ARTIFICIAL NEURAL NETWORK-BASED
MODELING AND INTELLIGENT CONTROL
OF TRANSITIONAL FLOWS

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ABSTRACT

Empirical eigenfunctions of transitional flow in a grooved channel are extracted
by proper orthogonal decomposition (POD). POD is applied to numerical solu-
tions of the governing Navier-Stokes partial differential equations at Reynolds
numbers \( Re = 430, 750, 1050 \) and at Prandtl number \( Pr = 0.71 \) (air flow).
For each value of \( Re \), a low-dimensional set of nonlinear ordinary differential
equations is derived by Galerkin projection. The Galerkin projection-based
low-order dynamical models are used to generate the data required to effi-
ciently train artificial neural networks in the range \( 400 \leq Re \leq 1200 \). Accurate arti-
ficial neural network-based models of the flow system are obtained. The study
demonstrates the potential use of Galerkin projection-based and artificial neural
network-based low-order models as valuable tools for flow modeling and for pre-
diction of short- and long-time behavior of transitional flow systems. A possible
real-time intelligent flow control scheme is briefly discussed.

1 Introduction

The use of artificial neural networks (ANNs) in the analysis of thermo-fluid
systems has received considerable attention in the past few years. The develop-
ment of powerful learning algorithms and the increasing number of applications
in many disciplines suggest that ANNs can provide useful tools for modeling prac-
tical flow and heat transfer problems. ANNs have already proved to be
successful in fault diagnosis, process modeling and control, pattern recognition
and process identification.

Application of ANNs in flow and heat transfer problems is a relatively new
research area. Jacobsen and Reynolds [1] studied active control of boundary
a conceptual study of active laminar flow control with ANNs. Recently, Gillies
and Anderson [3], using proper orthogonal decomposition (POD), studied low-
dimensional characterization and an adaptive non-linear flow control for a class
of unstable wake flows.

The grooved channel (Fig. 1) investigated in this study involves sharp cor-
ners leading to flow separation and temporal hydrodynamic instabilities that oc-
cur at low or moderate Reynolds numbers. The configuration arises frequently
in applications related to cooling of electronic equipment [4] where the protrusions are formed by chip modules. The geometry is assumed to be periodic in space, i.e., we consider the case that a large number of identical components (modules) are mounted periodically on one channel wall. Channel entrance effects are ignored. The flow in this configuration reaches a time-independent state when the Reynolds number is below a critical value, \( \text{Re}_c \). When the device operates above \( \text{Re}_c \), the flow exhibits self-sustained oscillations.

Deane et al. [5] and Sahan et al. [6] applied the POD methodology to derive low-order models (LOMs) for transitional flow in a periodically grooved channel. Sahan et al. [7] and Sahan [8] applied POD to simultaneous flow and heat transfer in a grooved channel. In the present study, we extract the most energetic eigenfunctions of air flow in a grooved channel (see Fig. 1) for several values of \( \text{Re} \). POD is performed at \( Pr = 0.71 \) and \( \text{Re} = 430, 750, 1050 \). At these values of \( \text{Re} \), the flow is time-dependent and, in the steady state, time periodic. For each Reynolds number, a low-dimensional set of nonlinear ordinary differential equations is derived by using Galerkin Projection (GP), leading to GP-based LOMs. The ability of the reduced models to describe the dynamics of the flow field is examined. Then, the developed GP-based LOMs are used to generate the data required to efficiently train ANNs in the range \( 400 \leq \text{Re} \leq 1200 \), resulting in accurate ANN-based dynamical models of the flow system. A real-time intelligent flow control strategy is discussed.

2 Formulation and Solution Method

2.1 Full Model: Flow in a Grooved Channel

Flow in the grooved channel geometry shown in Fig. 1 is studied. The incoming flow is assumed to be periodically fully-developed [4] and the fluid is considered to be incompressible. The partial differential equations (PDEs) governing constant-property, time-dependent flow can be written as follows:

\[
\nabla \cdot \mathbf{V} = 0,\tag{1}
\]

\[
\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\nabla P + \frac{1}{Re} \nabla^2 \mathbf{V} + \mathbf{F}.\tag{2}
\]

