# Fractal Geometry Applied To Fracture

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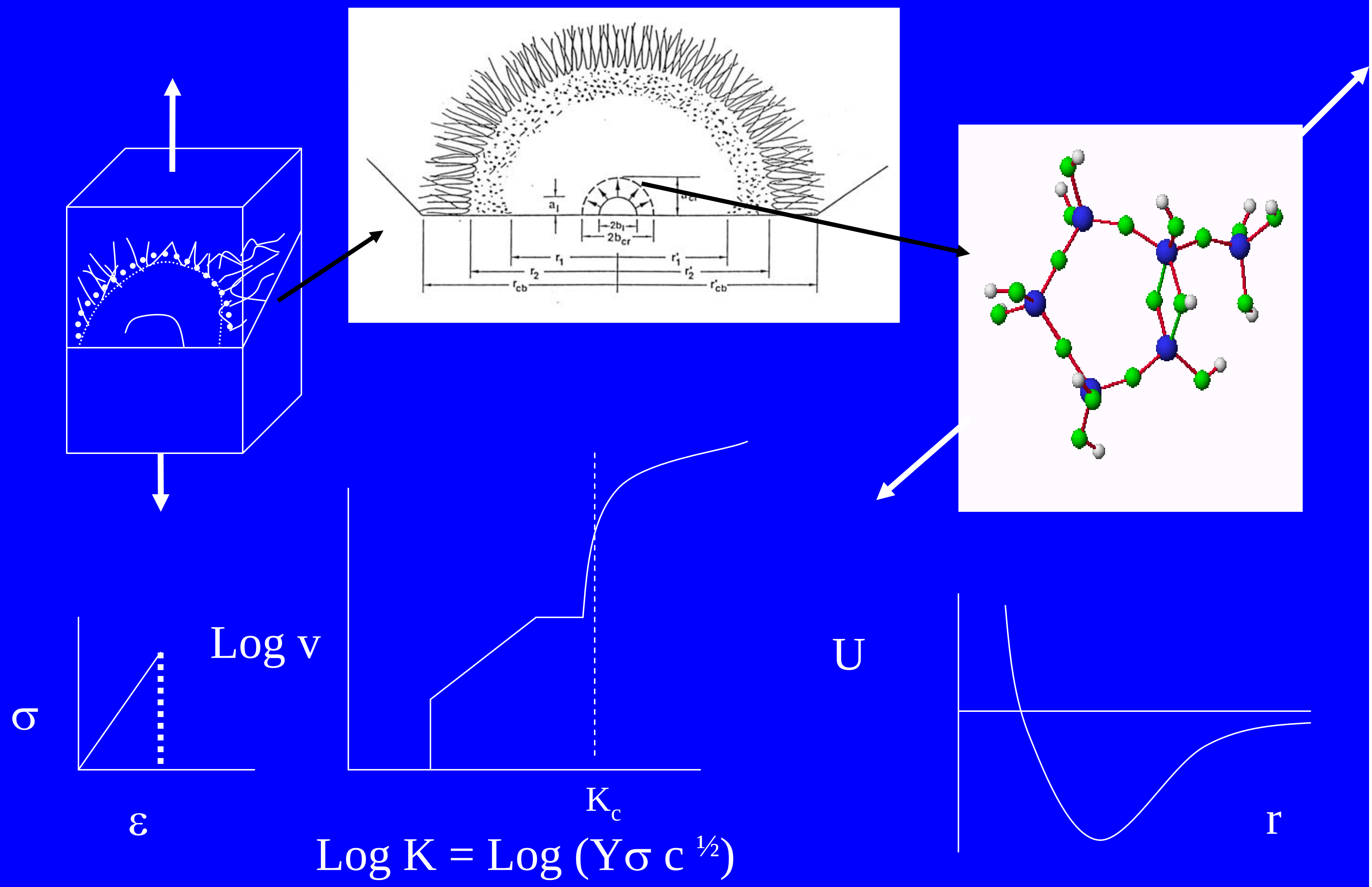
# Bond Breaking Leads to Characteristic Features



# Outline

- **Experimental Observations** - One View of Fracture

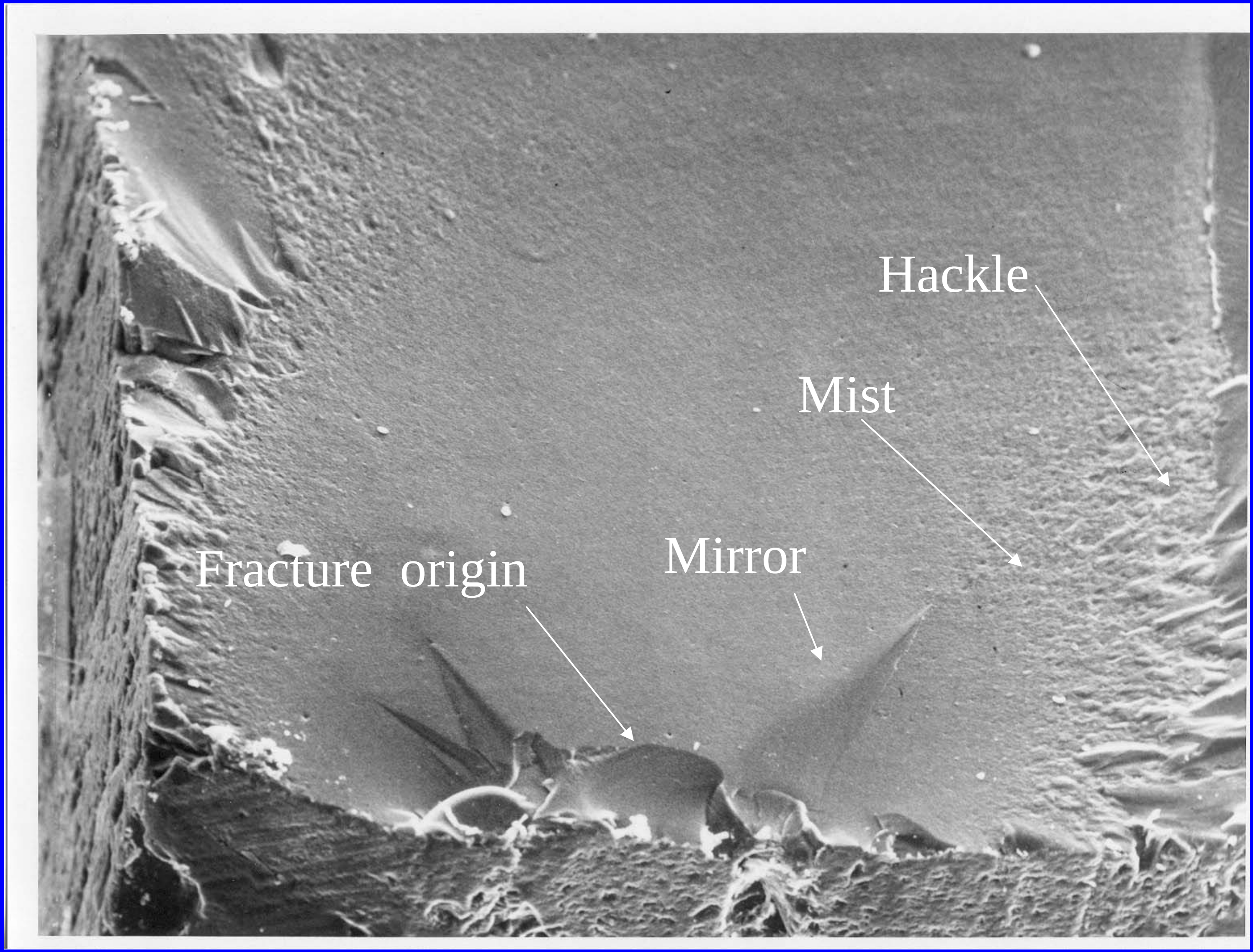
- **Fundamental Questions About Fracture** -
How does a crack propagate at all length scales?

- **Experimental Tools** - Fractography (FSA) , Fracture Mechanics (FM) & Fractal Analysis (FA)
Fractoemission (FE), Crack Velocity Measurements

- **Analytical Tools** - Quantum Mechanics (QM), Molecular Dynamics (MD), ab initio, Monte Carlo, FEM, FD

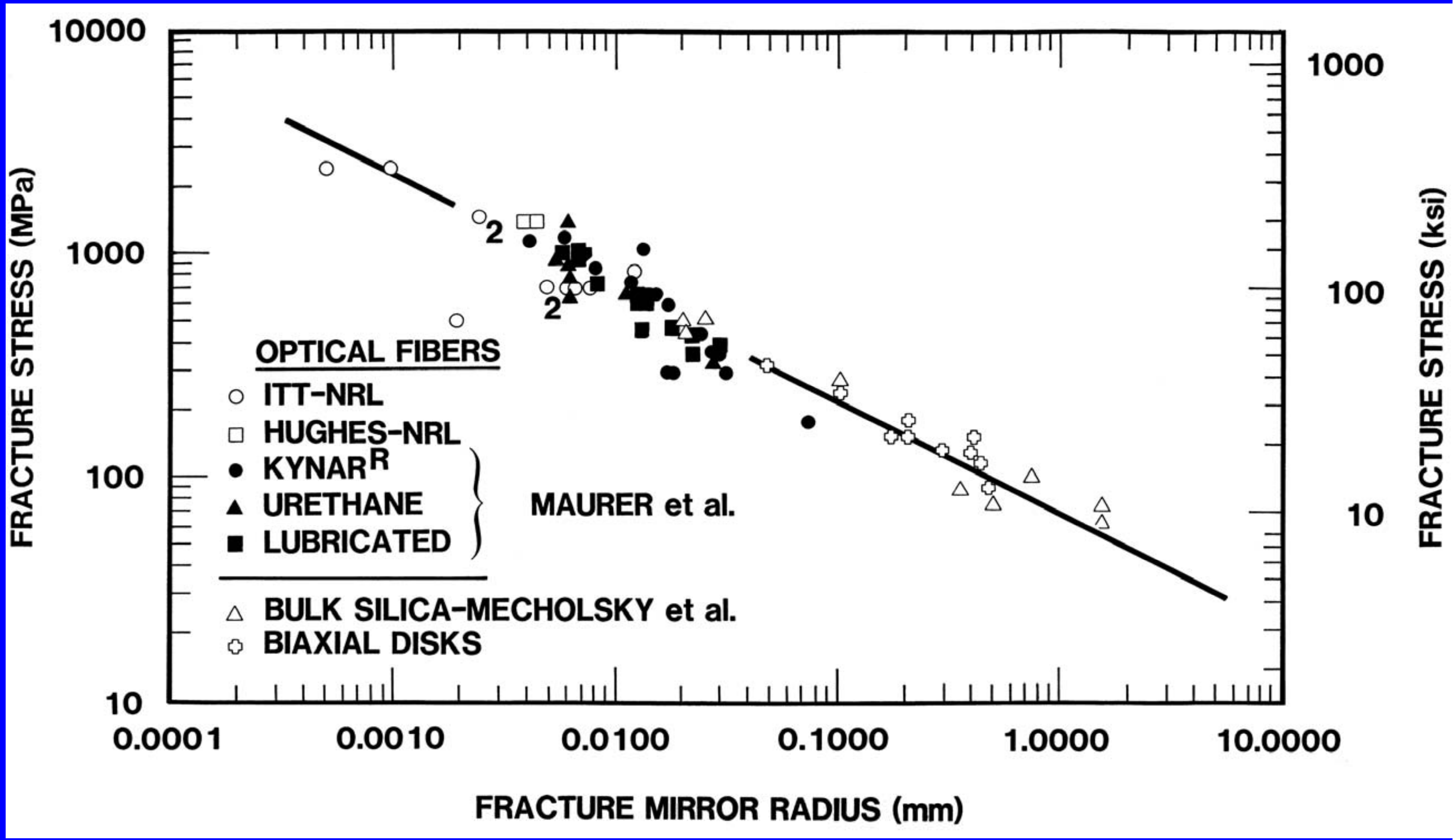
- **Conclusions** - FSA, FM, FA, MD & QM combine to form model of the scaled fracture process.

