# Corporate policy distortions and indirect costs of bankruptcy

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**Abstract**

Coupled with limited liability, asymmetric information between equityholders and debtholders is commonly invoked to explain distorting effects of debt on investment decisions. We propose a continuous-time, intertemporal decision model to investigate the role of bankruptcy risk on policies/activities of a firm in a symmetric information setting. No debt is in place and capital structure is set dynamically just prior to investment. The timing of debt financing and leverage are dependent on value of equity and the riskiness and magnitude of the investment opportunity. Under the threat of bankruptcy, we find limited liability alone is sufficient to cause distortions in firm activities. Even with perfect and symmetric information, limited liability can lead to both underinvestment and overinvestment and that the magnitude of this effect can be significant.

*Key words:* Real options; policy distortion; bankruptcy cost
*JEL classification*: D81, D92, G31, G33

## 1 Introduction

Conflicts of interest between bond and equity holders and the agency costs associated with use of debt have long been recognized in economic literature. It is often stated that, lacking countervailing incentives, managers of levered firms prefer risky projects over stable cash flows. Acting on behalf of shareholders, firm managers tend to *underinvest* in safer projects and *overinvest* in risky ones. Because of the limited liability feature of corporate borrowing, shareholders benefit from higher equity values as volatility increases, even though the value of existing debt may diminish. We show that this is not always the case.

*November 2002*

## 1.1 Overview

This article examines distortions in investment policies of firms brought on by leverage and, in particular, arising from possibility of bankruptcy. We isolate the impact of bankruptcy risk on investment decisions by taking away information asymmetries between firm insiders and outsiders and by removing debt overhang, characteristics which are commonly used to explain under- and overinvestment problems. We measure the effect of equity versus debt financing on the timing of an irreversible capital investment in the presence of default risk. The model of the firm we employ incorporates an endogenous bankruptcy trigger whereby equityholders choose the timing of default on debt obligations. Rooted in limited liability, this approach recognizes the ability of firm managers to determine the timing of default (Black and Cox, 1976; Mello and Parsons, 1992; Leland, 1994; Mella-Barral and Perraudin, 1997).

In a contingent claims framework, we compare investment and divestment decisions of a firm with assets in place and a growth option over an infinite horizon. We develop simple investment rules and examine them with and without the threat of bankruptcy, allowing for investment to take place at any future time. Incentive effects of bankruptcy appear in our model as differences in adoption times of a risky capacity expansion project and plant abandonment times. Numerical simulations indicate that with debt financing the investment threshold can be either lower or higher than equity financing depending on project characteristics. Information asymmetry does affect decisions, in general. However, our analysis reveals that the prospect of bankruptcy alone is sufficient to distort investment decisions. This holds even without debt overhang and if debt holders have full information about risks of investment opportunities available to firm managers.

## 1.2 Choice of capital structure and decision policy

The investment policy problem we address rests within a larger set of questions regarding the relevance of capital structure. Accordingly, our model draws both on corporate finance and real options theories. As pointed out by Modigliani and Miller (1958, 1963), firm value is invariant under choice of capital structure if leverage does not change cash flows. In their setting, firms select a mix of debt and equity financing that balances tax benefits of debt and *direct* costs associated with bankruptcy. Direct costs result from inefficient and costly liquidation of assets in bankruptcy. Numerous empirical studies have been devised to this "trade-off" hypothesis. As alluded earlier, the combination of debt and limited liability can lead to conflicts of interest between owners and creditors of a firm. These conflicts distort decisions and cash flows and, thus, constitute an *indirect* or agency cost of bankruptcy.

### *1.3 Importance of limited liability as a source of indirect cost*

Various agency problems can affect shareholders, managers, and debtholders of a firm. In this study, we restrict our attention to limited liability as a source of agency cost because we are interested in characterizing the role of bankruptcy risk.[1] Limited liability can significantly affect a firm's activities by inducing interaction among financing, investment, and operating decisions. For instance, since shareholders are not personally liable for the firm's debt obligations, they collectively hold embedded call options on asset value. The option is exercised by transferring the firm to creditors if value of assets falls below debt obligations. In their ground-breaking works, Black and Scholes (1973) and Merton (1974) point out the importance of limited liability in determining the value of corporate debt but, in accord with the view that capital structure is irrelevant, they assume firm value is exogenous. Challenging this assumption, Jensen and Meckling (1976) describe the potential for reduction in firm value through "asset substitution." They examine agency costs including incentive effects of debt on investment choices of owner-managers and assert that equityholders can extract value from debtholders by using existing debt funds to overinvest in risky projects. It is limited liability that gives shareholders greater value from an increase in risk exposure. They profit from the likelihood of larger gains at the expense of larger potential losses. Firm value is reduced and wealth is transferred from creditors to owners as bankruptcy risk grows. On the other hand, since shareholders are residual claimants to firm value after debt is paid, debtholders benefit more from a safe positive net value project than shareholders. An unlevered firm considers only the project cost. With leverage, managers invest in an equity-financed project if its value exceeds the sum of project cost and payments promised to creditors. Thus, a higher hurdle has to be overcome before a new project is undertaken. Myers (1977) argues that for this reason leverage causes a debt overhang or underinvestment problem.

Myers (1977) views the firm as a collection of existing assets, liabilities, and growth opportunities.[2] When financed with equity, the connection between assets and liabilities in this arrangement inhibits value-maximizing use of growth options. Only relatively recently attempts have been made to quantify the impact of this problem and early evidence has been mixed. Employing a model

[1] For excellent reviews of corporate finance and related agency problems see Brennan (1995) and Zingales (2000).

[2] We adopt a slight restatement of this definition and treat the firm as a collection of existing assets, liabilities, and real options to emphasize, in addition to growth opportunities, other operating and investment options are available to the firm.

in which firm managers maximize equity value after debt is in place, Leland (1998) finds that the magnitude of agency costs due to asset replacement is small. Parrino and Weisbach (1999) simulate the effect of shareholder-bondholder conflicts on a levered firm using discounted cash flow analysis where the growth option is a 'now or never' equity-financed investment in a risky project. They compute the expected transfer of wealth between bondholders and shareholders and conclude that, for most firms, the effect is small.

