

# Optical and Photonic Glasses

## Lecture 8: Mechanical Properties

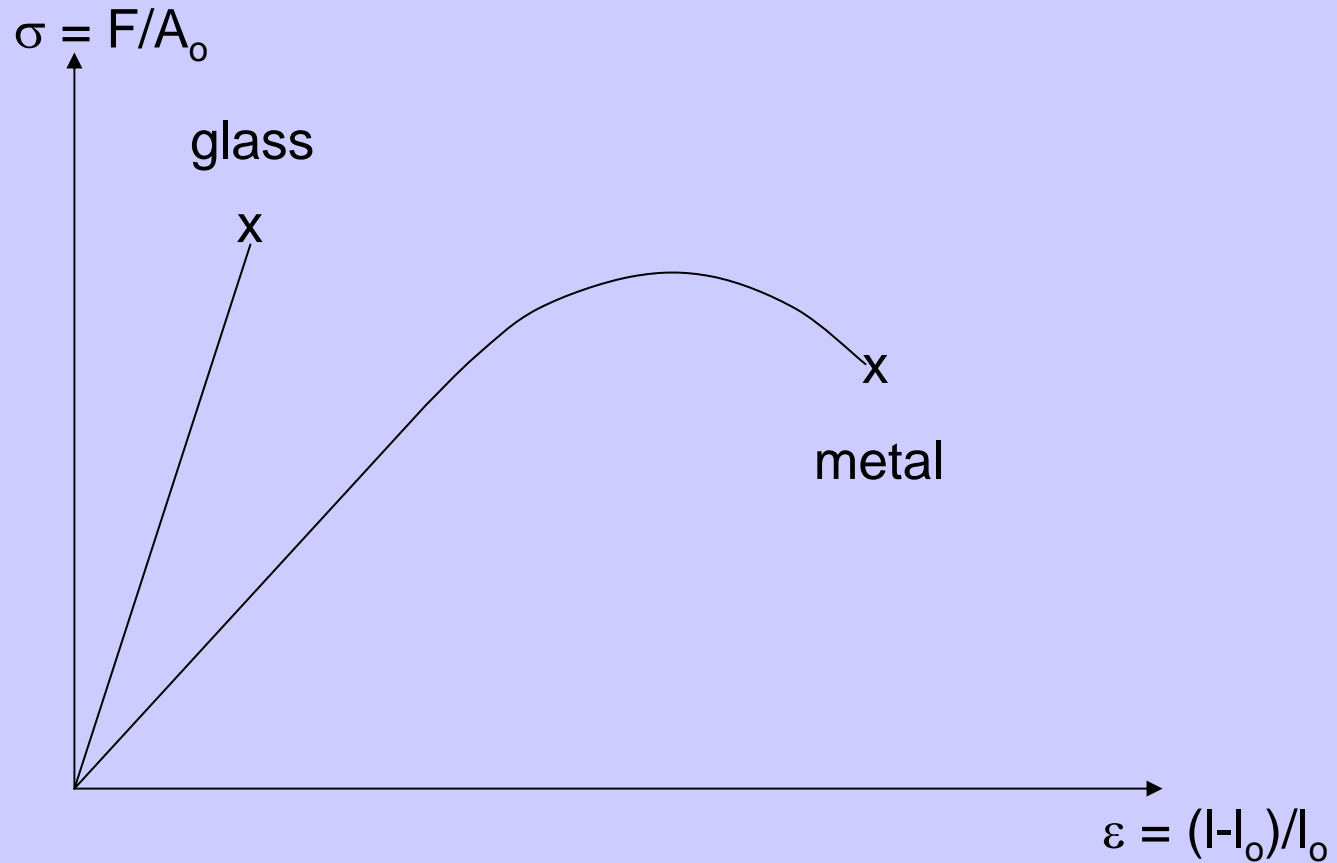
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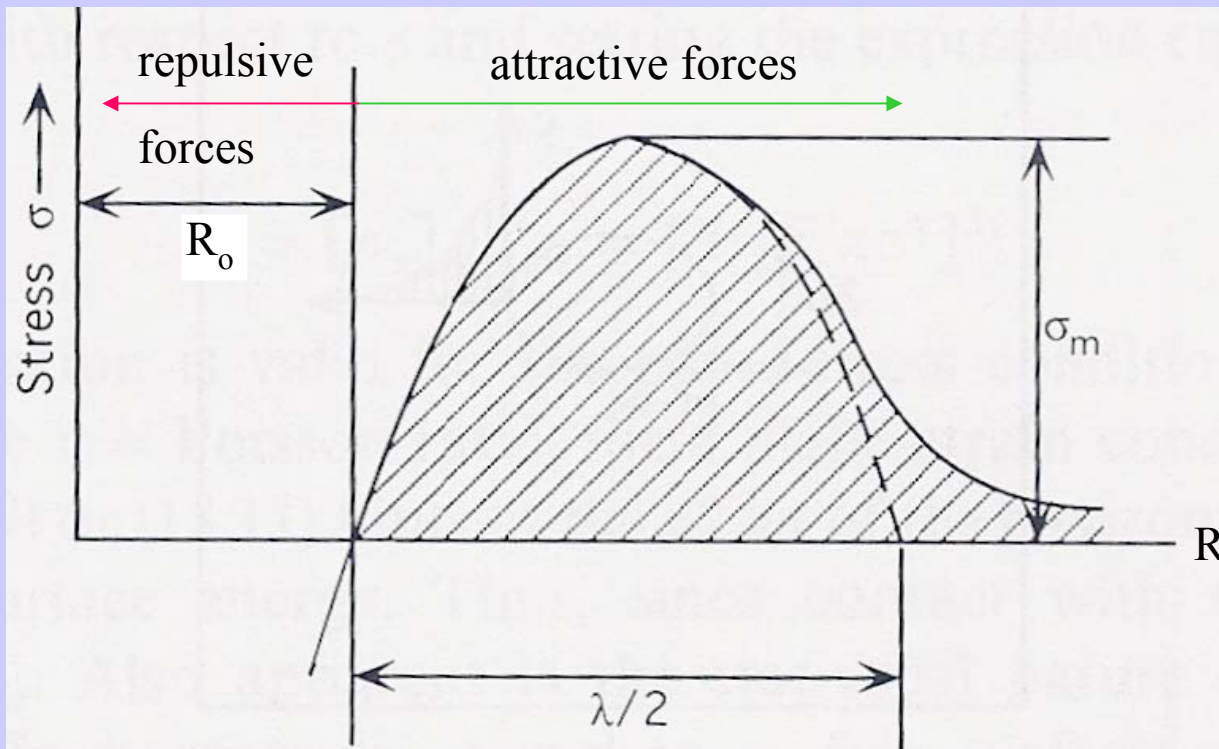
## Mechanical behavior of glass