# Characteristic Markings Are Observed on the Fracture Surface



# Relationship Holds For Large Size & Stress Range

$\sigma\, r^{1/2} = \text{constant}$




J.J. Mecholsky, Jr., Fractography of Optical Fibers, in ASM Engineered Materials Handbook, 4, Ceramics and Glasses, Section 9: Failure Analysis, (1992).

# Energy & Geometry Are Related

# In The Fracture Process

$$K_C = Y\sigma\,(c)^{1/2} = (2\,E\,\gamma)^{1/2}$$

$$2\,\gamma = a_0\ [ED^*]$$

$\gamma$ = fracture energy

E = Elastic Modulus

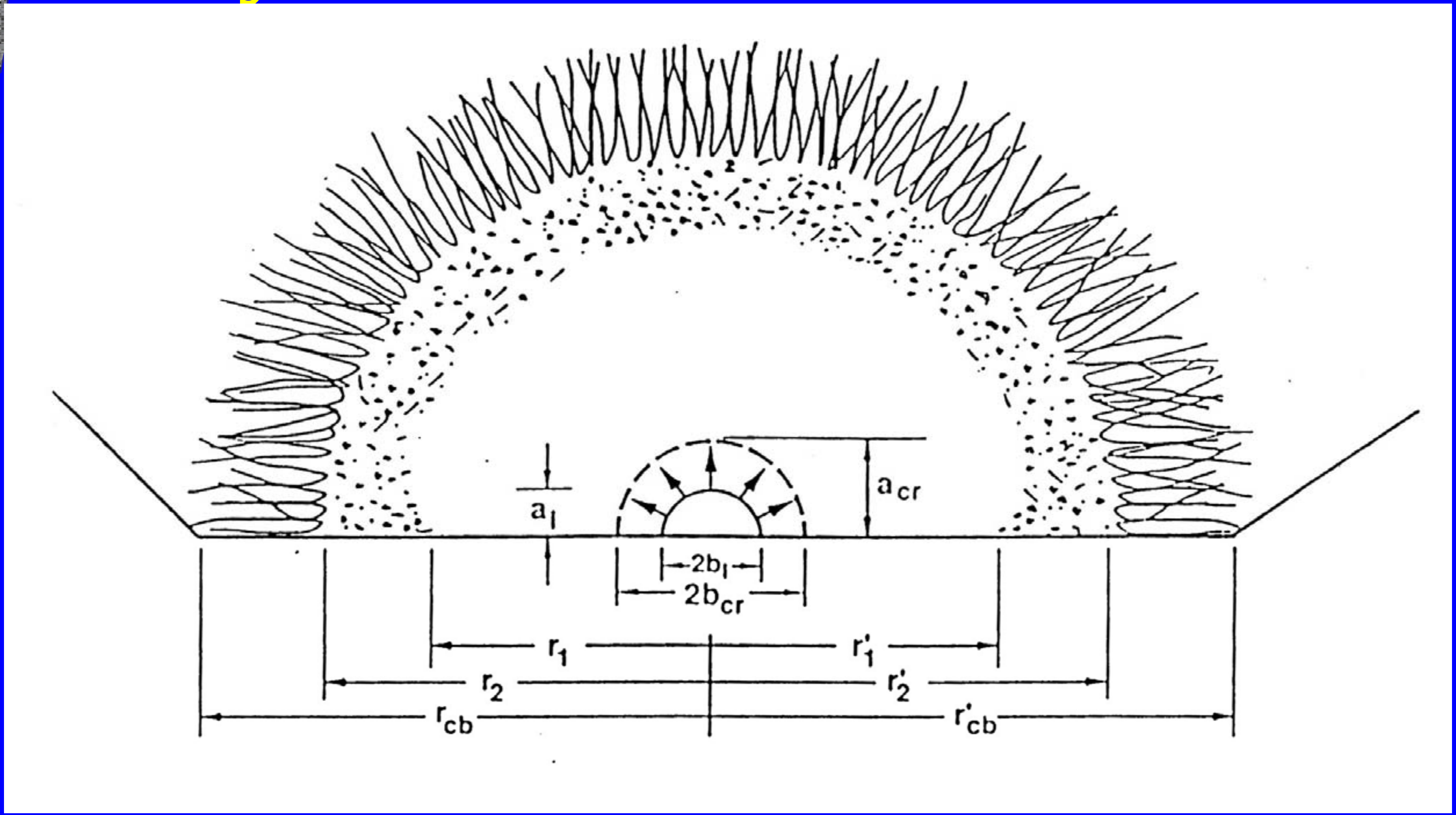
Y = Geometry & Loading Constant

$\sigma$ = Fracture Stress (Strength)

C = Critical Crack Size

$K_C$ = Fracture Toughness

# Characteristic Features Aid Failure Analysis




$$K_C = Y\,\sigma\,(c)^{1/2}$$
$$c = (a\,b\,)^{1/2}$$

$$K_{Bj} = Y\,\sigma\,(r_j)^{1/2}$$
$$r_j\,/\,c = \text{constant}$$

# Fracture Mechanics & Fractography Provide A Framework for Quantitative Analysis

$K_{IC} = Y\ \sigma\ c^{1/2}$ Crack Boundary

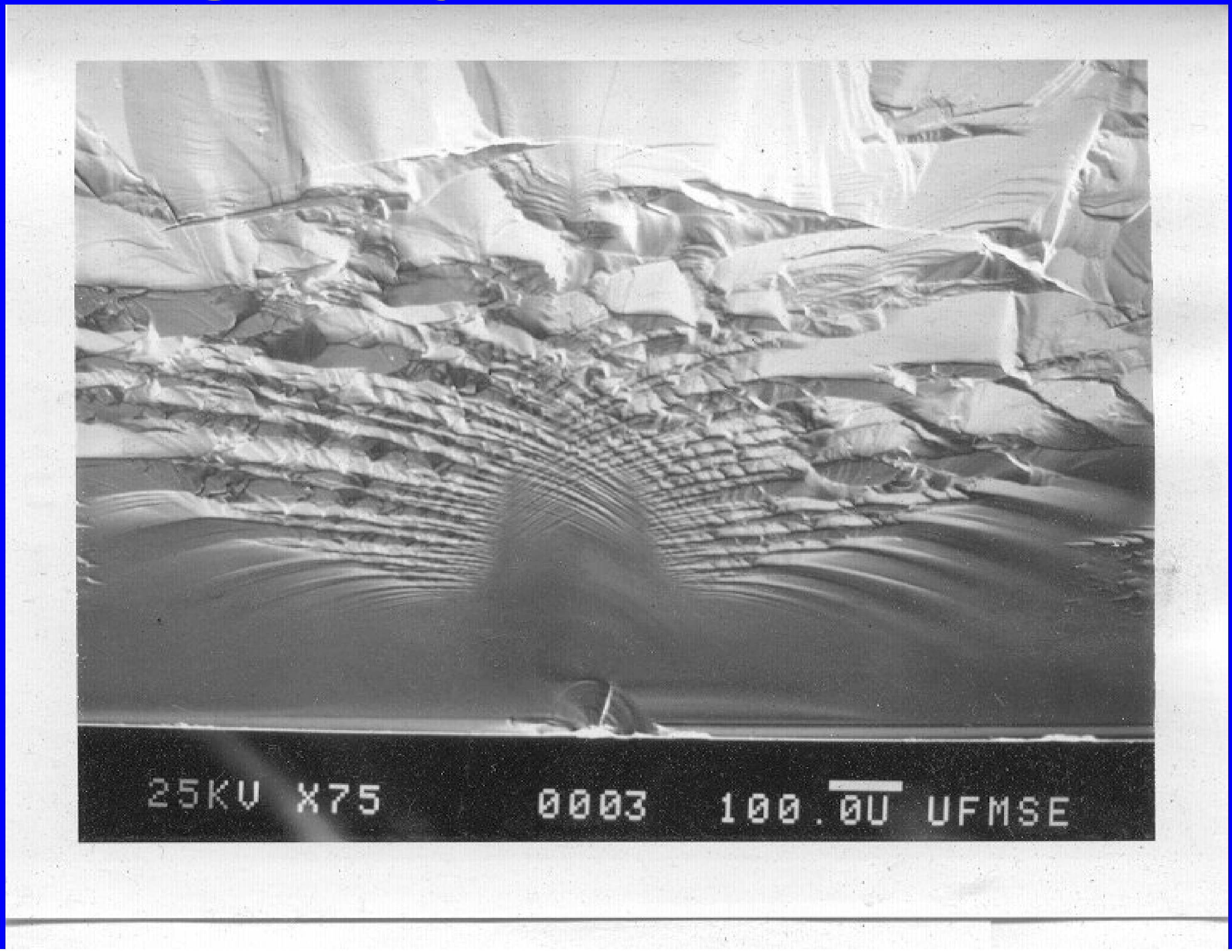
$K_{B1} = Y_1\ \sigma\ r_1^{1/2}$ Mirror-Mist Boundary