Several papers explore the interplay of financing and investment/operating policies in a dynamic decision environment and arrive at a different conclusion. Mello and Parsons (1992) build on the contingent claims model of Brennan and Schwartz (1986) of a mine to investigate how debt financing influences operating decisions and find that agency costs are comparable or greater than underwriting and other direct administrative expenses of leverage. Using a flexible discrete time, discrete state, finite horizon model, Childs et al. (2000) show that the agency costs of asset substitution and underinvestment have a substantial effect on optimal leverage and firm value. In the construction of Childs et al. (2000), the firm optimizes its initial leverage ratio. Titman and Tsyplakov (2002) study the incentive to underinvest by allowing managers to dynamically adjust the firm's leverage and observe significantly higher agency costs in comparison to values suggested by static debt models. To study the overinvestment problem, Mauer and Sarkar (2001) consider a firm whose only asset is a real option in the form of an opportunity to invest in a production facility. They optimize leverage, debt maturity, and timing of exercise of the option to find that the agency cost of debt is a significant fraction of firm value.

A common assumption in the above studies, in step with the insight of Jensen and Meckling (1976), is that investment is undertaken with debt in place. Indeed when investment or operating decisions are made under the shadow of existing debt, conflicts of interest between shareholders and lenders leads to agency problems. Underinvestment may be caused, at least in part, by overestimation of firm risk exposure. Likewise, overinvestment occurs because existing debt underestimates potential risk. Thus, existing debt contracts are not able to reflect the dynamic nature of investment opportunities. Would policy decisions be rid of distortions if debt could be dynamically and fairly priced to indicate the true cost of investment projects as opportunities arose?

Independent of *ex post* debt policy decisions, we demonstrate that the interaction between existing assets and real options can create agency costs in the presence of bankruptcy risk. In our model, debt is extended at the time of investment with full knowledge of the risk involved. The choice of capital structure and investment are combined into a single decision. By optimizing the time to invest, managers of the firm determine the value of the firm *and* set the ratio of debt to equity for the firm. However, our goal is not to study optimal capital structure. Instead we set out to isolate the effect of bankruptcy risk on capital investment and liquidation decisions. Our model's key predictions can be summarized as follows.

- Policy distortions respond nonlinearly to uncertainty in the output market and to book-to-market values of the firm.
- There is a higher potential for overinvestment in small (when comparing project cost with book value of assets) projects.
- Underinvestment is more likely in large investments.
- For medium sized projects, the tendency to underinvest or overinvest depends on project risk and volatility in existing operations.
- Distortion is lower during extremely volatile financial environments.

The remainder of this paper is organized as follows. Assumptions of the model are introduced in the next section followed by model development under equity financing and debt financing in Sections 2.1 and 2.2, respectively. Basic numerical solutions are gathered in Section 2.3. Discussion of results follows in Section 3 before concluding.

## 2 The model

Consider a risk-neutral firm that owns one plant and produces a unit of output which it sells for $P_t$ at time $t$. The market price $P_t$ evolves exogenously over time as a geometric Brownian motion

$$\mathrm{d}P_t = \mu P_t \,\mathrm{d}t + \sigma P_t \,\mathrm{d}B_t \tag{1}$$

where $\mu$ and $\sigma$ are constant parameters and $B_t$ is standard Brownian motion. Denoting by $\xi$ the unit cost of output produced, the firm's profit flow is $P_t - \xi$. Given a positive production cost ($\xi > 0$), the firm liquidates all assets the first time output price falls below some constant low level. At closure, the firm has a salvage value of $\gamma$.

The firm has an opportunity to expand operations by adding a new plant at cost $I$. The new plant, which produces one unit of output identical to the original plant, also has a salvage value $\gamma$ at closure. $\gamma$ is a fraction of the initial investment cost. The firm's decision problem is to determine when to invest in the additional plant. Once made, the investment is irreversible except that the firm can permanently cease all production activity by simultaneously shutting down both plants. The investment, if made, can be funded by issuing equity or debt. The firm has no preexisting debt. In addition, we assume managers of the firm are its owners. To focus on agency costs of debt on irreversible investments we do no distinguish between shareholders and managers. Both groups share the common objective of maximizing equity value. Once in operation, a plant must be run at full capacity. There is no possibility of scaling back production other than shutting down all operations and liquidating the firm. If funded by debt, bondholders immediately liquidate the firm upon bankruptcy. This effectively means that the direct cost of bankruptcy is sufficiently high to warrant closing the plant down.

### *2.1 Equity financing*

Let us begin with the investment decision under pure equity financing as shown in Figure 1. If profits are too low, the firm could be shut down before there is a chance to invest in the second plant. The firm owns several real options in the form of liquidation and plant expansion and exercises them such that equity value is maximized. The prospect of bankruptcy creates indirect costs because of inefficient investment and closure decisions. Let $W_2$ denote the value of the

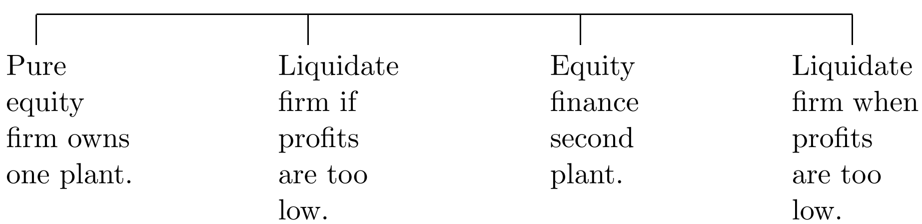


Fig. 1. **Actions of manager-owners under equity financing.**

pure equity firm that already owns two plants and has an income flow $2P_t - 2\xi$. Firm value is contingent on output price and satisfies the Bellman equation. If $W_2$ is twice differentiable, equation

$$rW_2(P) = 2P - 2\xi + \mu P W_2'(P) + \frac{1}{2}\sigma^2 P^2 W_2''(P) \tag{2}$$

holds for all output prices $P$. $W_2(P)$ satisfies several boundary conditions. At closure, the total value of firm assets must be equal to their salvage value, $W_2(P_{c_2}) = 2\gamma$. In the absence of speculative price bubbles, the limiting value of $W_2(P_t)$ for large $P_t$ is the expected discounted integral of future profit flows,

$$2\left(\frac{P_t}{r-\mu} - \frac{\xi}{r}\right)$$

where $r$ is the (risk-adjusted) discount rate. Equity holders choose the threshold price for closure, $P_{c_2}$, to maximize total firm value. This requires $W_2'(P_{c_2}) =$