Glasses are *brittle* materials: only recoverable strains, of the order of 0.1 %. There is no plastic deformation.



## Theoretical tensile strength

The theoretical *tensile strength* of glass,  $\sigma_t$ , corresponds to the stress needed to separate two atomic “planes”, when the *attractive forces* are at a *maximum*:



Variation of stress  $\sigma$  with increasing interatomic spacing.

(Adapted from: *Fundamentals of inorganic glasses*, A.K. Varshneya, Academic Press, 1994)

For a glass with an equilibrium interatomic spacing  $R_o$ , a Young's modulus  $E$  and a surface energy  $\gamma$ , it can be shown that:

$$\sigma_t = (E \gamma / 4 R_o)^{1/2}$$

For example, for v-SiO<sub>2</sub> ( $R_o=0.162$  nm,  $E=72$  GPa,  $\gamma=2.9$  J/m<sup>2</sup>), one has:

$$\sigma_t = 18 \text{ GPa} \sim E / 5$$

The actual experimental value (measured at 4 K, after flame polishing) was:

$$\sigma_t = 15 \text{ GPa}$$

Under normal practical conditions (at room temperature and without previous surface reconditioning), for v-SiO<sub>2</sub>, one measures only:

$$\sigma_t \sim 100 \text{ MPa}$$

more than 100 times less than the theoretical prediction.

