Center for Optical Technologies

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Scaling Laws for FSO Communications (I): Multiple Transmitters

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Motivation of the Work

- It is well-known that multiple free-space optical communication (FSOC) links improve both ergodic capacity and outage probability
- Motivated by need to understand
  - the **performance scaling laws** when multiple lasers and/or multiple detectors (MLMD) are deployed
  - the (closed-form) **bit error rate (BER)** expressions
- Consider MLSD, SLMD and MLMD

System Model

MLMD FSOC Channels

MLMD FSO system consists of **M** transmit and **N** receive apertures. The fading channel coefficient which models the channel from m-th transmit aperture to n-th receive aperture is assumed to follow a **lognormal** distribution

$$I_{mn} = I_0 \exp\left(2X_{mn} - 2\overline{X}_{mn}\right)$$

I_0 -- normalization factor

X_{mn} -- normal random variables

$$\overline{X}_{mn} = E[X_{mn}] = -\sigma_x^2$$

$$\sigma_x^2 = Var[X_{mn}^2]$$

The **joint distribution** of received signal **X** is

$$f_{\mathbf{x}}(\mathbf{x}) = \frac{1}{(2\pi)^{n/2}|C|^{n/2}}\exp\left(-\frac{1}{2}(\mathbf{x}-\overline{\mathbf{x}})^T |C|^{-1}(\mathbf{x}-\overline{\mathbf{x}})\right)$$

$$C = \begin{bmatrix} \sigma_x^2 & \sigma_x^2 b(d_{12}) & \cdots & \sigma_x^2 b(d_{1N}) \\ \sigma_x^2 b(d_{21}) & \sigma_x^2 & \cdots & \sigma_x^2 b(d_{2N}) \\ \cdot & \cdot & \cdots & \cdot \\ \sigma_x^2 b(d_{N1}) & \sigma_x^2 b(d_{N2}) & \cdots & \sigma_x^2 \end{bmatrix}_{N\times N}$$

$$b(d_{P_1,P_2}) = \frac{E[X(P_1)X(P_2)] - E[X(P_1)]E[X(P_2)]}{\sigma_x^2}$$

Approximation of Summation of Correlated Lognormal Random Variables

$$e^z \cong \sum_{k=1}^{K} e^{x_k}$$

$$\mu_z \approx \log K - \frac{\sigma_z^2}{2}$$

$$\sigma_z^2 \approx \frac{1}{K}\sigma_x^2 + \frac{1}{K^2}\sum_{k\neq l}\sigma_x^2 b(d_{kl})$$

Sum of ***k*** lognormal random variables can be **well approximated** by a **single** lognormal distributed random variable when σ_x^2 is small !

MLSD FSOC with Channel State Information

BER:
$$P_e = \int_{\mathbf{x}} f_{\mathbf{x}}(\mathbf{x})\, Q\left(\frac{\eta I_0}{2M\sigma_v}\sum_{m=1}^{M} e^{2x_m}\right)d\mathbf{x} \quad \leftarrow \text{vector integral}$$

$$\approx \int_{-\infty}^{\infty} N\left(x, \log M - \frac{2\sigma_x^2}{M}, \frac{4\sigma_x^2}{M}\right) Q\left(\frac{\eta I_0 e^x}{2M\sigma_v}\right)dx \quad \leftarrow \text{scalar integral}$$

$$= \int_{-\infty}^{\infty} N\left(x, -\frac{2\sigma_x^2}{M}, \frac{4\sigma_x^2}{M}\right) Q\left(\frac{\eta I_0 e^x}{2\sigma_v}\right)dx$$