0. The value of the pure equity firm with two plants is, then,

$$W_2(P) = \begin{cases} \frac{2P}{r-\mu} - \frac{2\xi}{r} + 2\left(\gamma - \frac{P_{c2}}{r-\mu} + \frac{\xi}{r}\right)\left(\frac{P}{P_{c_2}}\right)^{\lambda}, & P \geq P_{c_2} \\ 2\gamma, & P < P_{c_2} \end{cases} \quad (3)$$

where $\lambda = \min(\lambda_1, \lambda_2)$ is the negative root of the quadratic equation $\lambda(\lambda - 1)\sigma^2/2 + \lambda\mu - r = 0$ (with roots $\lambda_1$, $\lambda_2$) and $P_{c_2}$ is given by

$$P_{c_2} = -\frac{\lambda}{1-\lambda}\frac{\xi + r\gamma}{r}(r - \mu). \quad (4)$$

Now consider a firm that owns one plant and an option to invest in an identical plant at cost $I$. The investment can be made at any time and, if made, is entirely financed by equity. The new plant generates profit flows identical to the original plant. The value of this firm, denoted $W_1$, satisfies the equation

$$rW_1(P) = P - \xi + \mu P W_1'(P) + \frac{1}{2}\sigma^2 P^2 W_1''(P). \quad (5)$$

This has the general solution, valid when $P \geq P_{c_1}$,

$$W_1(P) = \frac{P}{r-\mu} - \frac{\xi}{r} + A_1 P^{\lambda_1} + A_2 P^{\lambda_2} \quad (6)$$

where $A_1$ and $A_2$ are constants. Suppose $P_e$ is the investment threshold for the pure equity financed investment. The boundary conditions for this problem are

$$W_1(P_e) = W_2(P_e) - I \quad (7a)$$
$$W_1'(P_e) = W_2'(P_e) \quad (7b)$$

and

$$W_1(P_{c_1}) = \gamma \quad (7c)$$
$$W_1'(P_{c_1}) = 0. \quad (7d)$$

These four equations determine the four unknowns—thresholds $P_e$, $P_{c_1}$ and coefficients $A_1$ and $A_2$—that solve the asset valuation, investment, and disinvestment problem for the equity financed firm.

### *2.2 Debt financing*

We now turn to the firm's investment decision under debt financing. Debt is issued just prior to investment in the new plant. Debtholders are informed about the financial state of the firm and its prospects and are, thus, able to determine the correct value of debt. The time line for the this case is shown in Figure 2. Again, we start the analysis with a firm that owns two plants and

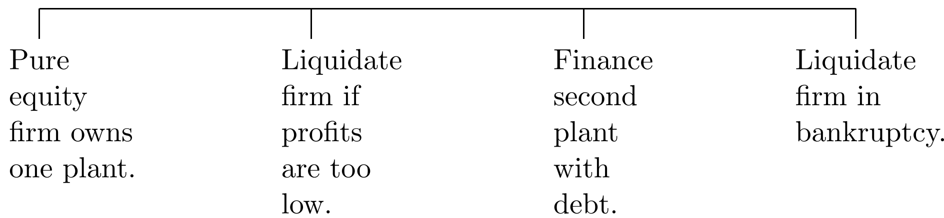


Fig. 2. **Actions of manager-owners under debt financing.**

has a profit flow $2P_t - 2\xi$, but now the firm has issued perpetual debt with principal $b/r$ and a contractual coupon flow $b$ per period of time. Managers operate the firm in order to maximize shareholder equity value, rather than the combined value of equity and debt. The levered firm's equity value $V_2$ satisfies the differential equation

$$rV_2(P) = 2P - 2\xi - b + \mu P V_2'(P) + \frac{1}{2}\,\sigma^2 P^2 V_2''(P). \tag{8}$$

Because of limited liability managers have some flexibility in choosing a default policy that maximizes equity value. Operationally, we assume that the owner-managers may cover the firm's operating losses by providing their own capital. Bankruptcy occurs when equity holders decide to stop injecting additional capital. They do this by selecting a bankruptcy trigger $P_b$ so as to maximize the value of equity. Equity holders declare bankruptcy the first time $P_t \leq P_b$ after adoption of the second plant. In bankruptcy, debtholders receive their share of tangible assets and shareholders receive the rest. The expression describing this requirement and the companion smooth pasting condition are

$$V_2(P_b) = \left(\widehat{W}_2(P_b) - \frac{b}{r}\right)^+$$
$$V_2'(P_b) = \left(\widehat{W}_2'(P_b)\right)^+$$

where $\widehat{W}_2$, assumed a differentiable function of price $P$, is the value of the firm under debt financing. Given the assumption that debt is risky, the above boundary conditions become $V_2(P_b) = 0$ and $V_2'(P_b) = 0$. In addition, ruling out speculative price bubbles, we have

$$\lim_{P\to\infty} V_2(P) = \frac{2P}{r-\mu} - \frac{2\xi + b}{r}.$$

If the total salvage value of the plants is less than the principal value of debt (*i.e.*, $2\gamma < b/r$), the solution to the firm's equity value is

$$V_2(P) = \frac{2P}{r-\mu} - \frac{2\xi + b}{r} - \left(\frac{2P_b}{r-\mu} - \frac{2\xi + b}{r}\right)\left(\frac{P}{P_b}\right)^{\lambda} \tag{9}$$

where $P_b$ is

$$P_b = -\frac{\lambda}{1-\lambda}\frac{2\xi + b}{2r}(r-\mu). \tag{10}$$

Now let us go back to the original firm that owns one plant and an option to invest in an identical plant at cost $I$. This time we assume that investment, if made, must be completely debt financed. Suppose the firm is able to issue perpetual debt with principal $b/r$ and a coupon flow $b$ per unit of time. The firm's value $V_1$ must satisfy the following differential equation

$$rV_1(P) = P - \xi + \mu P V_1'(P) + \frac{1}{2}\sigma^2 P^2 V_1''(P) \tag{11}$$

with general solution given by

$$V_1(P) = \frac{P}{r-\mu} - \frac{\xi}{r} + B_1 P^{\lambda_1} + B_2 P^{\lambda_2} \tag{12}$$

where $B_1$ and $B_2$ are constants. Let $P_d$ be the investment threshold for the completely debt financed investment and $P_c$ be the closure threshold before investment. Value of assets must satisfy the boundary conditions

$$V_1(P_d) = V_2(P_d) \tag{13a}$$
$$V_1'(P_d) = V_2'(P_d) \tag{13b}$$
$$V_1(P_c) = \gamma \tag{13c}$$
$$V_1'(P_c) = 0. \tag{13d}$$