$K_{B2} = Y_2\ \sigma\ r_2^{1/2}$ Mist-Hackle Boundary

$K_{B3} = Y_3\ \sigma\ r_3^{1/2}$ Crack Branching Boundary

$[c/r_j = \text{constant}]$

# FSA Can Be Applied To Single Crystals



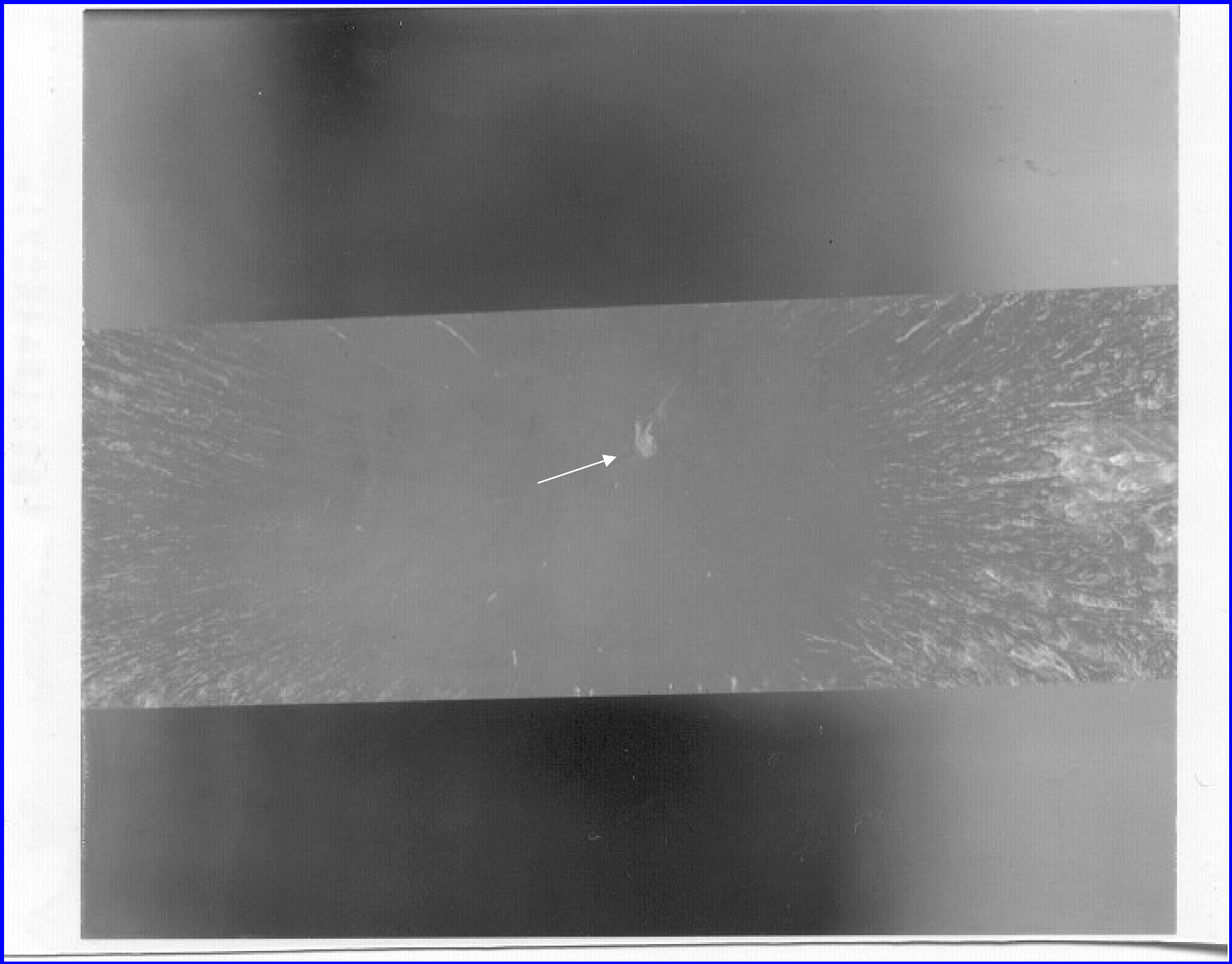
# Slate Fracture Surface On Window Sill at Kimbull Union Academy

Courtesy of Prof. Yet-Ming Chiang

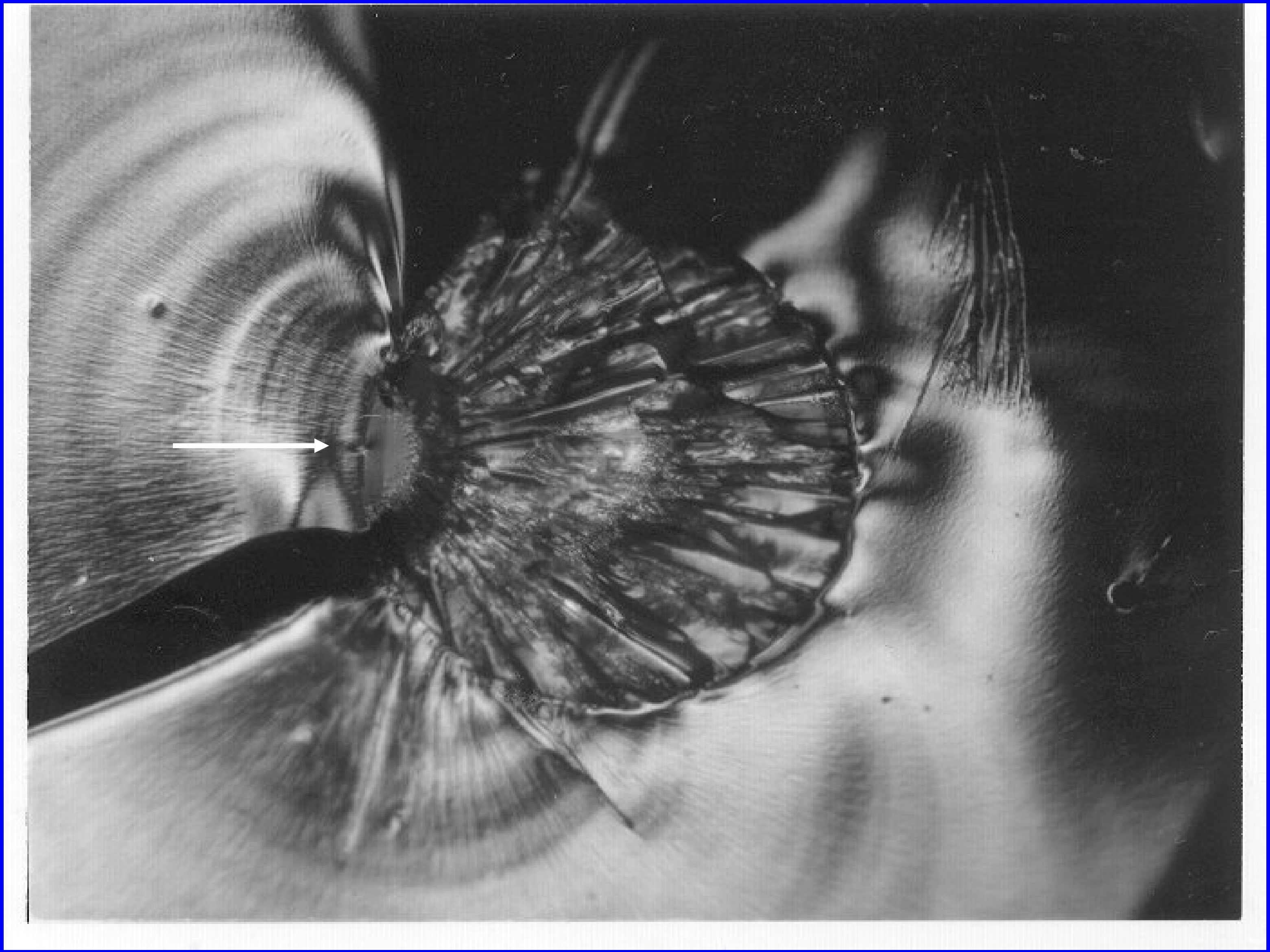
# Fracture Markings Can Last 4000 Years

Titi's Sarcophagus - Egypt c. 2500 BC   [Prof. Greenhut]