This large difference was attributed by *Griffith* (1920) to the occurrence of microscopic (or even nanoscopic, using today's terminology) *flaws* (or *cracks*) at the surface of ordinary glass specimens, which act as *stress concentrators*, causing glass fracture at an applied stress  $\sigma_a \ll \sigma_t$ . Such flaws are usually the result of handling or abrasion.

Griffith's model was based on a previous result of elasticity theory due to Inglis (1913), which yielded an expression for the maximum value of the concentrated tensile stress near the tip of an elliptical crack of major axis  $2c$  and crack tip radius  $\rho$ :

$$\sigma_{\max} = 2 \sigma_a (c/\rho)^{1/2}$$

Fracture will occur whenever  $\sigma_{\max} \geq \sigma_t$ .

Typical example of an elliptical flaw, where the crack tip radius:

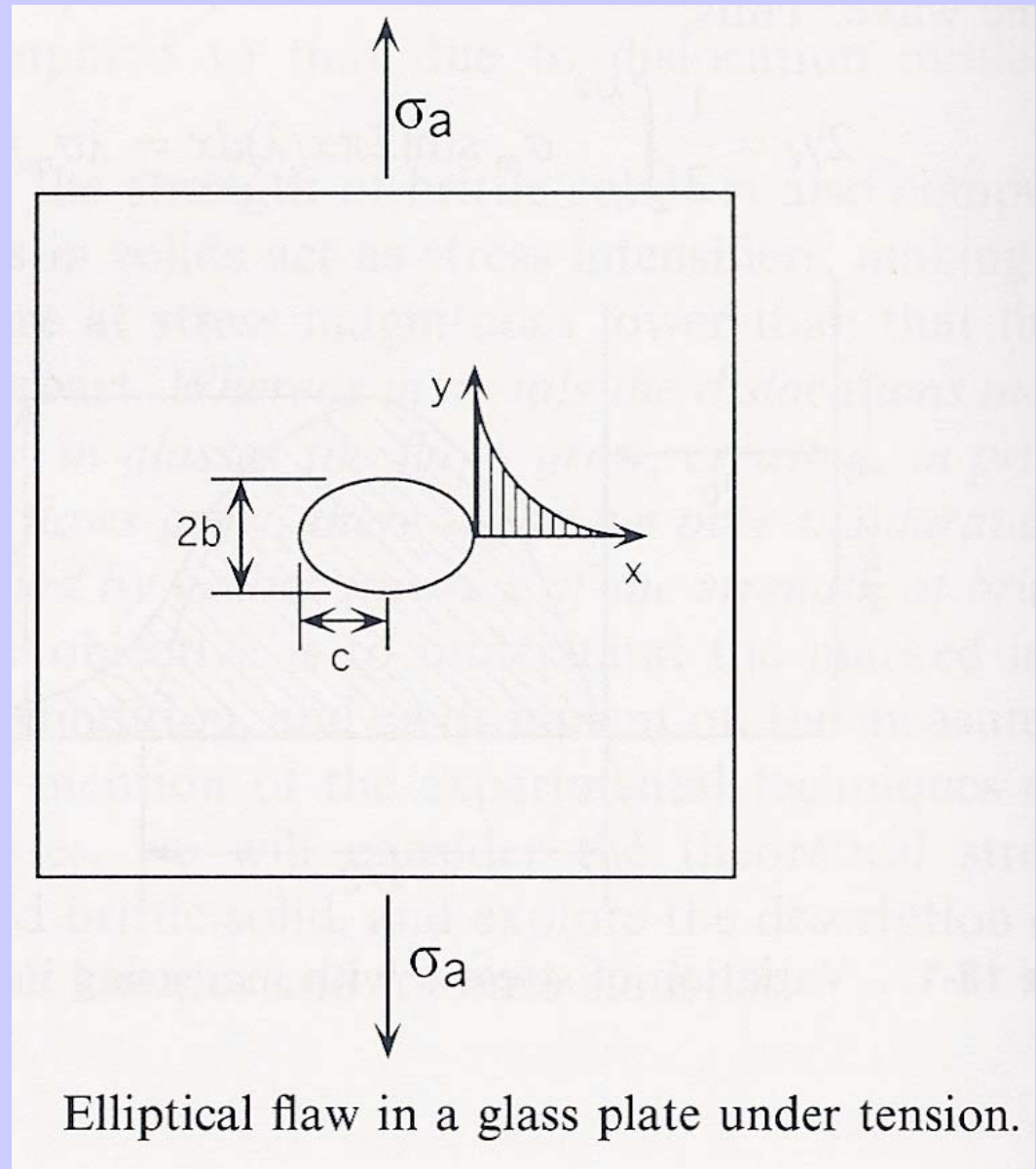
$$\rho = b^2 / c$$

is not too small.

However, for very sharp cracks (for which  $b/c \ll 1$ ), their tip radii may reach atomic dimensions. For example, if  $\rho \sim 0.2 \text{ nm}$  and  $c \sim 1 \text{ }\mu\text{m}$ , the *stress concentration factor*:

$$2 (c/\rho)^{1/2} = \sigma_{\text{max}} / \sigma_a$$

will be  $\sim 140$  .



(Adapted from: *Fundamentals of inorganic glasses*, A.K. Varshneya, Academic Press, 1994)

