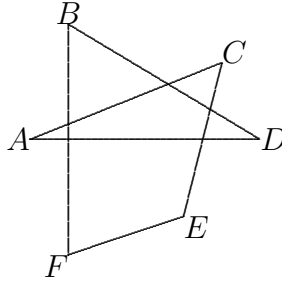


1. Write as a reduced fraction $\frac{1}{3} - (\frac{1}{4} - \frac{1}{5})$.
2. What is the area of a triangle with sides 10, 10, and 16?
3. Jack is three times as old as Jill. In 7 years, he will be twice as old as she. How old is Jack?
4. The price of an item is increased by 25%, and then there is another increase of 20%. What is the overall percentage increase?
5. If $f(x) = x^2 + 1$, what is the value of $f(f(f(f(0))))$?
6. A certain kind of candy was originally priced at 50 cents, but then the price was reduced, and all the candy bars, of which there were fewer than 1000, sold for a total amount of \$31.93. What was the new price, in cents? Since pennies are the smallest coins, the price in cents must be an integer.
7. What positive number N has the property that if the following steps are performed successively, beginning with N , the final result is 2? Divide by 3, then square, then subtract 52, then take square root, then add 8, then divide by 10.
8. Let N be a 3-digit positive number with distinct nonzero digits. What is the smallest possible value for the ratio of N to the sum of its digits?
9. Let a and b be positive numbers. Write a simple equation satisfied by a and b so that the three points where the parabola $y = ax^2 - b$ intersects the x - and y -axes are the vertices of an equilateral triangle. Simplify your equation as much as you can.
10. A function f satisfies $f(0) = 0$, $f(2n) = f(n)$, and $f(2n + 1) = f(n) + 1$ for all positive integers n . What is value of $f(2016)$?
11. A rectangular billiards table is 2 meters by 3 meters. A ball is on the short edge at distance $\frac{1}{3}\sqrt{3}$ from a corner. It is hit in such a way that it strikes each of the other walls once and returns to its starting point. How many meters did it travel? (You may consider the ball to have radius 0.)

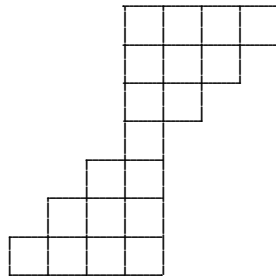
12. Let $Z = \sqrt[11]{10}$, the 11th root of 10. What is the smallest value of n such that the product of the numbers Z, Z^2, Z^3, \dots, Z^n exceeds 99,999?
13. In the diagram below, angle D equals 30 degrees. What is the sum of the angles at $A, B, C, E,$ and F ? Express your answer in degrees.



14. Alice and Bill live at opposite ends of the same street. They leave their houses at the same time and each walk, at constant speed, from their house to the other house and back. The first time they meet, they are 400 yards from Alice's house, and the second time they meet, they are 300 yards from Bill's house. Both times they are traveling in opposite directions. What is the distance, in yards, between the two houses?
15. Define a sequence by $a_1 = 5$, $a_{n+1} = a_n + 4n - 1$, for $n \geq 1$. What is the value of a_{1000} ?
16. How many 3-element subsets of $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ are there for which the sum of the elements in the subset is a multiple of 3?

17. Sam has two identical cups. The first is full of water, while the second is empty. He pours half the water from the first cup into the second. Then on the second transfer, he pours one third of the water that is in the second cup back into the first. He repeats this, alternating cups, pouring $1/(i+1)$ of the water that is in one cup back into the other on the i th transfer. What fraction of the water is in the first cup just before the 19th transfer?
18. Let A , B , and C be digits of base-7 numbers, with possible values $0, \dots, 6$. If $\underline{ACB}_7 + \underline{BCC}_7 = 1400_7$, what is the base-10 value of \underline{ACB}_7 ?
19. What is the remainder when $17^{17^{17}}$ is divided by 7?
20. What is the smallest positive value of $(4x^2 + 8x + 5)/(6x + 6)$ for all real numbers x ?
21. What is the solution set of the inequality $\frac{1}{x} + 2x \geq 3$?
22. Let ABC be a triangle with side lengths 5, 12, and 13. Let P be a point inside the triangle. What is the minimum value of $(AP)^2 + (BP)^2 + (CP)^2$, the sum of the squares of the distances from the point to the vertices?
23. A square is inscribed in a circle, and the region inside the circle but outside the square is shaded. A circle is then inscribed in the square, and a second square is inscribed inside this second circle. The region inside the second circle but outside the second square is shaded. If this process is continued *ad infinitum*, what fraction of the area of the original circle is shaded?
24. A triangle has a right angle at A with $AC = 3$ and $AB = 1$. The angle bisector at B meets AC at P . What is the length CP ?
25. Define integers $x_{i,j}$ for $i, j \geq 0$ by $x_{n,0} = n = x_{0,n}$, while $x_{i,j} = x_{i-1,j} + x_{i,j-1}$ if $i, j \geq 1$. Write a simple expression involving n which equals $\sum_{i=0}^n x_{i,n-i}$.