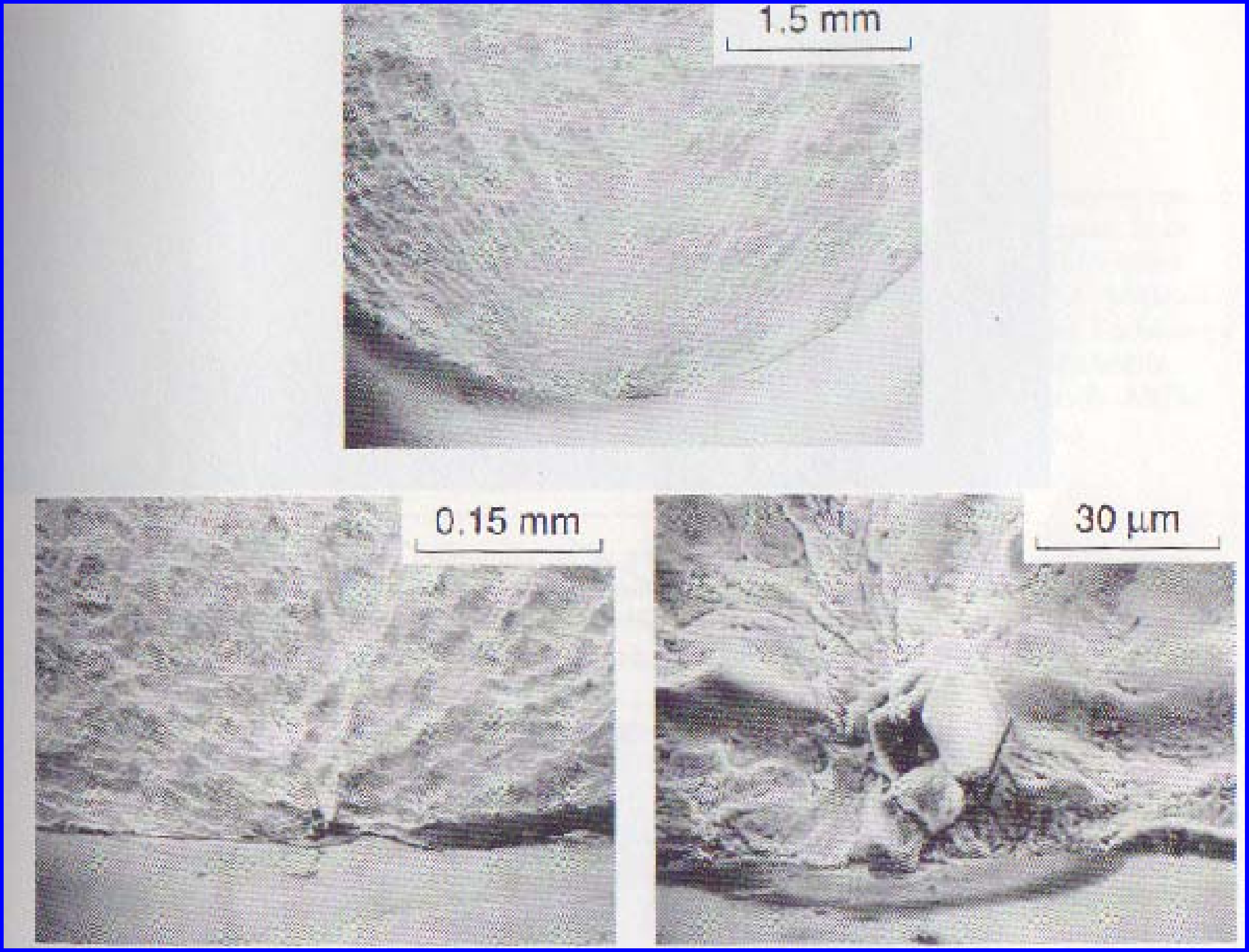
# Epoxy Fracture With Glass Fiber As Origin



# Glass Fiber Fractures Within Epoxy Matrix



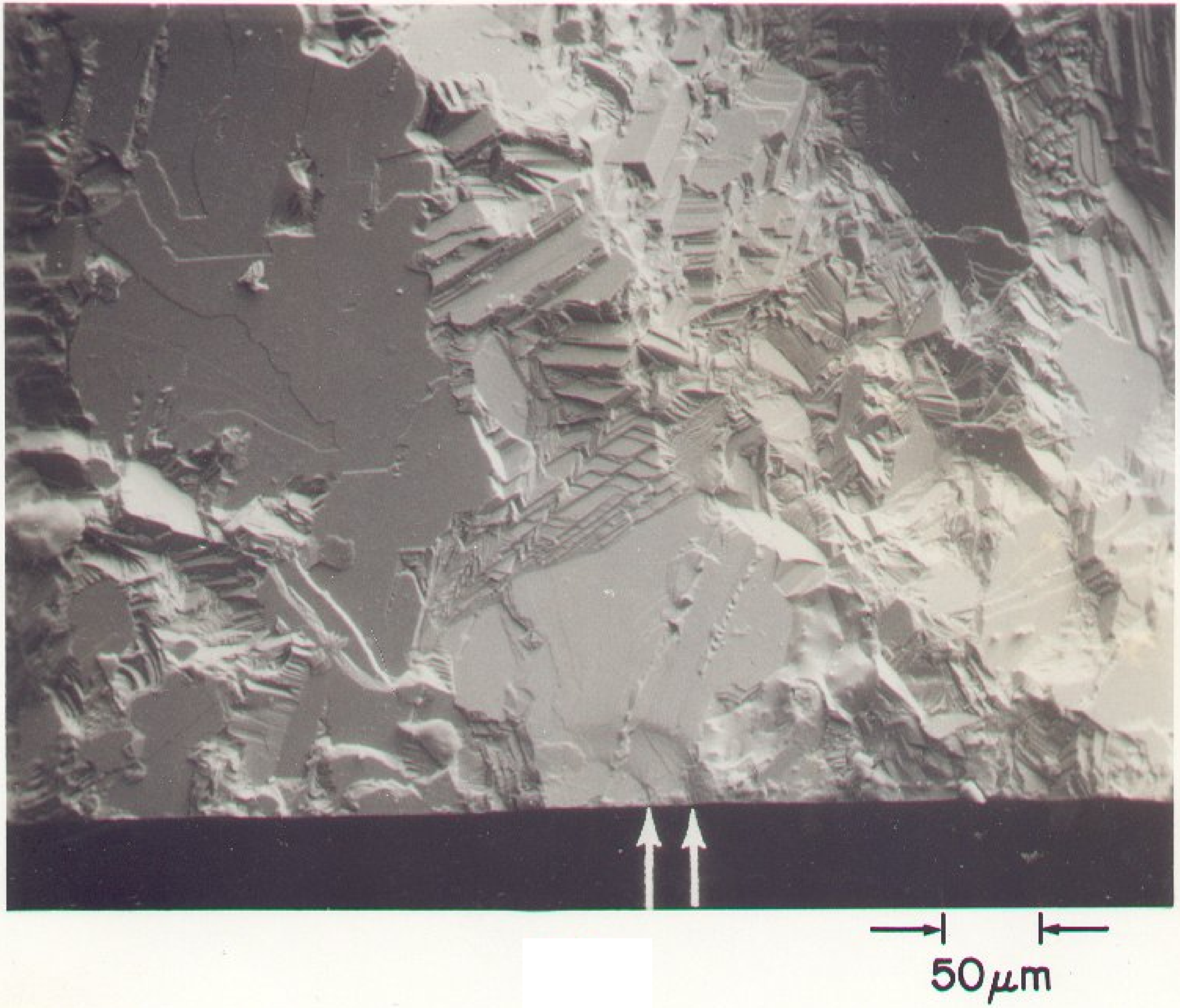
# Fracture In Steel Shows Characteristic Features