The dimensionless parameter appearing in Eq. (2), the Reynolds number \( \text{Re} \) is defined as \( \text{Re} = (U_{ref} h_1) / \nu \), where \( 2h_1 \) is the width of the by-pass part of the channel, \( \nu \) denotes the kinematic viscosity, and \( U_{ref} = (3/2) U_{av} \), where \( U_{av} \) is the average velocity at a channel cross-section. We consider the flow far from the channel entrance, and, taking into account the spatial periodicity of the channel, we solve the governing equations in one computational module by imposing periodic boundary conditions in the streamwise direction (see Fig. 1). The no-slip condition is applied at the solid-fluid interfaces. Equations (1)-(2) with the corresponding boundary conditions are solved by a spectral element method [9]. Implementation of the numerical method is based on a commercially available computer code [10]. In our simulations, 44 two-dimensional spectral elements were used. Numerical solutions were obtained for order of interpolants, \( N_1 = \)
4, 6, 8, 10, 12. Results are presented for $N_1 = 10$, $h_2/h_1 = 0.75$, $h_1/l_1 = 5.0$, $l_2/l_1 = 3.0$ (see Fig. 1). For these parameters, spontaneous, self-sustained oscillations appear at $Re_c \approx 300$.

2.2 Proper Orthogonal Decomposition

POD was developed in the context of statistical pattern recognition and has been used as a method for the extraction of large scale coherent structures in turbulent flows [11]. POD extracts empirical orthogonal eigenfunctions, identifies systematically coherent (spatio-temporal) structures, and leads naturally to efficient, low-order representations of transitional/turbulent flow systems [12-13]. Sirovich [13] proposed the snapshot POD in order to efficiently analyze large data sets. In applying the snapshot POD, the time-dependent data are first decomposed into time-averaged ($\overline{\mathbf{V}}$) and time-varying parts ($\mathbf{V}^\prime$). The time-averaged values are obtained as simple arithmetic means of $M$ snapshots of the velocity field. Thus, the fluctuating part of the velocity field can be computed by

$$\mathbf{V}^\prime(\mathbf{x},t) = \mathbf{V}(\mathbf{x},t) - \overline{\mathbf{V}}(\mathbf{x}).$$

(3)

The empirical eigenfunctions are constructed by an appropriate superposition of snapshots of the time-varying part of the velocity field [12], i.e.,

$$\phi_k(\mathbf{x}) = \sum_{i=1}^{M} A_{ki} \mathbf{V}^\prime(\mathbf{x},t_i), \quad k = 1, \ldots, M$$

(4)

where $A_{ki}$ denotes the $k$th eigenvector of the matrix eigenvalue problem

$$C A = \lambda A,$$

(5)

and matrix $C$ is defined as

$$C_{mn} = \frac{1}{M} \int_{\Omega} \mathbf{V}^\prime(\mathbf{x},t_m) \cdot \mathbf{V}^\prime(\mathbf{x},t_n) \, d\Omega.$$

(6)

Each eigenvalue $\lambda$ represents the contribution of the corresponding empirical eigenfunction to the total flow fluctuation energy. The eigenfunctions are orthogonal to each other and, when normalized properly, they form an orthonormal set of basis eigenfunctions. They also satisfy the boundary conditions of the problem. Furthermore, for incompressible flows, the velocity empirical eigenfunctions are divergence-free.

2.3. Galerkin Projection-Based Low-Order Dynamical Models

After the velocity empirical eigenfunctions have been obtained and normalized, we expand the time-varying part of the velocity field in terms of the eigenfunctions, i.e.,

$$\mathbf{V}^\prime(\mathbf{x},t) = \sum_{k=1}^{M_1} a_k(t) \phi_k(\mathbf{x}).$$

(7)

In the above equations, $\phi_k$ denotes the $k$th eigenfunction of the velocity field. The instantaneous velocity field can be expressed as

$$\mathbf{V}(\mathbf{x},t) = \overline{\mathbf{V}}(\mathbf{x}) + \sum_{k=1}^{M_1} a_k(t) \phi_k(\mathbf{x}).$$

(8)
To determine \( a_k(t) \), Eq. (8) is substituted into the dimensionless conservation of momentum equation, Eq. (2). The Galerkin procedure is applied and the orthonormality property of the empirical eigenfunctions is used. After rearranging the terms, we obtain the following \( M_1 \) non-linear ODEs for the expansion coefficients \( a_k(t) \) (the GP-based LOMs):

\[
\frac{da_k}{dt} = A_k + \frac{1}{Re} B_k + \sum_{i=1}^{M_1} C_{ki} a_i + \frac{1}{Re} \sum_{i=1}^{M_1} D_{ki} a_i + \sum_{i=1}^{M_1} \sum_{j=1}^{M_1} E_{kij} a_ia_j.
\]

In general, the number of retained modes in the truncated series expansion is much smaller than the number of snapshots, i.e., \( M_1 \ll M \). The above system of equations is integrated numerically to obtain \( a_k(t) \) using a fourth order Runge-Kutta solver.