26. A deck of cards consists of four suits with 13 distinct cards in each suit. A “hand” is a subset of these cards consisting of exactly 6 cards. Let N be the number of hands containing at least one card of each suit. What is the largest prime factor of N ?
27. If a and b are positive real numbers satisfying $(a - b)^2 = 4(ab)^3$, what is the smallest possible value of $\frac{1}{a} + \frac{1}{b}$?
28. The longer leg of a right triangle equals the hypotenuse of a 30-60-90 triangle. The two triangles have equal perimeters. What is the tangent of the smallest angle of the first triangle?
29. In the grid below, how many paths from the top row to the bottom row are there, where a path consists of 6 steps, and each step is a segment connecting the center of a square to the center of a square which shares at least one vertex with the current square and lies in the row below it? The paths can start in any square in the top row and end in any square in the bottom row.



30. There are two values of m so that the equation

$$x^4 - (3m + 2)x^2 + m^2 = 0$$

has four real roots in arithmetic progression. What is the smaller of these two values of m ?

31. In parallelogram $ABCD$, side AD is extended beyond D to point F . The segment FB intersects side CD at G , and intersects diagonal AC at E , in such a way that $FG = 4$ and $GE = 1$. What is the length EB ?

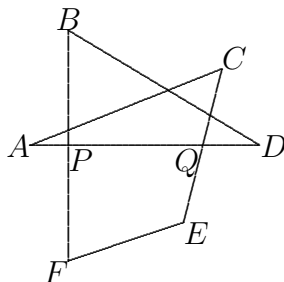
32. In a sequence of 0's and 1's, a "run" is defined to be a string of consecutive 1's or 0's, including runs of length 1. For example, the sequence 00100011 has four runs. In a random sequence containing 15 0's and 9 1's, what is the average (or expected) number of runs?
33. If a complex number z satisfies $z + \frac{1}{z} = 1$, then what is the value of $z^{80} + \frac{1}{z^{80}}$?
34. What is the smallest integer larger than $(\sqrt{5} + \sqrt{3})^6$?
35. In square $ABCD$ of side length 3, P and Q trisect side AB . Let E be a point on an edge of the square, but not on AB , which maximizes angle PEQ . What is the area of triangle PEQ ?
36. Triangle ABC has $AB = 6$, $AC = 5$, and $BC = 4$. Points P_1 , P_2 , and P_3 on BC satisfy $BP_1 = P_1P_2 = P_2P_3 = P_3C = 1$. What is the value of $(AP_1)^2 + (AP_2)^2 + (AP_3)^2$?
37. You have three bowls, each of which contains 6 balls. A "move" consists of selecting a random bowl and then a different random bowl and moving a ball from the first bowl to the second. What is the probability that after 5 such moves, all bowls will again have 6 balls?
38. Let \mathcal{P} denote the set of all subsets of $\{a, b, c\}$, including the empty set. How many functions f are there from \mathcal{P} to \mathcal{P} which satisfy that, for all A and B in \mathcal{P} , $f(A \cup B) = f(A) \cup f(B)$?
39. If $a_0 = 0$, $a_1 = 1$, and $a_{n+1} = 1 + \frac{a_n^2 - a_n a_{n-1} + 1}{a_n - a_{n-1}}$ for $n \geq 1$, what integer is closest to a_{12} ?
40. A triangle has sides of length 5, 7, and 8. Of all the points inside the triangle, what is the maximum value of the product of the lengths of the altitudes to the three sides of the triangle?

SOLUTIONS TO 2016 CONTEST, annotated with the number, out of the 54 scorers, of people answering it correctly.