Appropriate accounting of bankruptcy risk requires equating investment cost $I$ with debt value at the threshold price (see, for example, Mella-Barral and Perraudin, 1997). It can be shown that this requirement is given by

$$I = \frac{b}{r} + \left(2\gamma - \frac{b}{r}\right)\left(\frac{P_d}{P_b}\right)^{\lambda} \tag{14}$$

which along with Equations (10) and (13) provide the six equations that determine $P_d$, $P_c$, $B_1$, $B_2$, $b$, and $P_b$.[3]

[3] One may be tempted, instead, to set $I = b/r$ and solve Equations (13) for the general solution. That would underestimate foreclosure risk and overvalue debt, in

Table 1
Investment thresholds $P_e$ and $P_d$ for an investment cost $I = 30$

| $\sigma$ | $P_e$ | $P_d$ | $\frac{P_e - P_d}{P_d}\%$ |
|---|---|---|---|
| 0.01 | 1.55 | 1.55 | 0.00 |
| 0.05 | 1.77 | 1.78 | −0.28 |
| 0.10 | 2.09 | 2.11 | −1.14 |
| 0.15 | 2.46 | 2.49 | −1.31 |
| 0.20 | 2.88 | 2.90 | −0.95 |
| 0.25 | 3.35 | 3.36 | −0.36 |
| 0.30 | 3.87 | 3.86 | 0.27 |
| 0.35 | 4.45 | 4.42 | 0.86 |
| 0.40 | 5.09 | 5.03 | 1.37 |

*$P_e$ and $P_d$ are investment threshold levels for equity, and value debt financed investments, respectively. The firm invests in the additional plant as soon as output price reaches the corresponding investment threshold level.*

When the debt principal, $b/r$, is less than the salvage value of assets, $2\gamma$, bankruptcy does not occur. Timing of firm closure is then efficient, leverage imposes no agency costs, and the value of the firm is then the same as it would be under pure equity financing, $\widehat{W}_2(P) = W_2(P)$. Since debt is free of risk, the firm is effectively operated as an all-equity interest. Equity value is given by

$$V_1(P) = W_1(P)$$
$$V_2(P) = W_2(P) - D(P)$$

where debt value $D(P) = I = b/r$.

## *2.3 Numerical results*

To examine how the prospect of bankruptcy affects decision making, we compute investment thresholds $P_e$ and $P_d$ from Equations (7), (10), (13), and (14). Since the equations defining the investment threshold and the closure threshold are highly nonlinear and do not have closed form solutions, numerical methods are needed. We single out the role of bankruptcy risk by comparing

which case debt would be (erroneously) treated as if it were riskless and the optimal investment threshold would be lower than that of the equity financed investment. If valued in this manner, debt creates an incentive for overinvestment through asset substitution.

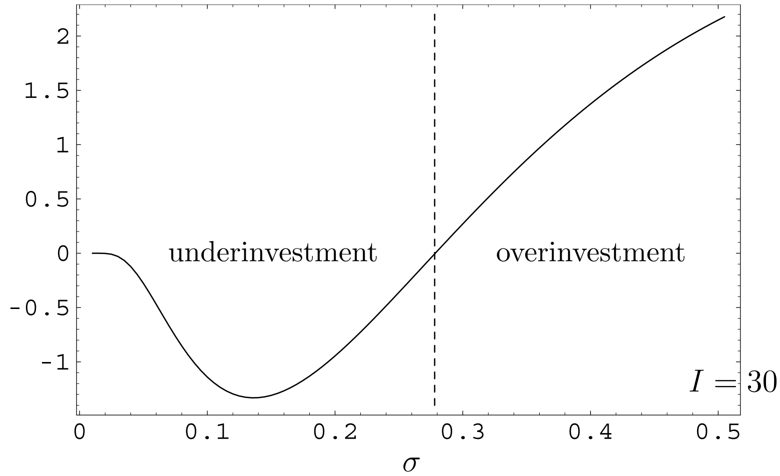


Fig. 3. **Difference in investment thresholds (percent).** Percentage difference $(\frac{P_e - P_d}{P_d} \times 100)$ between equity and debt financed investment thresholds for $I = 30$ as a function of price volatility ($\sigma$). When the project is financed by debt, the threat of bankruptcy induces under- and over-investment incentives.

the investment threshold under threat of bankruptcy to the investment threshold without bankruptcy risk. To do so, we let the volatility change, holding other parameters constant. Specifically, we let $\sigma$ vary from 10 percent to 40 percent and fix other parameters as follows: a salvage value, $\gamma$, of 2, a risk-free interest rate of 4.5 percent, and a flow cost $\xi = 0.15$. Finally, we set $\mu = 0$ and $I = 30$.

Table 1 shows values of investment thresholds for various price volatilities. All values of investment thresholds are higher than $\xi + rI$ ($= 1.5$), the full cost of investing and operating the additional plant (Dixit, 1989). This is due to the effect of profit uncertainty on the investment option. In addition, investment thresholds increase as volatility increases. This is consistent with the general finding in real options literature that the higher the uncertainty, the higher the investment threshold (McDonald and Siegel, 1986; Dixit and Pindyck, 1994).

Comparing the trigger prices for adopting the second plant with and without the risk of bankruptcy, we find that when volatility is low, $P_e < P_d$. There is a crossover level where the investment threshold for debt financing starts higher but becomes lower than the equity financing threshold as $\sigma$ increases. As $\sigma$ increases, the negative gap between $P_e$ and $P_d$ reaches an apex before narrowing to zero and changing sign. The percentage difference in investment thresholds between the debt and equity financing is illustrated in Figure 3. It implies that the prospect of bankruptcy distorts real investment decisions in a direction that depends on the magnitude of volatility. [4]

[4] In contrast, $P_e > P_{d-}$ as long as debt is risky, where price $P_{d-}$ is the threshold level assuming debt financing treats the project as risk-free, corresponding to over-