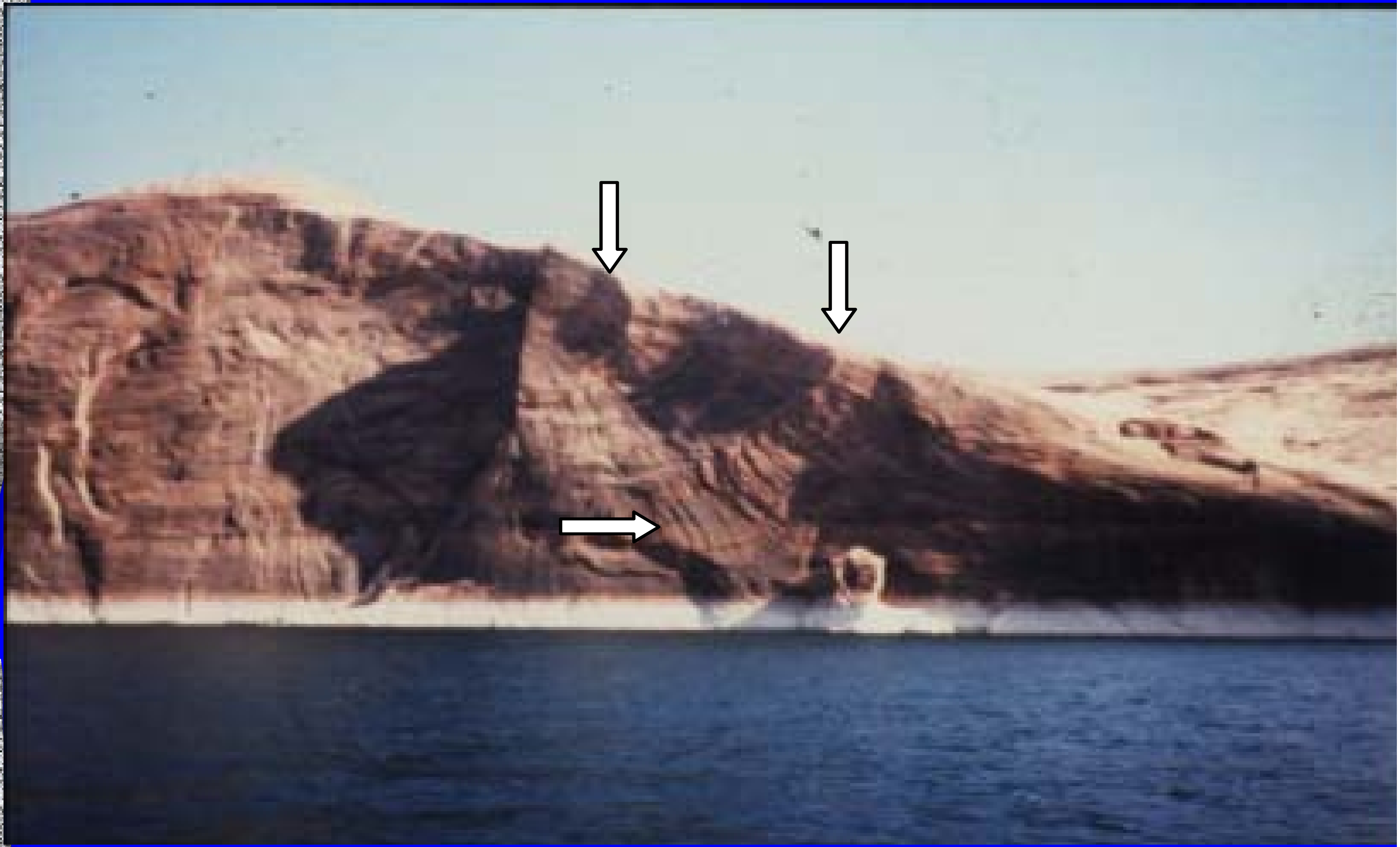

AISI
4340

$\sigma$ =790MPa

# Brittle Fracture Is Observed At Many Length Scales

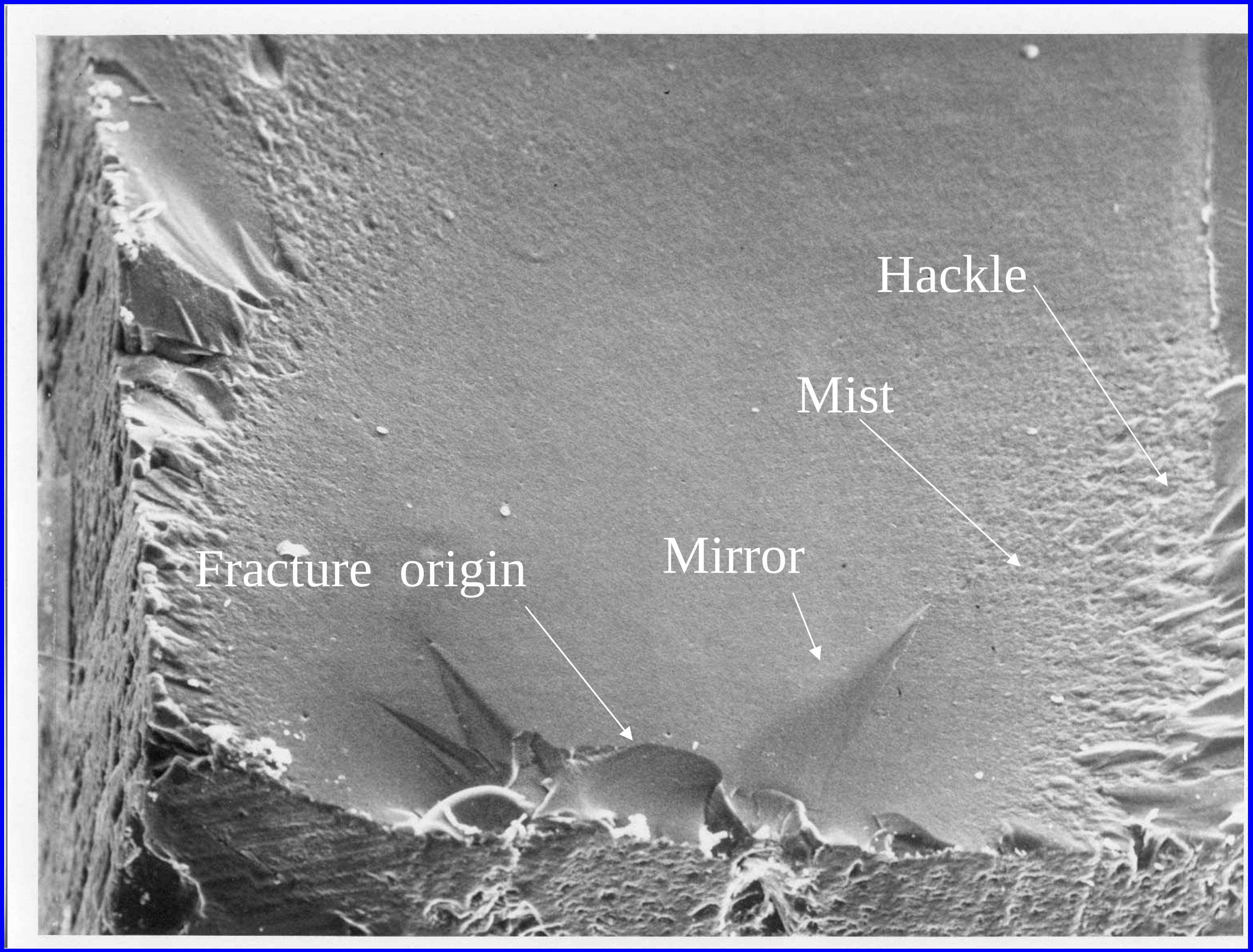


# Brittle Fracture Can Be Observed At Many Length Scales

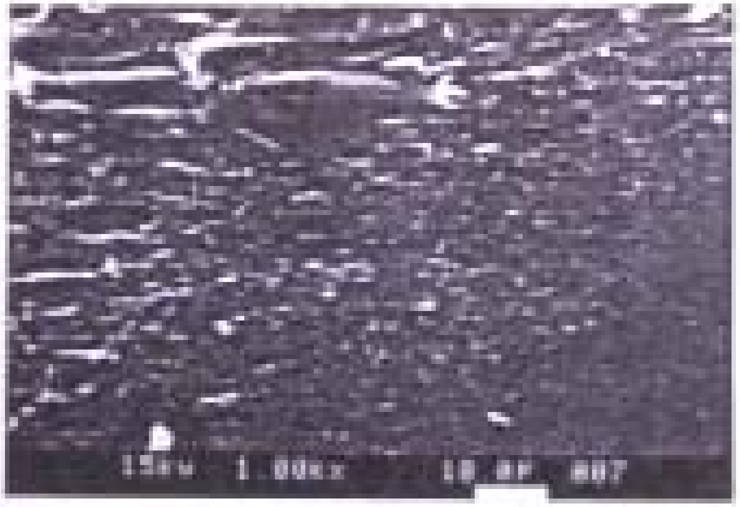


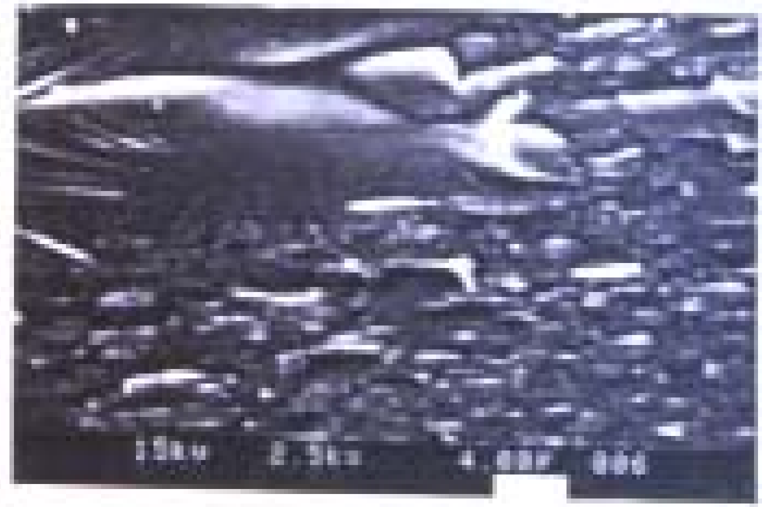
Courtesy of Dr. Darryl Butt [UF]

# Characteristic Markings Are Observed on the Fracture Surface



# Mist and Hackle Appear Similar in Shape



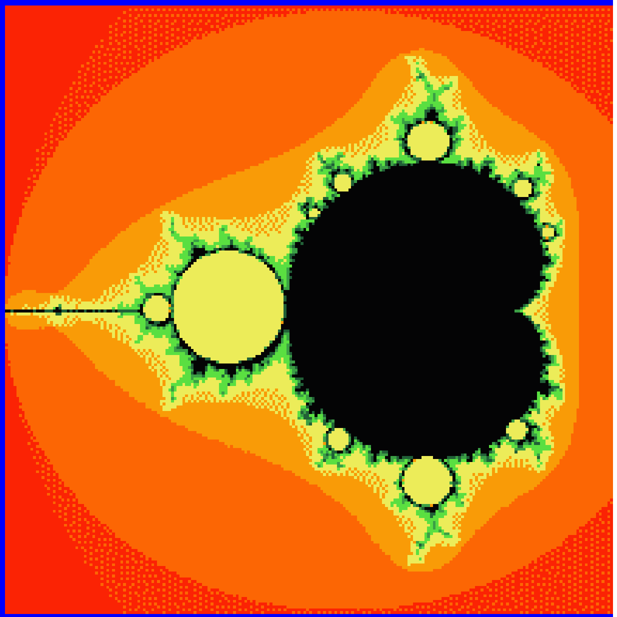


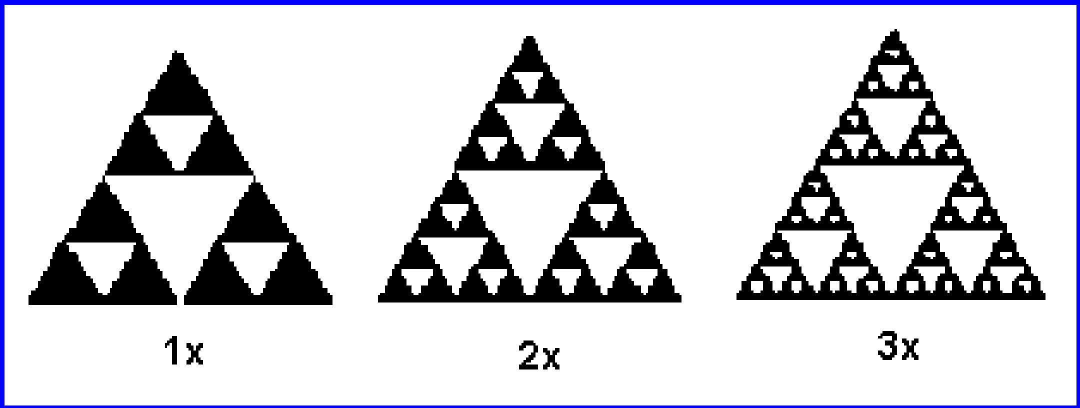
# Many Observations Lead To The Conclusion Of Fractal Fracture

- Scaling Behavior
- Self-Similarity In A Plane
- Scale Invariance

# Fractal Geometry – A Renewed Math

- Characteristics:
  - Non-differentiable
  - Defined by a Fractional Dimension (i.e. 1.3, 2.4, etc.)
  - Self-similar features
  - Scale invariant features