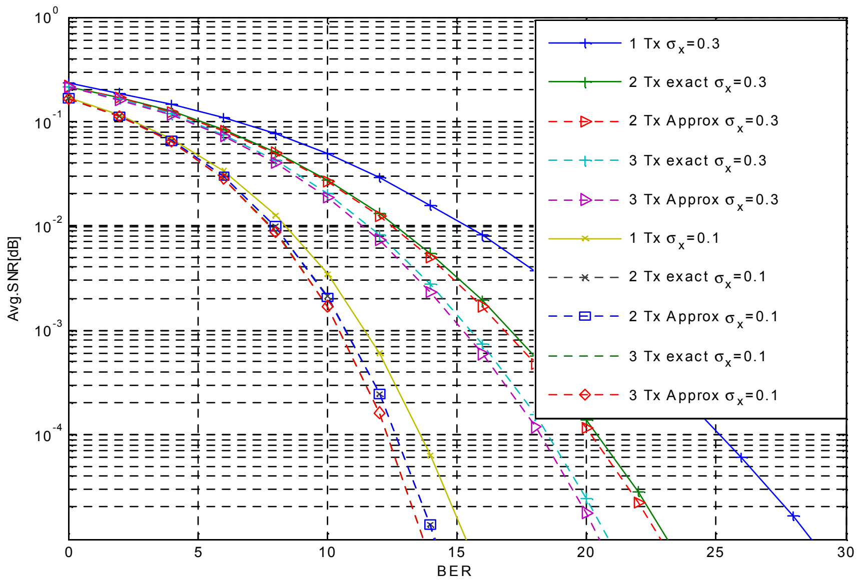

Fig.1. Comparison of exact and approximate BER for the MLSD link with CSI

Figure 1 ➔ the approximated expressions well reflect the true performance when Channel State Information is available at the receiver

MISO FSOC without perfect CSI

When Channel State Information is not available at the receiver, the **likelihood function** is given by

$$\Lambda(r) = \int_{\mathbf{x}} f_{\mathbf{x}}(\mathbf{x})\exp\left\{-\frac{1}{N_0}\left[\left(r - \eta I_0 \frac{1}{M}\sum_{n=1}^{M} e^{2x_n}\right)^2 - r^2\right]\right\}d\mathbf{x} \quad \leftarrow \text{vector integral}$$

$$\approx \int_{-\infty}^{\infty} N\left(x, -\frac{2\sigma_x^2}{M}, \frac{4\sigma_x^2}{M}\right)\exp\left(-\frac{(r-\eta I_0 e^x)^2 - r^2}{N_0}\right)dx \quad \leftarrow \text{scalar integral}$$

BER is evaluated by integrating the tailing probability of the likelihood function.

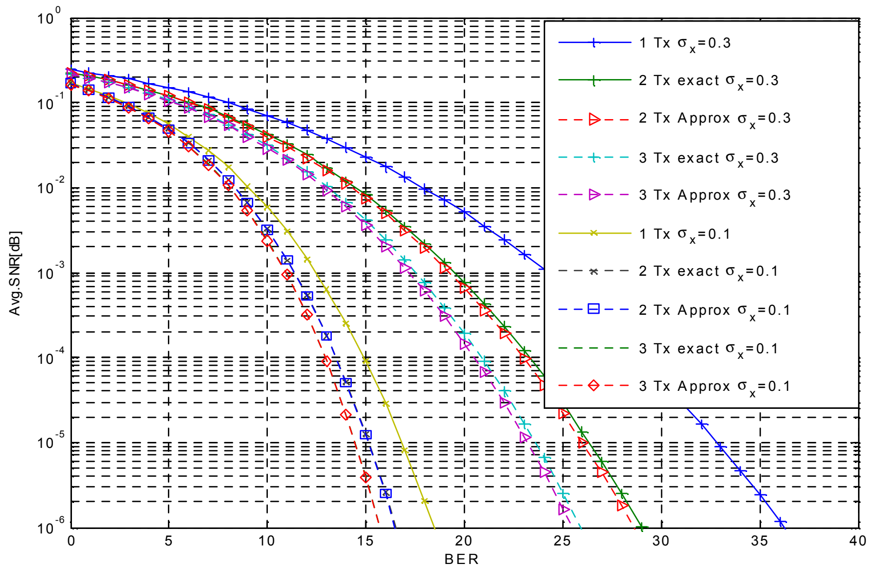

Fig. 2. Comparison of exact and approximate BER for a MLSD link without CSI.

Figure 2 ➔ the approximated expressions well reflect the true performance when Channel State Information is NOT available at the receiver

Conclusion

- FSO links with transmit diversity (MLSD) can be efficiently represented by equivalent SLSD systems with appropriate scaling in the channel variance.
- The effect of spatial diversity manifests itself as a decrease of the channel variance.