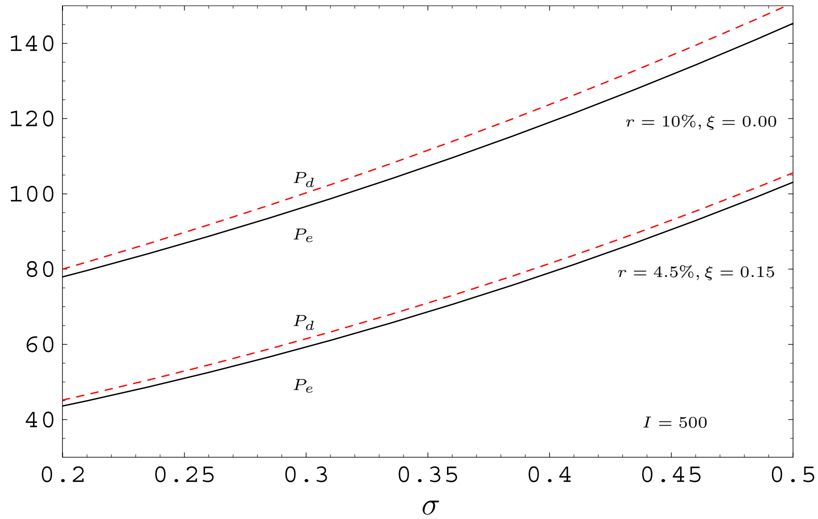


Fig. 4. **Investment threshold levels.** Comparison of investment threshold levels $P_e$ and $P_d$) under equity and debt financing, respectively.

Investment threshold levels for equity and debt financing are compared in Figure 4 for different interest rate and production cost scenarios. There is motivation for delayed investment under debt financing when $P_d$ is greater than $P_e$. The opposite is true when $P_e > P_d$.

An even more interesting picture emerges when the amount of financing is considered. Figure 5 shows that a levered firm may be either more or less likely to make an irreversible investment than a pure equity firm depending on the level of uncertainty and the amount of capital at risk (project size minus salvage value, or market-to-book value of the firm). The combination of these two factors determines whether debt induces early or late investment compared with equity financing. It is interesting that distortions tend to vanish in highly volatile environments.

## 3 Discussion

To understand the behavior of threshold prices it helps to establish some facts about the investment decision and choice of financing. We begin with a discussion of debt and equity values. It is worth noting that at the time the second plant is adopted, whenever the optimal time may be, the cost of investment as a fraction of firm value remains nearly constant for a wide range of costs. That is, for fixed $\sigma$, the debt-equity ratio is stable across a wide range of investment costs (see Figure 6).

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investment in all cases. This is to be expected since bankruptcy risk is undervalued in obtaining $P_{d-}$.

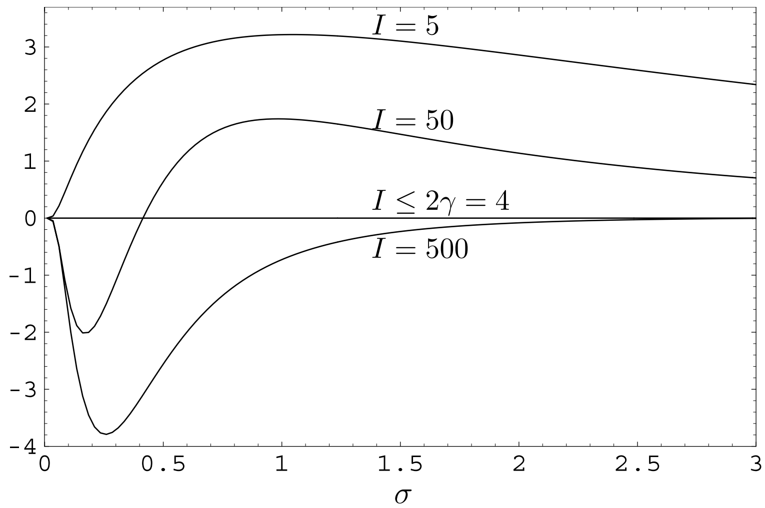


Fig. 5. **Difference in investment thresholds (percent).** Comparing the percentage difference between equity and debt financed investment thresholds for small and large capital requirements. When the investment amount does not exceed salvage value of assets (*i.e.*, $I \leq 4$), debt and equity financing are interchangeable. All curves approach zero for sufficiently high price volatilities.

### *3.1 Equity and debt values*

The timing of the investment determines firm value and simultaneously picks the optimal debt to equity ratio. Thus, capital structure and investment choice are collapsed into a single decision.

How is it possible for equity value of the firm to increase as investment cost increases? This is because investment is made only if the firm's output price is high enough to warrant it. The ratio of debt to equity can even be a decreasing function of investment cost. For expensive projects, the firm postpones the investment longer and waits for higher output prices. Equity value of the firm, meanwhile, increases as the output price increases.

Figure 7 shows another view of the debt-to-equity ratio at the time when debt financing is initiated. The ratio is a decreasing function of price volatility $\sigma$. [5] For large investment projects (and capital at risk), the proportion of debt to equity is higher for low to high levels of volatility when compared with smaller projects. Under very high levels of uncertainty, the reverse is true, slightly larger leverage ratios for smaller projects.

The effective interest rate on the risky debt, $b/I$, or *cost of capital*, is shown in Figure 8. As $\sigma$ approaches zero, the ratio of coupon flow to investment

[5] Mauer and Sarkar (2001) obtain a similar result in agreement with empirical findings of Barclay and Smith (1995) and Stohs and Mauer (1996).

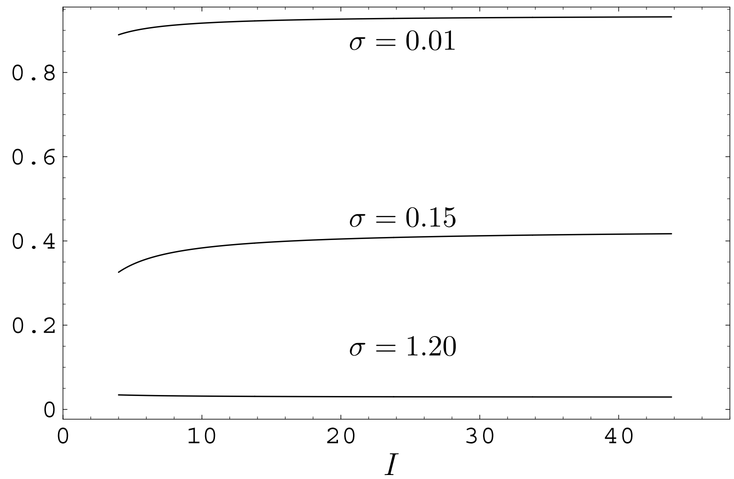


Fig. 6. **Leverage versus investment cost.** Debt-to-equity ratio ($I/V_2(P_d)$) ratio at the time of investment (*i.e.*, when output price is equal to the investment threshold price, $P = P_d$) as a function of investment cost ($I$) for $I \geq 2\gamma$.