–http://math.rice.edu/~lanius/frac/

## SCALING CAN BE DESCRIBED BY FRACTAL GEOMETRY

**Fractal Geometry** is

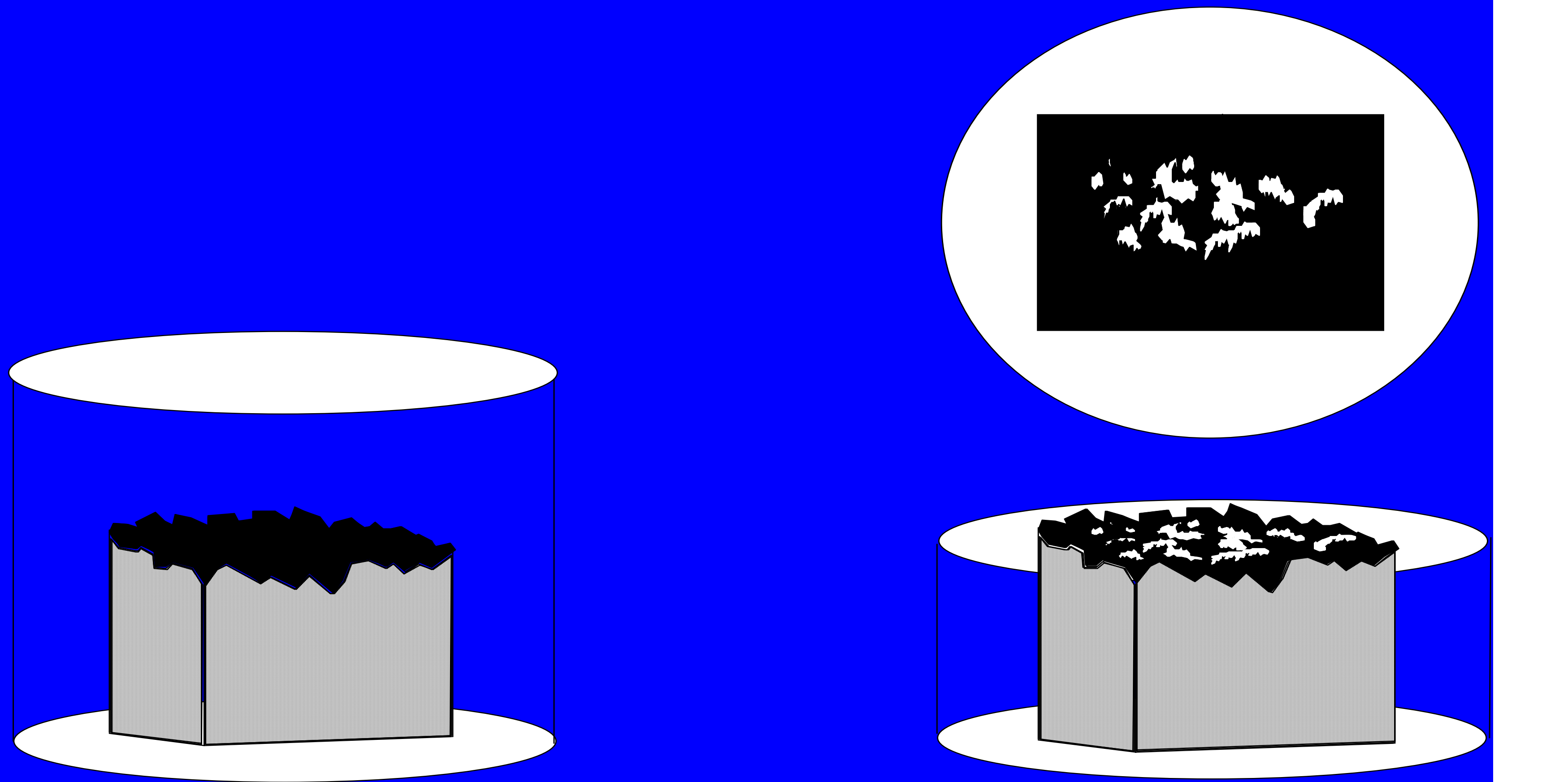
- a non-Euclidean geometry that exhibits

- **self-similarity** (or self-affinity) &

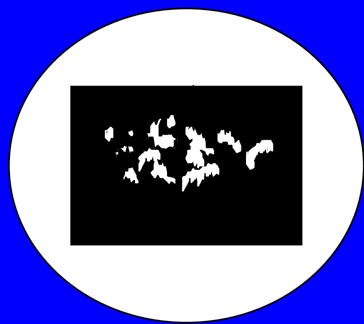
- **scale invariance** and is characterized by the

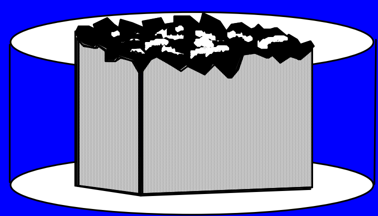
- **fractal dimension, D.**

# Replicas Provide Multiple Slit Islands for Analysis

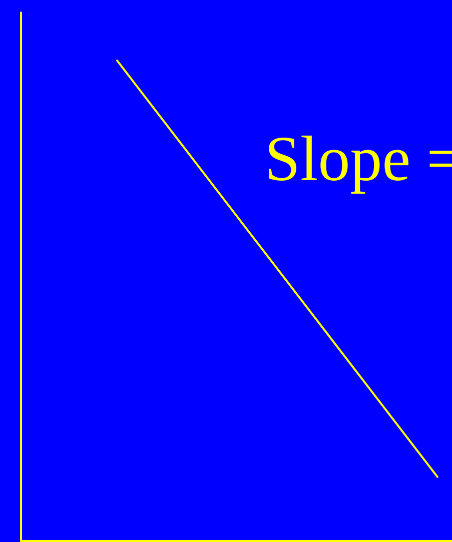


# FRACTAL DIMENSION IS MEASURED ALONG CONTOUR



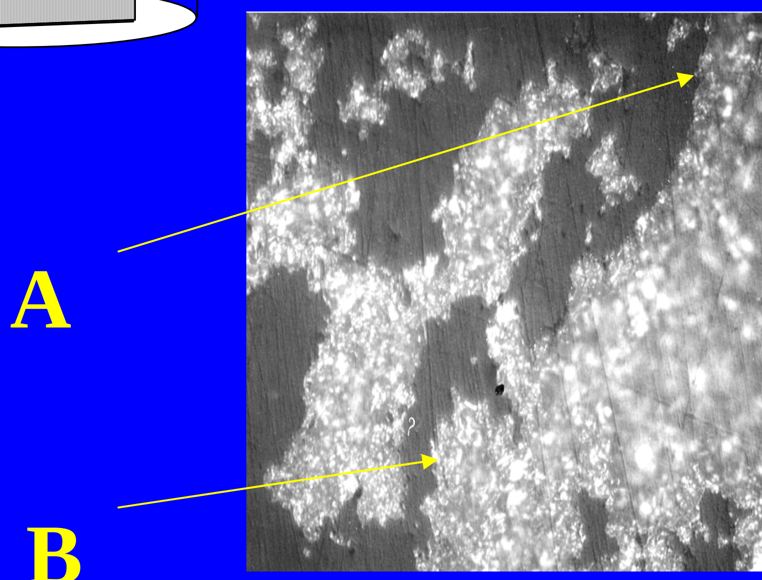


Log Length (A-B)



Slope = -D*

Log (Ruler Length)




A-B = Slit Island Contour

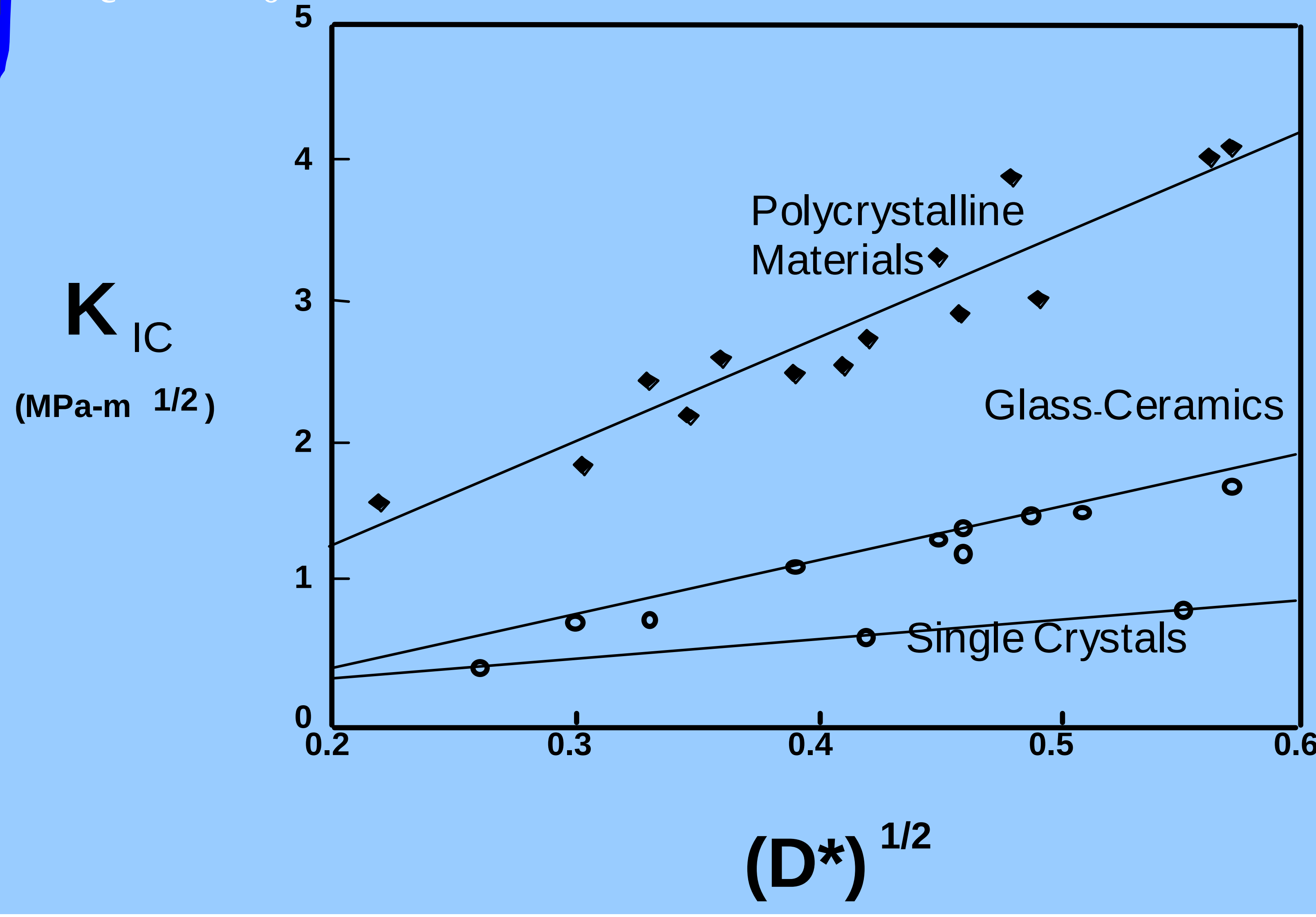
# Fractal Dimension Varies For Different Materials

| Material Class | D* (dimensionless) |
|---|---|
| Single Crystals | 0.07-0.12 |
| Glasses | 0.07-0.1 |
| Glass Ceramics | 0.06-0.3 |
| Polycrystalline Ceramics | 0.06-0.35 |
| Polymers | 0.2-0.29 |
| Metals | 0.06-2.5 |

J. J. Mecholsky, Jr., Fractography, Fracture Mechanics and Fractal Geometry: An Integration, Ceram. Trans. 64, in Fractography of Glasses and Ceramics III, eds. J. P. Varner, V.D. Frechette, & G. D. Quinn, Am. Ceram. Soc. (1996).