1. 17/60. [54] It is $\frac{1}{3} - \frac{1}{20}$.
2. 48. [54] It is composed of two 6-8-10 right triangles. They can be rearranged to form a 6-by-8 rectangle.
3. 21. [54] If x is Jill's age, then $3x + 7 = 2(x + 7)$, so $x = 7$, and Jack's age is $3 \cdot 7$.
4. 50 or 50%. [53] The price is multiplied by $5/4$ and then that is multiplied by $6/5$. The net effect is to multiply the price by $3/2$.
5. 26. [54] The successive values are 1, 2, 5, 26.
6. 31. [53] The total amount in cents factors as $103 \cdot 31$. It must be that there were 103 candy bars sold at 31 cents apiece.
7. 42. [53] Working backwards from the 2, the numbers must be 20, 12, 144, 196, 14, 42. Note that if it had not been specified that we were looking for a positive number, then -42 would also work.
8. 10.5. [19] You would want the first digit to be 1 and the other digits to be as large as possible. The number should be 189. Note, for example, that using $N = 179$ gives a ratio of $10 + \frac{9}{17}$, which is larger than 10.5.
9. $ab = 3$. [27.5] The points are $(0, -b)$ and $(\pm\sqrt{b/a}, 0)$. Thus $2\sqrt{b/a} = \sqrt{b^2 + \frac{b}{a}}$. Thus $3\frac{b}{a} = b^2$, and so $ab = 3$.
10. 6. [50] You can easily work backwards, or can note that this function gives the number of 1's in the binary expansion of n . $f(2016) = f(63) = 6$ since $63 = 111111_2$.
11. $2\sqrt{13}$. [31] The $\frac{1}{3}\sqrt{3}$ is irrelevant. The easiest way to work the problem, by far, is to draw a grid of 2-by-3 rectangles. By considering reflections, the path can be viewed as a straight line

path on this grid from the starting point to the corresponding point two blocks over and two blocks up. The distance is $\sqrt{4^2 + 6^2}$.

12. 10. [51] Since the product equals $Z^{n(n+1)/2}$, we require $10^{n(n+1)/22} > 10^5 - 1$, so $n(n+1) \geq 110$.
13. 360. [39] Since $P = B + D$ and $Q = A + C$ (exterior angles of triangles), and $P + Q + E + F = 360$, the answer follows. Note that the information about angle D was irrelevant.



14. 900. [44] If their speeds are S_A and S_B , respectively, and the distance is D , then $\frac{400}{S_A} = \frac{D-400}{S_B}$ and $\frac{D+300}{S_A} = \frac{2D-300}{S_B}$. Thus $\frac{D-400}{400} = \frac{2D-300}{D+300}$. We obtain $D^2 - 100D = 800D$, so $D = 900$. Note that other solutions are possible if they might have been traveling in the same direction the second time that they met, if one was much faster than the other.
15. 1997006. [39] $a_{1000} = 5 + 3 + 7 + 11 + \dots$, with 1000 terms including the 5. Thus it equals $5 + 3 \cdot 999 + 4(1 + 2 + \dots + 998) = 3002 + 2 \cdot 998 \cdot 999 = 3002 + 2(1000 - 2)(1000 - 1) = 3006 + 2 \cdot 10^6 - 6000$.
16. 57. [44] The mod 3 values of the elements in the subset can be $\{0, 0, 0\}$, $\{1, 1, 1\}$, $\{2, 2, 2\}$, or $\{0, 1, 2\}$. The number of such subsets is $\binom{3}{3}$, $\binom{4}{3}$, $\binom{4}{3}$, and $3 \cdot 4 \cdot 4$, respectively. So the answer is $1 + 4 + 4 + 48$.
17. 10/19. [31] If you work out the first few, it appears that after an odd numbered transfer, it will be half and half, while just

before the $(2k + 1)$ st transfer, it will be $(k + 1)/(2k + 1)$ in the first, and $k/(2k + 1)$ in the second. This can easily be proved by induction.

18. 319. [50] The units digit implies that $B + C = 7$. Then the 7's digit implies $2C + 1 = 7$, so $C = 3$ and $B = 4$. Now the 7²-digit implies $A + 4 + 1 = 11$ so $\underline{ACB}_7 = 634_7 = 319_{10}$.
19. 5. [38] We want the mod 7 value of $3^{17^{17}}$. Since $3^6 \equiv 1 \pmod{7}$, we note that, mod 6, $17^{17} \equiv (-1)^{17} \equiv 5$. Thus, mod 7, $3^{17^{17}} \equiv 3^5 \equiv 5$.
20. $2/3$. [34] The expression simplifies to $\frac{4(x+1)^2+1}{6(x+1)} = \frac{2}{3}(x+1) + \frac{1}{6} \frac{1}{x+1}$. By the AM-GM Inequality, the minimum value of $X + Y$, for positive numbers X and Y , is $2\sqrt{XY}$, achieved when $X = Y$. In this case, the minimum is $2\sqrt{2/18}$, achieved when $x = -1/2$.
21. $(0, \frac{1}{2}] \cup [1, \infty)$. [39] The inequality can be written as $(2x - 1)(x - 1)/x \geq 0$. This is satisfied if all three factors are nonnegative, or if only one is nonnegative. The solution $x = 0$ is excluded since $1/x$ is not defined.
22. $338/3$. [12] Let the vertices be $(0, 0)$, $(12, 0)$, and $(0, 5)$, and let (x, y) be the point. The desired sum is