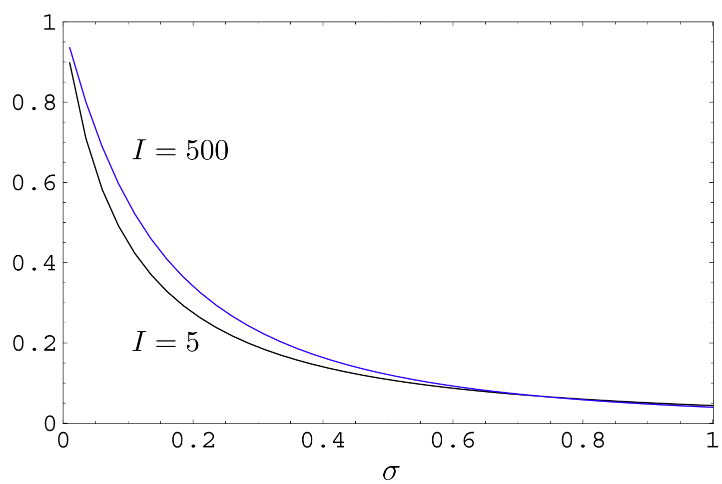


Fig. 7. **Leverage versus price volatility.** Ratio of the firm's debt to equity value at the time of investment. The ratio $I/V_2(P_d)$ is plotted a function of price volatility ($\sigma$).

cost approaches the risk-free rate of $r = 4.5\%$. For both cases shown in the figure, we have $\frac{b}{r} \geq I \geq 2\gamma$. Cost of capital, which incorporates likelihood of bankruptcy, is monotonic in price volatility and investment cost. For small $\sigma$ ($\leq 10\%$), it is approximately equal to the risk-free interest rate exhibiting little sensitivity to investment cost. Capital cost increases sharply for larger $\sigma$ as $I$ increases.

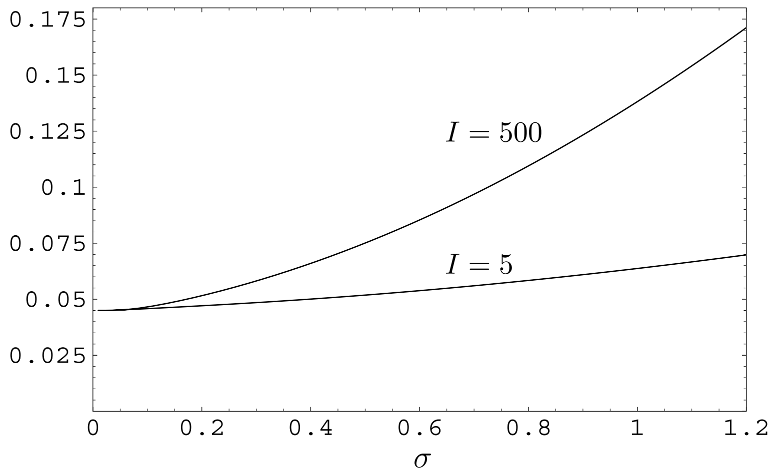


Fig. 8. **Cost of capital in debt financing.** Coupon flow divided by investment cost, $(b/I)$, is the effective rate of interest charged by the debt holder.

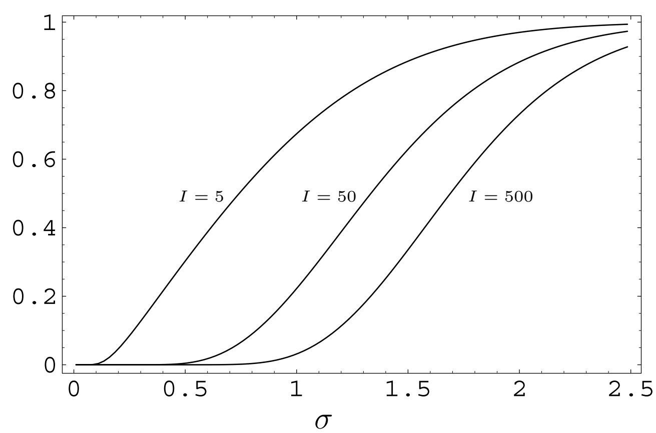


Fig. 9. **Likelihood of investing in the new plant.** Probability of reaching the investment threshold under equity financing ($P_e$) within five years, starting at an initial output price of $P_0 = 0.25$.

### *3.2 Investment and bankruptcy*

The probability of reaching the investment threshold is an increasing function of uncertainty. The more volatile the output price, the more likely it is for the second plant to be adopted in a fixed time horizon. For a given $\sigma$, this probability is a decreasing function of $I$, but for large values of $\sigma$, it approaches one. See Figure 9. Larger projects are more likely to be delayed but less so in times of high price uncertainty.

There is a chance for bankruptcy if the new plant is financed with debt. The

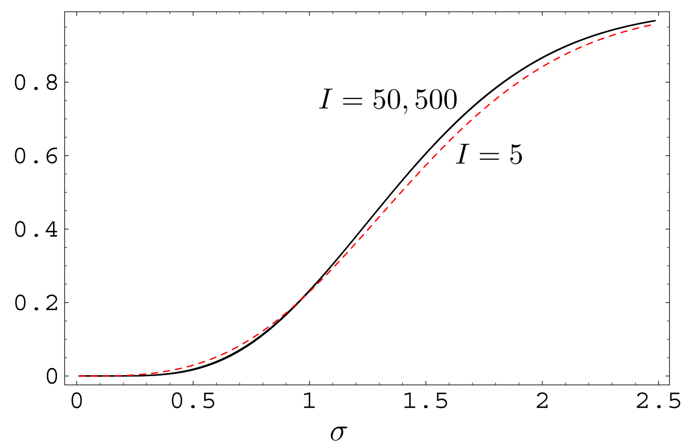


Fig. 10. **Likelihood of bankruptcy.** Five-year probability of bankruptcy from time the investment is made in the second plant.

