Week 3:

1. 1.4 Special Matrices
2. 1.5 Determinants
3. 1.6 Properties of Dets
4. 2.1 Vector Spaces

We compute $\det \begin{pmatrix} 2 & 1 & 5 \\ 4 & 2 & 3 \\ 9 & 5 & 1 \end{pmatrix}$ using the (first) row expansion (by minors):

$$det(A) = 2 \begin{vmatrix} 2 & 3 \\ 5 & 1 \end{vmatrix} - 1 \begin{vmatrix} 4 & 3 \\ 9 & 1 \end{vmatrix} + 5 \begin{vmatrix} 4 & 2 \\ 9 & 5 \end{vmatrix}$$

$$= 2(2 - 15) - (4 - 27) + 5(20 - 18)$$

$$= 2(-13) - (-23) + 5(2) = -26 + 33 = 7.$$

Problem

$$\text{Reduce } A = \begin{pmatrix} 2 & 1 & 3 & 5 \\ 3 & 0 & 1 & 2 \\ 4 & 1 & 4 & 3 \\ 5 & 2 & 5 & 3 \end{pmatrix}$$

to an upper triagular matrix and use the reduction to find $det(A)$.

Solution:

$$A \to \begin{pmatrix} -1 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \\ 4 & 1 & 4 & 3 \\ 5 & 2 & 5 & 3 \end{pmatrix} \quad (r_1 \to r_1 - r_2)$$

$$\to \begin{pmatrix} -1 & 1 & 2 & 3 \\ 0 & 3 & 7 & 11 \\ 0 & 5 & 12 & 15 \\ 0 & 7 & 15 & 18 \end{pmatrix}$$

$(r_2 \to r_2 + 3r_1, r_3 \to r_3 + 4r_1, r_4 \to r_4 + 5r_1)$

$$\to \begin{pmatrix} -1 & 1 & 2 & 3 \\ 0 & 3 & 7 & 11 \\ 0 & 5 & 12 & 15 \\ 0 & 1 & 1 & -4 \end{pmatrix} \quad (r_4 \to r_4 - 2r_2)$$

$$\to \begin{pmatrix} -1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -4 \\ 0 & 5 & 12 & 15 \\ 0 & 3 & 7 & 11 \end{pmatrix} (r_2 \leftrightarrow r_4)$$

$$\to \begin{pmatrix} -1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & 7 & 35 \\ 0 & 0 & 4 & 23 \end{pmatrix}$$

$$(r_3 \to r_3 - 5r_2, r_4 \to r_4 - 3r_2)$$

$$\to \begin{pmatrix} -1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & -1 & -11 \\ 0 & 0 & 4 & 23 \end{pmatrix} (r_3 \to r_3 - 2r_4)$$

$$\to \begin{pmatrix} -1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & -1 & -11 \\ 0 & 0 & 0 & -21 \end{pmatrix} = U \qquad (r_4 \to r_4 + 4r_3).$$

Now we have $\det U = (-1)(1)(-1)(-21) = -21$,

and $\det A = (-1)\det U = 21$, since all of the 3rd elementary operations change the determinant by a factor of (1) – so no change - there are no 2nd EROs and exactly one 1st ERO, with each 1st ERO changing the determinant by a factor of (-1).