

# Lehigh University Center for Optical Technologies

## On the Capacity of SLSD and MLMD Free-Space Optical Communication Systems

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### Motivation of the Work

- Motivated by the need to understand the ergodic channel capacity and the outage probability of free-space optical communications (FSOC)
- Evaluate optimal power allocation among multiple channels using the water-filling method

### System Model

#### SLSD FSOC Channels

**Log-Normal Scintillation Channels:** Optical signals experience atmospheric loss and interference along propagation path are dominated by **atmospheric turbulence**.

$$y_k = \eta U_k x_k + n_k = I_k x_k + n_k$$

$x_k$  -- transmitted signal at time instant  $k$

$y_k$  -- received signal

$\eta$  -- optical-to-electrical conversion coefficient

$U_k$  -- normal fading coefficient

$n_k$  -- AWGN  $N(0,1)$

$I_k$  -- instantaneous intensity gain

$$f_I(z) = \frac{1}{2z\sigma_z\sqrt{2\pi}} \exp\left(-\frac{(\ln z)^2}{8\sigma_z^2}\right)$$

$$\bar{\gamma} = \frac{E[I^2]E[x^2]}{N_0} = \frac{e^{8\sigma_z^2}}{2N_0}$$

$\bar{\gamma}$  -- average SNR

#### MLMD FSOC Systems

- $M$ -by- $M$  MLMD FSOC system forms  $M$  parallel SLSD FSOC channels.
  - No cross-over channels (due to the nature of FSOC systems)
  - Cannot exploit space-time codes, beam-forming etc
- Consider 2-by-2 systems → 2 independent log-normal fading channels

### Ergodic Capacity

#### Ergodic Capacity for SLSD systems

- With perfect channel side information (CSI), the instantaneous SLSD FSOC channel is essentially a band-limited AWGN channel

$$C_{SLSD}(\gamma) = \int_0^\infty \log_2(1 + I^2 E[x^2] / N_0) f_I(I) dI$$

$$= \int_0^\infty \frac{\log_2(1 + z^2 e^{-8\sigma_z^2} \gamma)}{2z\sigma_z\sqrt{2\pi}} \exp\left(-\frac{\ln^2 z}{8\sigma_z^2}\right) dz$$

$$\gamma = e^{8\sigma_z^2} E[x^2] / N_0 \quad \text{-- instantaneous SNR}$$

Capacity increases monotonically with SNR, but decreases with  $\sigma_z$ . However, with perfect CSI, the effect of  $\sigma_z$  is minor.

#### MLMD Systems and Optimal Power Allocation

- Power distribution given, MLMD system has the sum capacity of SLSD systems
- Optimal power allocation to maximize the sum capacity?
  - Water-filling theorem

$$C_{MLMD} = \max_{0 \leq \rho \leq 1} C_{SLSD, \sigma_{z1}}(\rho\gamma) + C_{SLSD, \sigma_{z2}}((1-\rho)\gamma).$$

Observation: optimal power allocation is not much different from uniform power allocation (Table 1)

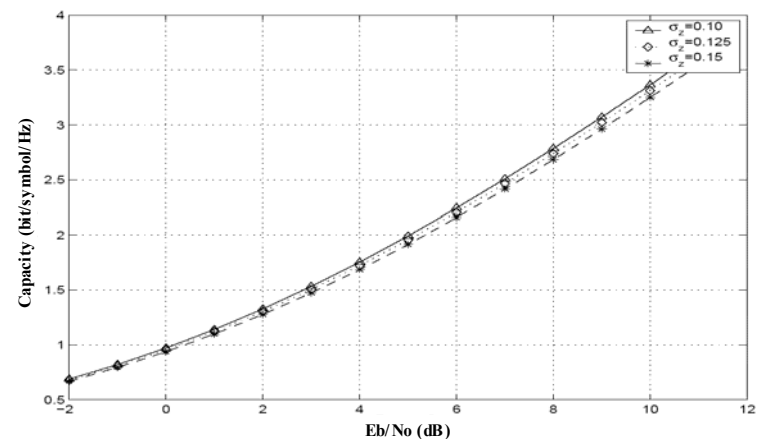


Figure 1. Ergodic Capacity with perfect channel state information

Table 1: Capacity of Water-filling Between 2 Parallel FSOC Channels (SNR=3dB)

Channel 1		Channel 2		$C_{total}$ opti. alloc.	$C_{total}$ even alloc.
$\sigma_{z1}$	power	$\sigma_{z2}$	power		
0.100	48.1%	0.050	51.9%	1.964809	1.96453
0.100	48.8%	0.075	51.2%	1.95575	1.955658
0.100	50%	0.100	50%	1.943416	1.943416
0.100	51.4%	0.125	48.6%	1.928042	1.927978
0.100	53.8%	0.150	46.2%	1.910418	1.90955

### Outage Probability

- Outage probability is a statistical measure for the quality of fading channel
- It specifies the time the instantaneous channel quality is below a threshold

$$P_{SLSD, outage}(C) = \int_0^C f_c(x) dx,$$

$$P_{MLMD, outage}(C) = \int_0^C (f_c(x) * f_c(x)) dx,$$

where \* denotes the convolutional operation.,  $f_c(c)$  is the probability density function of the instantaneous ergodic capacity of a single channel

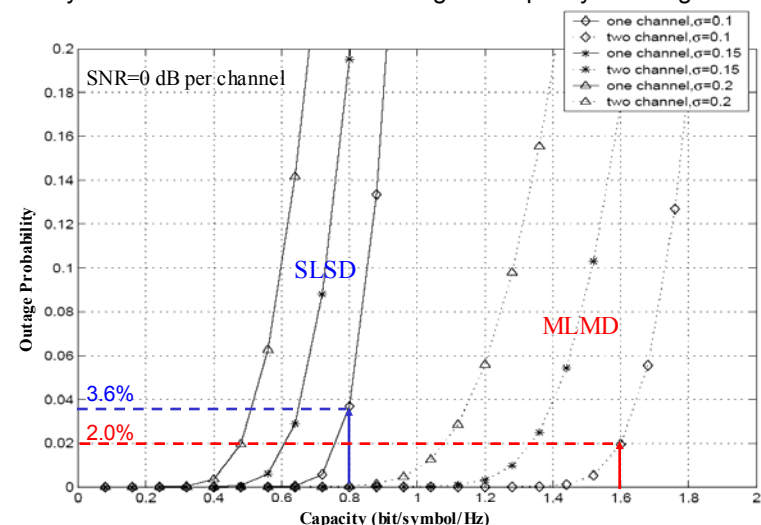


Figure 2. Outage probability of SLSD FSOC and 2-by-2 MLMD systems

### Conclusion

- Capacity (with perfect CSI): the effect of turbulence is minor for SLSD, optimal power allocation is practically the same as uniform power allocation (on average)
- Outage probability: MLMD noticeably better than SLSD