}$.
Substitution in the DE gives

$$0 = y'' - 4y' + 3y = r^2 e^{rx} - 4re^{rx} + 3e^{rx}$$

$$= (r^2 - 4r + 3)e^{rx}.$$

Now $r^2 - 4r + 3 = (r-3)(r-1) = 0$ has roots $r_1 = 1$ and $r_2 = 3$, so we get solutions $y_1 = e^x$ and $y_2 = e^{3x}$, so the general solution is $y = c_1 e^x + c_2 e^{3x}$.

Constant Coef, Homogeneous DE

When the DE $y'' + a_1 y' + a_2 y = 0$ has coefficients a_1 and a_2 that are constant, the two linearly independent solutions y_1 and y_2 in the general solution $y_H = c_1 y_1 + c_2 y_2$ may be determined using the roots of the characteristic polynomial $P(r) = r^2 + a_1 r + a_2$.

Three cases:

(1) For distinct real roots r_1 and r_2,
$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$.

(2) For a repeated real root r_1 the 2nd independent solution is $y_2 = x e^{r_1 x}$, and $y = c_1 e^{r_1 x} + c_2 x e^{r_1 x}$.

(3) For a pair of complex roots $r = a + bi$ and $r = a - bi$ with a and b real, $b > 0$,
$y = c_1 e^{ax} \cos(bx) + c_2 e^{ax} \sin(bx)$.

Wed: Recitation on 4.2.

Problem 2:

Solve $y'' - 6y' + 34y = 0$.

Solution:

The characteristic polynomial is $r^2 - 6r + 34$, so the quadratic formula gives roots $\dfrac{6 \pm \sqrt{6^2 - 4 \cdot 34}}{2}$

$= 3 \pm 5i$. (why?)

So $y = e^{3x}(c_1 \cos 5x + c_2 \sin 5x)$ is the general solution.

————————-

***Postscript:** We may

solve Example 5 with the IVP $y'' + 4y' + 4y = 0,\ y(0) = 1, y'(0) = 4$.

We also may solve the IVP for Problem 8 (time

permitting). In any case, the calculation of constants

c_1 and c_2 to solve an IVP look rather different in

(at least) four cases: the three types of roots in the

homogeneous DE, and

(fourth) the non-homogeneous DE.

————————-