$$\begin{aligned} & x^2 + y^2 + x^2 + (5 - y)^2 + (12 - x)^2 + y^2 \\ &= 3(x^2 - 8x + 48) + 3(y^2 - \frac{10}{3}y + 25) \\ &= 3((x - 4)^2 + 32) + 3((y - \frac{5}{3})^2 + \frac{50}{9}), \end{aligned}$$

whose minimum is $96 + \frac{50}{3}$.

23. $2(\pi - 2)/\pi$ or $2 - \frac{4}{\pi}$. [46] Let the radius of the initial circle equal 1. Then the sidelength of the inscribed square is $\sqrt{2}$, and so the area shaded in the first round is $\pi - 2$. The radius of the second circle is $\frac{1}{2}\sqrt{2}$, and so the area shaded in the second round is $1/2$ times the area shaded in the first round. Similarly, each successive round has half as much area shaded as the preceding

round. Thus the total area shaded is $(\pi - 2)(1 + \frac{1}{2} + (\frac{1}{2})^2 + \dots) = 2(\pi - 2)$, so the fraction is as claimed.

24. $(10 - \sqrt{10})/3$. [45.5] Let θ equal angle PBA . Then $3 = \tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$. Solving the quadratic equation for $\tan \theta$, we obtain $AP = \tan \theta = \frac{\sqrt{10}-1}{3}$ and $CP = 3 - AP$.
25. $2^{n+1} - 2$. [19] The easiest way is to make a table of the first few rows and observe the pattern. It can be proved inductively as follows:

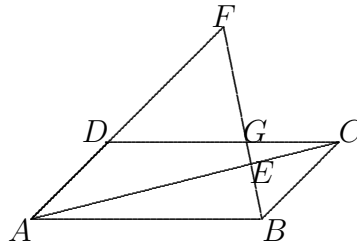
$$\sum_{i=0}^n x_{i,n-i} = 2n + \sum_{i=1}^{n-1} (x_{i-1,n-i} + x_{i,n-i-1}) = 2n + 2(2^n - 2 - (n-1)) = 2^{n+1} - 2.$$

26. 19. [14] The number of ways in which there are two suits with two cards is $\binom{4}{2} \binom{13}{2}^2 13^2$, while the number with three cards in one suit is $4 \binom{13}{3} 13^3$. The sum of these can be factored as $13^4 \cdot 2^3(3^3 + 11) = 2^4 \cdot 13^4 \cdot 19$.
27. $2\sqrt{2}$. [9] $\frac{1}{a} + \frac{1}{b} = m$ is equivalent to $a + b = mab$. The given equation says $4(ab)^3 = (a + b)^2 - 4ab = m^2 a^2 b^2 - 4ab$. Thus $4(ab)^2 - m^2 ab + 4 = 0$, which is only satisfied for some $ab > 0$ if $m^2 \geq 8$, so $m \geq 2\sqrt{2}$. Equality is obtained for $\{a, b\} = \{\sqrt{2} \pm 1\}$.
28. $\frac{1}{4}(3 - \sqrt{3})$. [27] We may assume that the sides of the second triangle are 1, $\sqrt{3}$, and 2. The first triangle has sides 2, x , and $\sqrt{4 + x^2}$, with $2 + x + \sqrt{4 + x^2} = 1 + \sqrt{3} + 2$. Thus $4 + x^2 = (1 + \sqrt{3} - x)^2$, and so $x = \frac{\sqrt{3}}{1 + \sqrt{3}} = \frac{3 - \sqrt{3}}{2}$. Thus the tangent is $\frac{3 - \sqrt{3}}{4}$.
29. 169. [33] Working up or down from the middle square, we insert the number of paths from each square to the center square. There are $4 + 5 + 3 + 1$ paths from the top row to the center

square, and 13 paths from the center square to the bottom row.