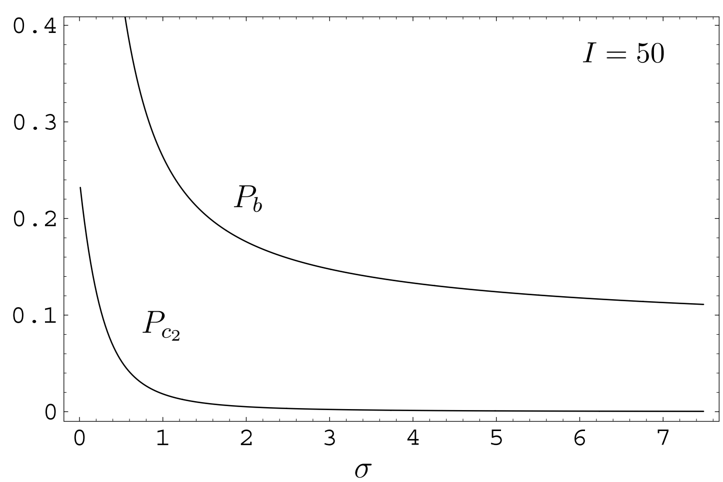


Fig. 11. **Closure thresholds.** The bankruptcy price and the closure price thresholds of a two-plant equity financed firm. Comparison of bankruptcy level ($P_b$) and liquidation threshold ($P_{c_2}$) indicates there is inefficiency in leverage if $P_b > P_{c_2}$ since the firm would be liquidated prematurely.

likelihood of bankruptcy within five years from investing in the second plant is shown in Figure 10.

The firm initiates bankruptcy when output price falls below a certain level. This threshold price is decreasing in output price volatility but increasing in investment cost. A comparison with the threshold price for liquidation of the all-equity firm reveals that a bankruptcy would always occur earlier than liquidation under equity financing (Figure 11). Thus, there is inefficiency in leverage from the perspective of shareholders.

Investment threshold levels $P_e$ and $P_d$ are increasing functions of investment cost ($I$) and revenue volatility ($\sigma$). The *difference* between equity and debt financed threshold levels $P_e - P_d$ can be positive or negative depending on $I$ and $\sigma$ but approaches zero for very large $\sigma$ for all investment costs $I$. As volatility increases, the debt-financing investment threshold increases and the equity share of total firm value increases. Therefore, it is not surprising that differences in threshold prices (Figure 5) converge to zero as volatility increases. The largest distortion in investment, corresponding to underinvestment, occurs for very large investments. It is also the case, although not shown in the graphs, that all investment thresholds decrease as the equity to debt ratio is increased.

The incentive for under- and overinvestment can be explained in terms of two countervailing forces: (1) Firm managers are reluctant to take on projects that transfer equity value to debtholders even though total firm value increases, and (2) for low debt levels, the likelihood of bankruptcy is low allowing shareholders to transfer wealth to themselves from debtholders by using debt rather than equity to pay for the project (*i.e.*, they overinvest).

For low to moderate values of $\sigma$, the ratio of debt to equity increases as larger investment outlays are funded with debt. Since equityholders are residual claimants to the value of the firm, they have lower incentive to take on higher cost projects compared to the case of all equity financing. Thus, $P_e < P_d$ because of premature closure due to bankruptcy. For very large $\sigma$, while the chance of bankruptcy increases, debt to equity ratios converge to zero for all investment sizes (Figure 7). Debt becomes a less significant portion of firm value as growth options gain in value. Thus, we have $\lim_{\sigma \to \infty} P_d = P_e$.

When equity to debt ratio is very high (corresponding to values of $I \leq 2\gamma$), $P_e$ and $P_d$ are the same. The investment can be treated as an equity investment since there is no bankruptcy risk and salvage value covers the loan. When debt to equity ratio is low (moderate costs $I$), $P_d < P_e$. As $\sigma$ increases from zero, the debt financed investment threshold becomes lower than the threshold under pure equity financing. However, as $\sigma$ continues to increase, the gap between the thresholds narrows, converging once again.

As volatility increases, the chance of bankruptcy increases. But from the perspective of debt holders, the amount of capital at risk is small. It is possible for debt service flow to offset this risk when the investment is made. With higher leverage, there is more capital at risk. Bankruptcy risk depends both on the value of the firm at the time of investment as well as on uncertainty in future revenues. These act to offset each other. For very high leverage ratios,