# Toughness Increases With Fractal Dimension

$K_c = E\, a_0^{1/2}\ D^{*1/2}$



## D* Is Related To Flaw-to-Mirror Size Ratio

$$K_{IC} = E\,(a_0^{1/2})\,D^{*1/2} = Y\,\sigma_a\,(c)^{1/2}$$

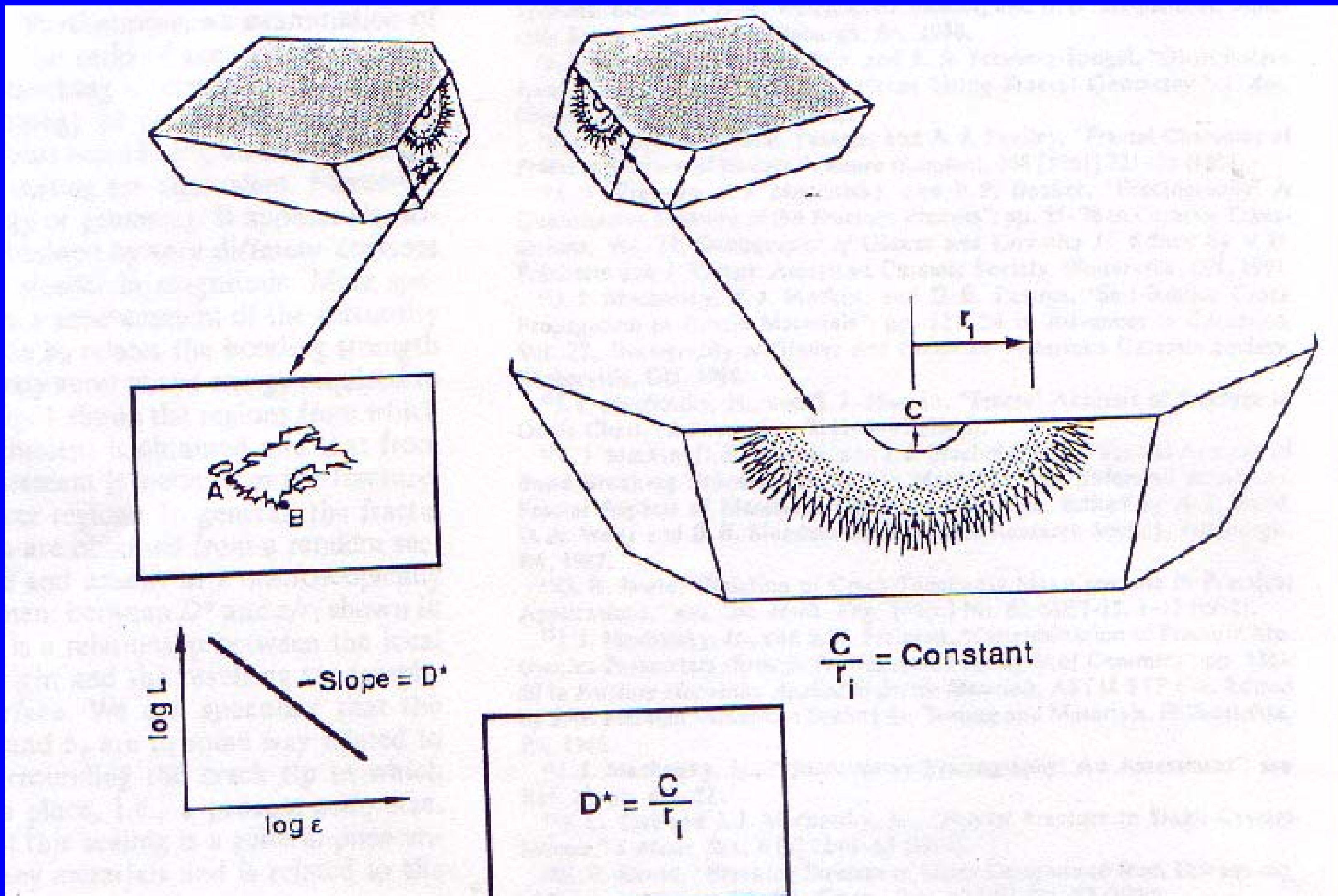
$$K_{B1} = E\,(b_0^{1/2}) = Y_1\sigma_a\,(r_1)^{1/2}$$

[if $a_0 = b_0$ and $Y = Y_1$]