			4	5	3	1
			2	2	1	
			1	1		
			1			
			1	1		
			1	2	2	
			1	3	5	4

30. $-6/19$. [8] Let the roots be $a \pm t$ and $a \pm 3t$. Then $((x-a)^2 - t^2)((x-a)^2 - 9t^2) = x^4 - (3m+2)x^2 + m^2$ is an identity. We must have $a = 0$, $10t^2 = 3m + 2$, and $9t^4 = m^2$. Hence $m = \pm 3t^2$ and $2 = (10 \pm 9)t^2$. Now $t^2 = 2$ or $2/19$, and $3m = 20 - 2$ or $\frac{20}{19} - 2$. Finally $m = 6$ or $-6/19$.
31. $\sqrt{5}$. [17] Let $EB = x$, $DG = y$, and $DC = b$. Since triangles EBA and EGC are similar, $\frac{x}{1} = \frac{b}{b-y}$. Since triangles GFD and GBC are similar, $\frac{4}{y} = \frac{1+x}{b-y}$. The first equation yields $\frac{y}{b} = \frac{x-1}{x}$, while the second says $\frac{1+x}{4} = \frac{b}{y} - 1$. We obtain $\frac{1+x}{4} = \frac{x}{x-1} - 1$, which simplifies to $x^2 = 5$.



32. 12.25. [0] Let X_i be the random variable which is 1 if the i th entry starts a run, and 0 otherwise, and let $E(X_i)$ be its expected value. Then $E(X_1) = 1$. If $i > 1$, then the probability that $X_i = 1$ is $\frac{15}{24} \cdot \frac{9}{23} + \frac{9}{24} \cdot \frac{15}{23}$, so $E(X_i) = \frac{2 \cdot 15 \cdot 9}{24 \cdot 23}$. The expected number of runs is $E(X_1) + \cdots + E(X_{24}) = 1 + 23 \frac{2 \cdot 15 \cdot 9}{24 \cdot 23} = 1 + \frac{45}{4}$.

33. -1. [29] Let $w_n = z^n + z^{-n}$. Expanding shows that $w_{n+m} = w_n w_m - w_{|n-m|}$. Also $w_0 = 2$ and $w_1 = 1$. From this it follows by induction that $w_{2n} = -1$ for $n \geq 1$. Next $w_{3,2n} = w_{2n+1} w_{2n} - w_{2n} = 1 - (-1) = 2$ for $n \geq 1$. Finally $w_{5,2n} = w_{2n+2} w_{2n} - w_{3,2n} = 1 - 2 = -1$ for $n \geq 1$. **Alternatively:** $z = e^{i\pi/3}$, so the answer is $e^{80i\pi/3} + e^{-80i\pi/3} = 2 \cos(2\pi/3) = -1$.

34. 3904. [8] Note that

$$\begin{aligned} & (\sqrt{5} + \sqrt{3})^6 + (\sqrt{5} - \sqrt{3})^6 \\ &= 2((\sqrt{5})^6 + 15(\sqrt{5})^2(\sqrt{3})^2((\sqrt{5})^2 + (\sqrt{3})^2) + (\sqrt{3})^6) \\ &= 2(125 + 15 \cdot 15 \cdot 8 + 27) = 3904. \end{aligned}$$

Since $\sqrt{5} - \sqrt{3} < 1$, this is the desired answer. The actual value is roughly 3903.98.

35. $\frac{1}{2}\sqrt{2}$. [5] For any point E , angle PEQ subtends the minor arc of chord PQ in the circle passing through P , E , and Q . To make this angle as large as possible, we would want the radius of this circle to be as small as possible. The circle through P and Q tangent to sides BC and AD accomplishes this, as no smaller circle through P and Q meets another edge. The line through E parallel to PQ will be a diameter of the circle. The right triangle determined by P , the center of this circle, and the midpoint of PQ shows that the altitude in triangle PEQ from side PQ has length $\sqrt{(\frac{3}{2})^2 - (\frac{1}{2})^2} = \sqrt{2}$. Since the base of triangle PEQ has length 1, its area is $\frac{1}{2}\sqrt{2}$.