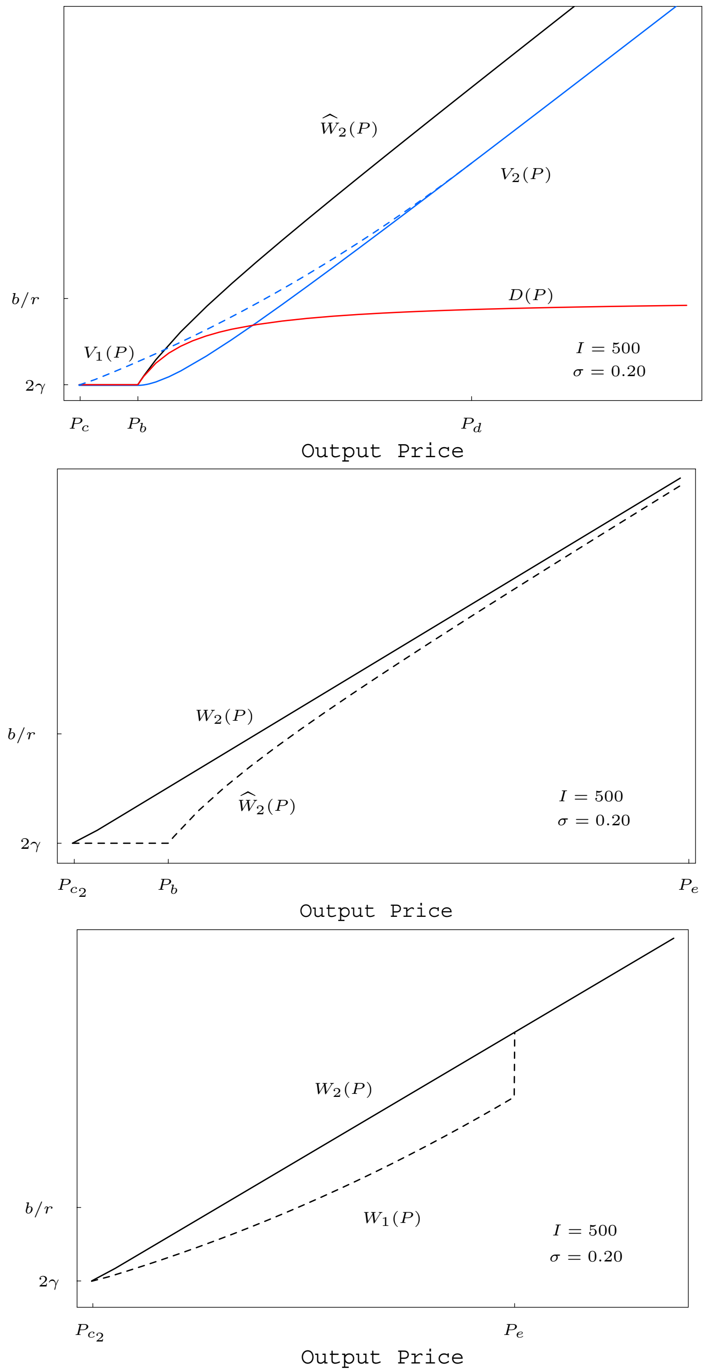


Fig. 12. **Asset values.** Representative firm, equity, and debt values as functions of output price. $D(P)$ denotes value of debt, $V_1(P)$ and $V_2(P)$ are equity values with one and two plants, respectively, under debt financing, $W_1(P)$ and $W_2(P)$ represent firm values of all equity financed for the one and two plant operation, respectively, and $\widehat{W}_2(P)$ is value of the firm with two plants under debt financing.

the amount of capital at risk is so high that the value of the option to wait dominates: $P_e \leq P_d$ for all $\sigma$.

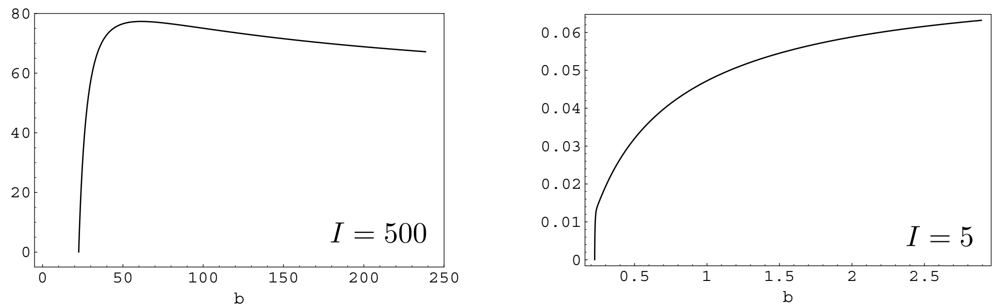


Fig. 13. **Comparison of firm value with equity and debt financing.** The plots show $W_2(P_d) - \widehat{W}_2(P_d)$, the difference between equity financed and debt financed values of the two-plant firm as a function of coupon flow $b$ for two different investment costs.

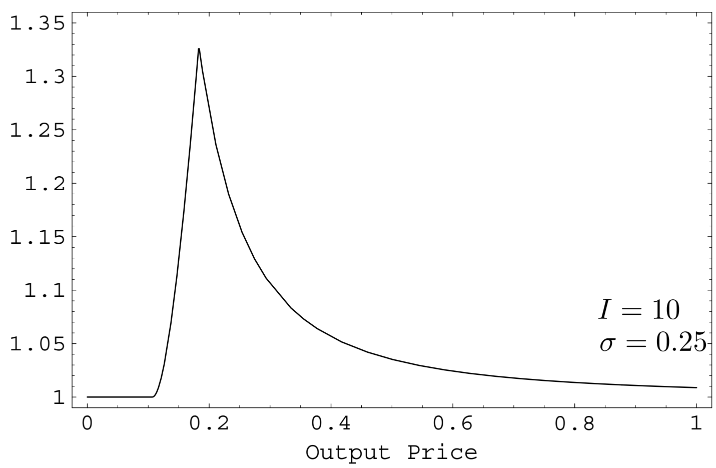


Fig. 14. **Loss of firm value.** Fraction of firm value under equity financing over firm value under debt financing. In the example shown, firm value can be up to 30% lower under debt financing when output prices are low.

Distortions in investment and closure decisions affect firm value. Figures 12 and 13 illustrate the impact of bankruptcy risk that accompanies debt financing. Inefficient decisions can have a lasting effect. Figure 14 shows firm values can be substantially lower under debt financing, especially when a firm is in financial distress.

## 4 Concluding remarks

Agency costs of debt have long been recognized to affect financing and investment activities of corporations. That leverage may distort real investment decisions is pointed out by Jensen and Meckling (1976) and Myers (1977). This paper quantifies the impact of bankruptcy risk separate from leverage on investment and liquidation timing. We compare equity versus debt financing in a capital investment expansion decision and find that the investment threshold may be lower under risky debt financing. Our results are in accord with the view that although uncertainty can create value for firms to wait, threat of bankruptcy reduces the value of waiting. However, perhaps surprisingly, this characteristic is not uniform. Depending on the level of uncertainty and the net worth of the project, bankruptcy risk can discourage as well as encourage early investment. The driving force behind this distortion in investment policy is the trade-off between assets in place and growth options in the presence of limited liability introduced through debt financing. We end the discussion of our findings with three predictions.

- When there is little capital at risk for the lender, debt financing (if priced correctly to reflect project risk) is likely to induce early investment.
- For moderate levels of capital at risk, there may be early or late investment by the firm depending on the uncertainty in future revenues. In low/high revenue volatility environments, we would expect investments to be made earlier/later than they would be under equity financing.
- Excluding competitive forces, projects are more likely to be postponed and for longer periods when they are financed with debt.

The model considered here can be extended in several ways. We have not studied the interaction of corporate taxation and bankruptcy risk. The tax benefits of debt tend to offset agency costs of bankruptcy. On the other hand, we have assumed that default leads to liquidation of the firms assets. In practice, a financially distressed firm may either liquidate its assets or decide to renegotiate its debt obligations. Including this possibility in the analysis will affect investment decisions. In addition, policy distortions may be even greater if conflicts of interest between managers and shareholders of the firm are taken into account. Choice of debt maturity is yet another variable that firm managers can use to increase equity value at the expense of firm value.

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