A combination of Griffith's and Inglis' theories eventually leads to the equation:

$$\sigma_t \sim (4 E \gamma / \rho)^{1/2}$$

Comparing this equation with the theoretical estimate of  $\sigma_t$ , one obtains the following estimate of the crack tip radius:

$$\rho \sim 16 R_0$$

In the case of uniaxial compression, Griffith's model predicts that the theoretical compressive strength,  $\sigma_c$ , equals 8 times the tensile strength, for infinitely sharp cracks ( $b/c \rightarrow 0$ ).

The mechanical strength of brittle materials like glass has a *statistical* nature, with varying populations of cracks from sample to sample (number and size), which account for the usual scattering in measured strength data.

A statistical criterion derived by Weibull (Weibull statistics, similar but not equal to the normal gaussian distribution) is the most suitable in this case.

In 1957, Irwin introduced the *stress intensity factor*,  $K$ :

$$K = Y \sigma_a c^{1/2} \quad (\text{in Pa}\cdot\text{m}^{1/2}, \text{ or Nm}^{-3/2})$$

where  $Y$  is a shape factor; for surface flaws in a glass,  $Y \sim \pi^{1/2}$  and  $K \sim \sigma_a(\pi c)^{1/2}$ .

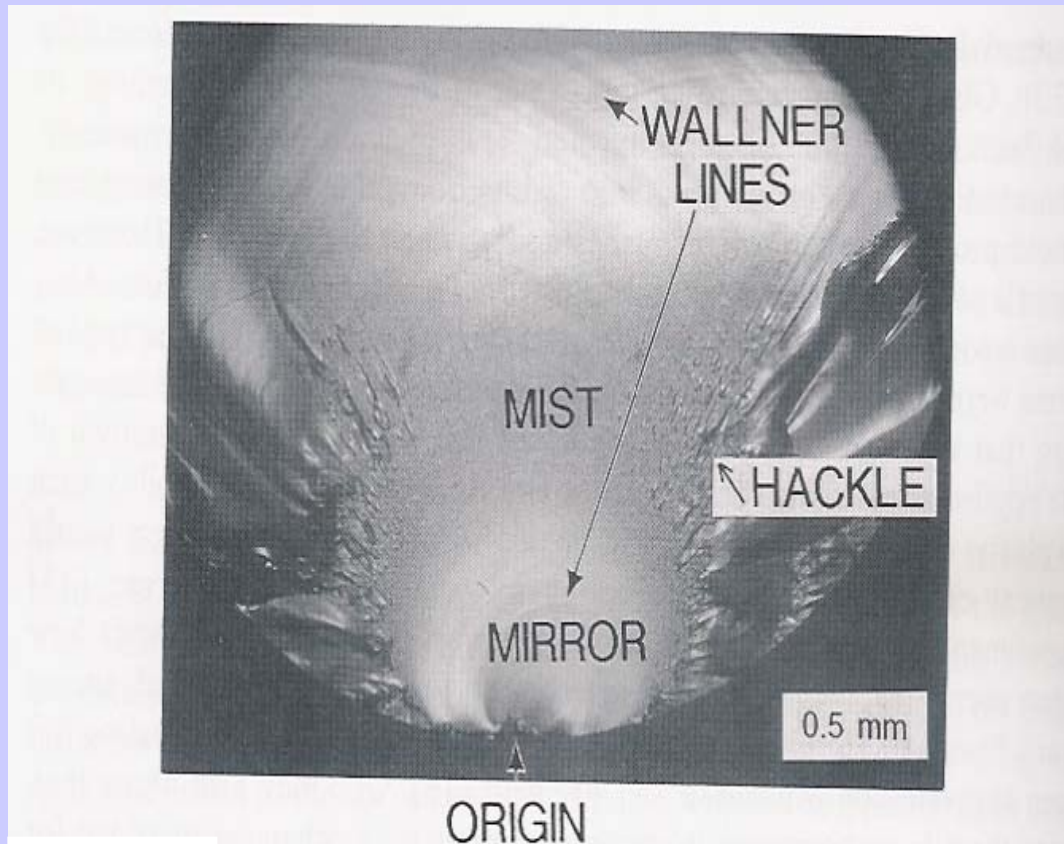
$K$  is, in fact, a measure of the stress intensity near the crack tip. Taking  $\pi^{1/2} \sim 2$ , Inglis' equation becomes:

$$\sigma_{\max} \sim K / \rho^{1/2} \quad \Leftrightarrow \quad K \sim \sigma_{\max} \rho^{1/2}$$

indicating that, for a given applied stress and crack length  $2c$ , a smaller tip radius leads to glass failure at a lower stress level.



**Typical shell-like shape of glass fracture.** Next to the origin of the catastrophic failure, where the *critical flaw* was located, lies a shiny region called the *mirror*, which ends in a misty texture called the *mist*. The mist begins to form when the fracture front approaches its terminal velocity, usually  $\sim 60\%$  of the transverse acoustic velocity,  $v_t \sim 3$  km/s. The rougher *hackle* region surrounding the mist represents the motion of the fracture front at terminal velocity, after which it decelerates, producing *Wallner lines*.



Fracture surface of a soda lime silica glass rod broken in bending at room temperature. (80 $\times$ .)

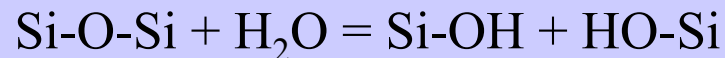
The *fractographic* analysis of broken glass may yield very important information regarding the cause of the fracture.

(Adapted from: *Fundamentals of inorganic glasses*, A.K. Varshneya, Academic Press, 1994)

For glass specimens with pre-existing surface flaws (usually due to handling), fracture is controlled by **crack propagation**. When a critical stress level, the applied fracture stress,  $\sigma_a^{\text{fr}}$ , is reached for the largest flaw with proper orientation (called the *critical flaw*, of length  $2c_{\text{cr}}$ ), Griffith's criterion may be expressed as  $\sigma_a \geq \sigma_a^{\text{fr}} = (E\gamma/c_{\text{cr}})^{1/2}$ , indicating *catastrophic failure*.

However, if  $\sigma_a < \sigma_a^{\text{fr}}$  (for example, for a glass window under a static load), there will be no failure. Unless the glass part exhibits *static fatigue*, or delayed failure, which consists of *slow crack growth*, under a sub-critical, static applied stress, until the most severe crack reaches the length  $2c_{\text{cr}}$  and brittle fracture occurs.

Such phenomenon, in common silicate glasses, is usually attributed to *stress corrosion* at the crack tip, where strained bonds are broken, often by the combined action of stress and atmospheric humidity:



forming a gel-like region where the crack is able to propagate under sub-critical conditions. It thus becomes important to follow the velocity of crack propagation as a function of the stress intensity factor  $K$ , for the three possible independent *modes of crack propagation*.