then,

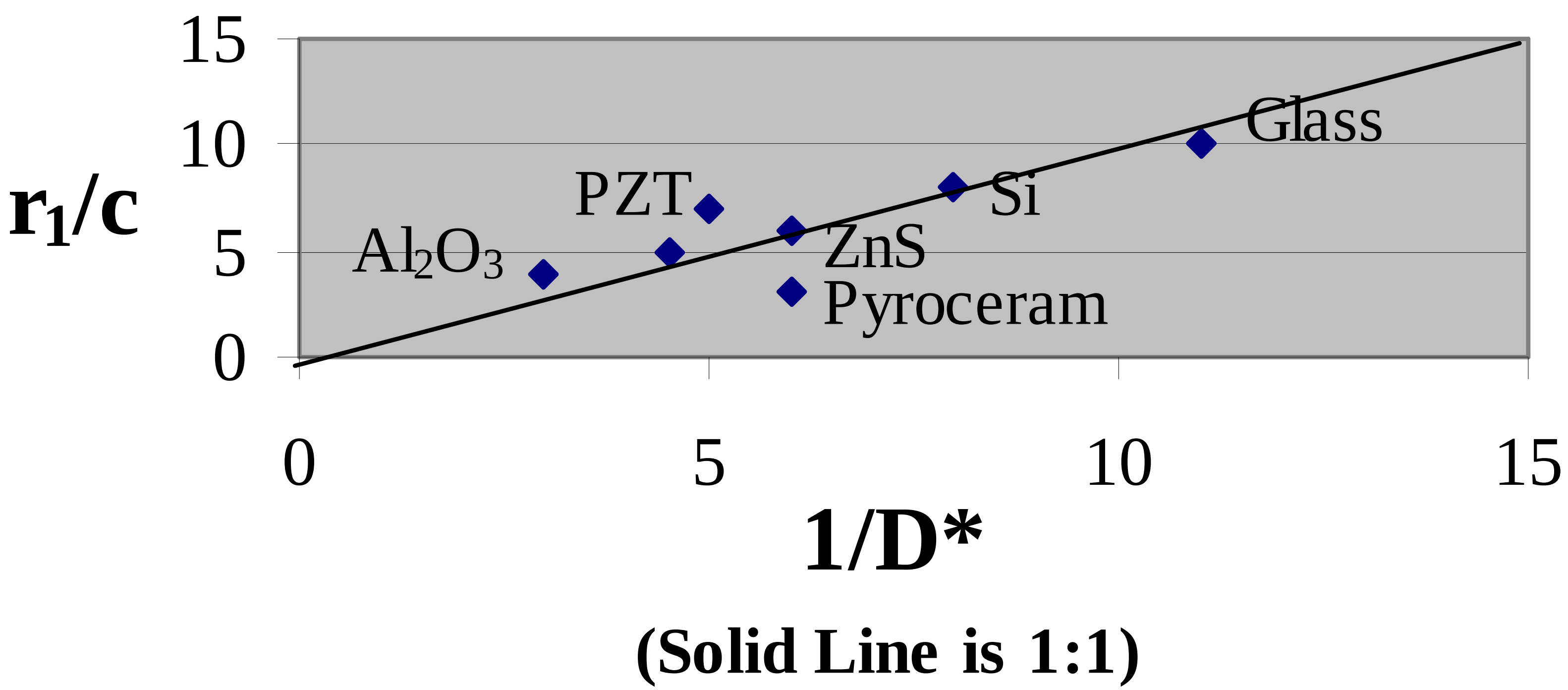
$$D^* = c/r_1$$

# D* Is Related To Flaw-to-Mirror Size Ratio



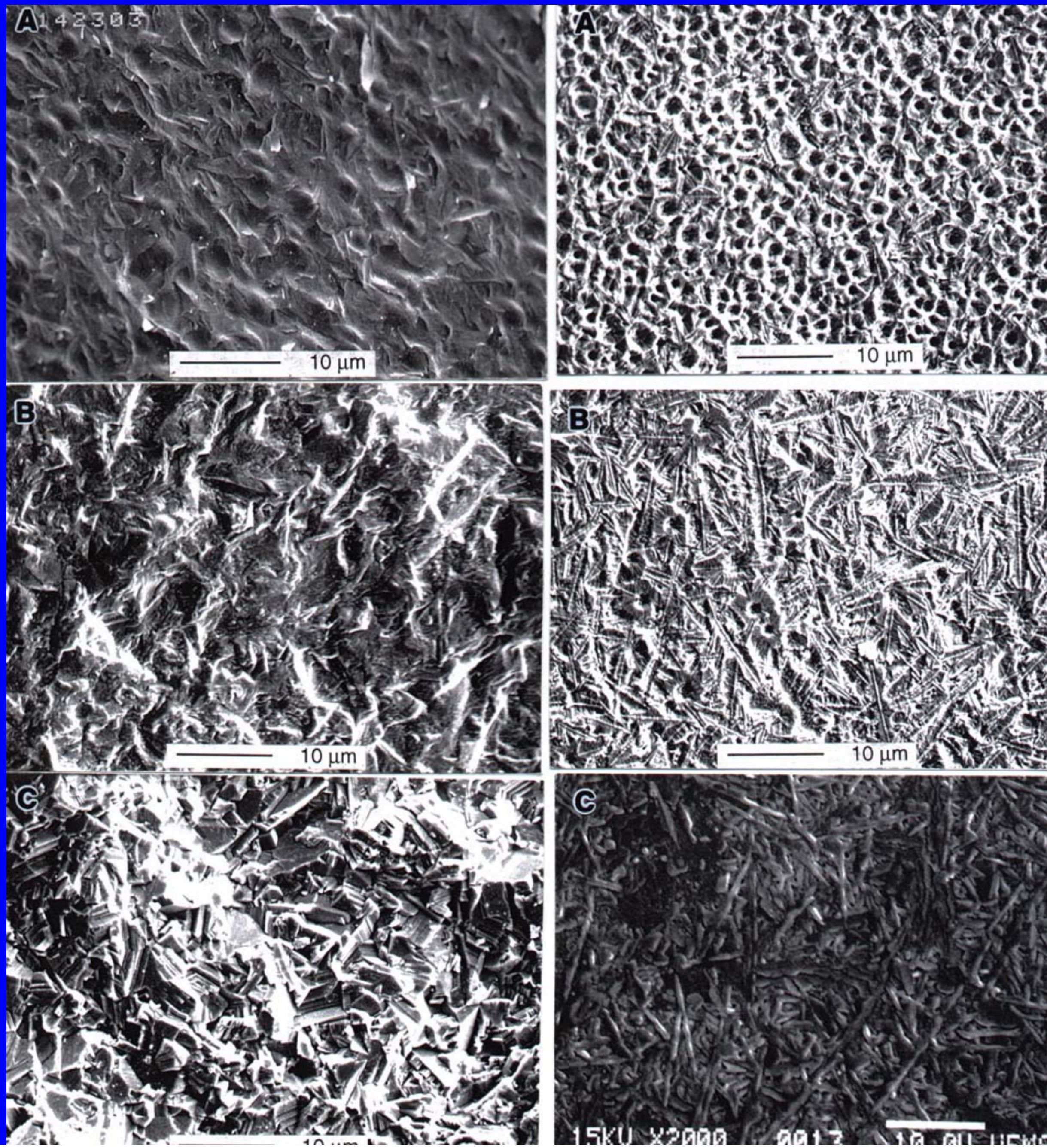

Mecholsky & Freiman J ACerS 74[12]3136 (1991)

# D* Is Related to Mirror-to-Flaw Size Ratio




Mecholsky & Freiman J ACerS 74[12]3136 (1991)

Fracture Surfaces Polished Surfaces




D*=0.13

$K_c$=1.0 I

AR= 1.4

D*=0.16

$K_c$=1.3 I

AR= 3.6

D*=0.26

$K_c$=2.2 I

AR = 8

I =$MPam^{1/2}$

$3BaO•5SiO_2$

Glass:

$K_c$ = 0.7 I

D* = 0.1

# Energy & Geometry Are Related

# In The Fracture Process

$$K_C = (Ea_0^{1/2})\ D^{*1/2} = (2\,E\,\gamma)^{1/2}$$

$$2\,\gamma = a_0\ [ED^*]$$

$\gamma$ = fracture energy

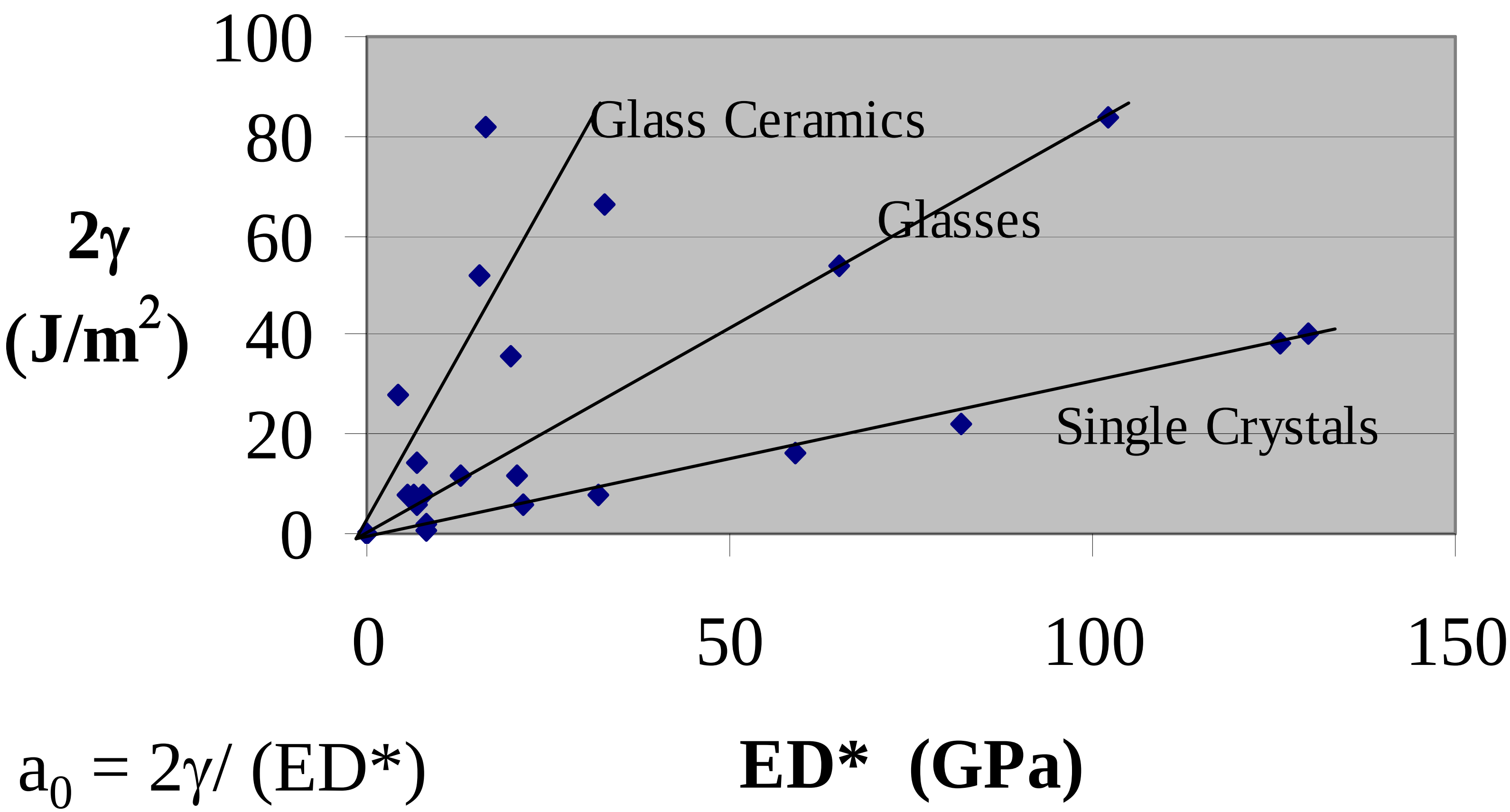
E = Elastic Modulus

$D^*$ = Fractal Dimensional Increment

$a_0$ = Characteristic Structural Parameter

$K_C$ = Fracture Toughness

# Fracture Behavior Appears Related To Material Class




$a_0 = 2\gamma / (ED^*)$

# Fractal Geometry Has Been Used In Failure Analysis

- Silicon [J. Mater Res **6**,1248 '91 ]
- Ocala Chert [J Mater Sci Ltrs **7**, 1145 '88]
- Intermetallics [J. Mater. Sci.**6**,1856 '91]
- $Si_3N_4$ [J. Mater. Sci. **32** 6317 '97]
- Dental Glass Cer. [JACerS **78** 3045 '95]
- Epoxy Resin [Scanning **20** 99 '98]

$\mathbf{2\,\gamma = a_0\ [ED^*]}$ ; $K_c = a_0^{1/2}\, ED^{*\,1/2}$ ; $D^* = c/r_1$

# Energy & Geometry Are Related

# In The Fracture Process

$$\mathbf{K_C = (E a_0^{1/2})\ D^{*1/2} = (2\,E\,\gamma)^{1/2}}$$

$$2\,\gamma = \mathbf{a_0\ [ED^*]}$$

$\gamma$ = fracture energy

E = Elastic Modulus

D* = Fractal Dimensional Increment

$a_0$ = Characteristic Structural Parameter

$K_C$ = Fracture Toughness

# Modeling Has to Explain Several Observations

- Scaling
  - topography (mirror, mist, etc.)
  - self similarity (self affinity)
- Fracto-emission
- Velocity (Chaotic) Behavior

# Much evidence exists for chaotic and fractal scaling behavior

**Fracto-emission : light, particles, molecules, etc.**

[e.g., cf. Langford, et. al., J.Mat. Res. 4, 1272 (1989)]

**Dynamic instability : chaotic crack velocities at branching**

[e.g., cf. Fineberg, et. al., Phys.Rev.Ltrs 67,4(1991)]

**Self-similar fracture surfaces : crack branching**

[e.g., Ravi-Chandar & Knauss, Inter.J.Fracture 26,65-80(1984)
Kulawansa et al., J. Mater. Res.9,2476 (1994)
Mecholsky et al., Adv. In Cer.22 ACerS (1988); J. Materials Res.13 ,11 (1998).]

**Scaling : Energy ($\gamma_c$) & Geometry ($a_0$, D*)**

[e.g., cf. Passoja, Adv. In Cer. 22, 101 (1988) ACerS; Mecholsky, Cer.Trans. 64 385-93, ACerS (1996); West et al., J. Non-crystalline Solids, 260 (1999) 99-108; Y. Fahmy, J. C. Russ and C. C. Koch, J. Mater. Sci. 6, 1856-1861 (1991).]

# Fracture In Materials

**There are several fundamental questions that need to be answered:**

- How do bonds break?
- Once a bond “breaks”, how do the ensembles of “broken” bonds propagate?

- Is there a mathematical formulation which permits insight into the fracture process at all scales?