36. $163/2$. [6] Let θ denote the angle at B . Then $\cos \theta = \frac{6^2+4^2-5^2}{2 \cdot 4 \cdot 6} = \frac{9}{16}$. We have

$$\begin{aligned} (AP_1)^2 &= 6^2 + 1^2 - 2 \cdot 1 \cdot 6 \cos \theta \\ (AP_2)^2 &= 6^2 + 2^2 - 2 \cdot 2 \cdot 6 \cos \theta \\ (AP_3)^2 &= 6^2 + 3^2 - 2 \cdot 3 \cdot 6 \cos \theta. \end{aligned}$$

Thus $(AP_1)^2 + (AP_2)^2 + (AP_3)^2 = 3 \cdot 6^2 + 1^2 + 2^2 + 3^2 - 2 \cdot 6 \cdot 6 \cdot \frac{9}{16} = 122 - \frac{81}{2} = \frac{163}{2}$.

37. $5/108$. [1] There are 6 choices for each move (3 for the bowl from which the ball is chosen, and 2 for the bowl into which it is placed), and so there are 6^5 possible sequences of moves. Suppose the bowls are labeled A, B, and C. It must be the case that 2 bowls have two balls removed, and one bowl has one removed. There are 3 choices for the bowl from which only one ball is removed. Assume that is bowl A. There are 5 choices for the turn on which that removal will take place, and 2 for the bowl into which that ball will be moved. If the ball from bowl A is moved into bowl B, then it must be that the ball that is moved into bowl A must come from bowl C, for if it came from B, it would be impossible to end up with an equal number of balls in bowls B and C. There are now 4 choices of the turn on which the ball is moved from C to A. On the other three moves, there must be two from B to C, and one from C to B. There are three choices for the turn on which we switched from C to B. So there are $3 \cdot 5 \cdot 2 \cdot 4 \cdot 3$ choices altogether. The ratio of this to 6^5 is $5/108$.
38. 729. [0] First note that $f(\emptyset \cup A) = f(\emptyset) \cup f(A)$ implies that $f(\emptyset) \subset f(A)$ for all A . Also, f is determined by $f(\emptyset)$, $f(\{a\})$, $f(\{b\})$, and $f(\{c\})$. If $|f(\emptyset)| = 3$, then $|f(A)| = 3$ for all A , so there is only one such f . If $f(\emptyset) = \{a, b\}$, there are 2^3 choices for f , depending on which of $f(\{a\})$, $f(\{b\})$, and $f(\{c\})$ contain c . The situation is similar for $f(\emptyset) = \{a, c\}$ or $\{b, c\}$, so there are $3 \cdot 8$ possible f 's for which $|f(\emptyset)| = 2$. For $f(\emptyset) = \{a\}$, there are 4^3 possible functions f , depending independently on whether or not b and/or c is in (independently) $f(\{a\})$, $f(\{b\})$, and $f(\{c\})$. So there are $3 \cdot 4^3$ functions f for which $|f(\emptyset)| = 1$. Finally, if $f(\emptyset) = \emptyset$, then there are, independently, 8 possibilities for each of $f(\{a\})$, $f(\{b\})$, and $f(\{c\})$, so 8^3 possible f with $|f(\emptyset)| = 0$. The total number of f is $1 + 24 + 192 + 512 = 729$ functions altogether.

39. 19. [2] Let $b_{n+1} = a_{n+1} - a_n$. Then $b_1 = 1$ and $b_{n+1} = 1 + \frac{1}{b_n}$. Thus $b_n = \frac{f_{n+1}}{f_n}$, the ratio of successive Fibonacci numbers.

$$a_{12} = b_1 + \cdots + b_{12} = 1 + 2 + \frac{3}{2} + \frac{5}{3} + \cdots + \frac{144}{89} + \frac{233}{144}.$$

The values of b_n approach the golden ratio $\phi \approx 1.618$. Beginning with b_4 , they are quite close to that value, and so $a_{12} \approx 4.5 + 9 \cdot 1.618 = 19.062$. The actual value is roughly 19.098.

40. $200\sqrt{3}/63$. [1] If x , y , and z are the lengths of the altitudes, then the area of the triangle equals $\frac{1}{2}(5x + 7y + 8z)$. By Heron's formula, it equals $\sqrt{10 \cdot 5 \cdot 3 \cdot 2} = 10\sqrt{3}$. Thus $5x + 7y + 8z = 20\sqrt{3}$. By AM-GM Inequality, $(5x + 7y + 8z)/3 \geq \sqrt[3]{(5x)(7y)(8z)}$. Thus $280xyz \leq (\frac{20}{3}\sqrt{3})^3$ and so $xyz \leq 200\sqrt{3}/63$. Equality is obtained when $x = 4\sqrt{3}/3$, $y = 20\sqrt{3}/21$, and $z = 5\sqrt{3}/6$.