3 Development of Artificial Neural Network-Based Dynamical Models

In this study it is desired that a neural network learn the relationship between Reynolds number \( Re \), time \( t \), the present (and past if necessary) states \( \{a_i(t)\text{ and } a_i(t-q), i = 1, \ldots, M_1\} \) of air flow, and the future state of air flow \( \{a_i(t+q), i = 1, \ldots, M_1\} \) where \( q \) denotes the time step. The ANN should make predictions of the future state when given estimates of Reynolds number, time, and the present state. ANNs employing the backpropagation algorithm were chosen here due to their ability to process time-series efficiently and their strong theoretical base.

In this investigation, ANNs exhibiting various architectures and learning properties were developed and tested in an attempt to achieve optimal neural network predictive capabilities over varying ranges of \( Re \). A neural network software package [14] was used in order to train ANNs and to develop ANN-based LOMs. The GP-based low-order dynamical models developed in Sec. 2.3 are capable of capturing the dynamical attributes of the flow system [7,15-17]. Because they are much easier to solve numerically than the full model equations, they are used in this work as reference models in order to generate the data for ANN training. The data have been generated as follows. First the range of prediction in time and \( Re \) were chosen. The final prediction time for all neural networks was chosen to be \( t_f = 200 \), so as to reach steady state periodic self-sustained oscillations in time. An ANN capable of predicting the short and long-time dynamical behavior of the flow within the range of \( 400 \leq Re \leq 1200 \) was developed. For a time step \( q = 0.5 \), data sets were generated from GP-based LOMs of the flow for numerous \( Re \) within the above range. Data were separated into training and testing sets, with 22\% of the data randomly placed into testing sets and the rest placed into a training set.

In all calculations, it was assumed that the number of processing elements in each layer were equal. The ANN architecture consists of six inputs \( \{Re, t, \text{ and } a_i(t), i = 1, \ldots, 4\} \) and four outputs \( \{a_i(t+q), i = 1, \ldots, 4\} \). The final form of the developed ANN consisted of three hidden layers and fourteen processing
elements per layer. Testing and training set RMS errors were 4.38% and 3.78% respectively. The developed ANN-based LOM embodies dynamical information and physical attributes of the full system. The ANN-based LOM also serves as another ODE solver for the temporal expansion coefficients within the range $400 \leq Re \leq 1200$.

4 Results and Discussion

4.1 POD Analysis

The decomposition was performed at $Re = 430$, 750 and 1050. For these values of $Re$ the long term attractor of the flow corresponds to a limit cycle. We take $M = 20$ snapshots in a period. Here we summarize the important outcomes of the POD analysis. Detailed results can be found in Sahan et al. [6-7] and Sahan [8].

4.2 Eigenvalues

Table 1 lists the cumulative contribution of the four most energetic modes to the flow fluctuation energy. For the values of $Re$ considered in this study, the modes occur in pairs with eigenvalues of comparable magnitude.

For each $Re$, the first velocity modal pair (first and second velocity eigen-modes) captures major portion of the total flow fluctuation energy. As $Re$ increases, the percentage of energy contained in the first two modes (the first modal pair) decreases while the relative contribution of the third and fourth modes to the total fluctuation energy increases.

4.3 Eigenfunctions and spatio-temporal structures

For each $Re$ the two most energetic eigenfunctions contain the large scale features of each field while higher modes (of lower energy level) capture the small scale features of the field. The eigenfunctions also occur in pairs. Within each pair the eigenfunctions are phase shifted by approximately a quarter-wavelength in the by-pass region of the channel.

The temporal expansion coefficients, obtained by direct projection of the input velocity data on the computed eigenfunctions, are plotted in Figure 2(a) for $Re = 1050$. The temporal expansion coefficients corresponding to a pair, $(a_1, a_2), (a_3, a_4), \ldots$ etc., are also phase shifted by a quarter of a period. The first dynamical (spatio-temporal) coherent structures of velocity, $\tilde{\zeta}_1(\bar{X}, t)$, may be defined as follows:

$$\tilde{\zeta}_1(\bar{X}, t) = \sum_{i=1}^{2} a_i(t) \tilde{\varphi}_i(\bar{X})$$

Based on the phase relations described above, $\tilde{\zeta}_1$ is identified as a travelling wave.
4.4 Galerkin projection-based low-order model predictions

The ultimate goal of this investigation is to develop POD-based LOMs to capture the stability and bifurcation behavior of the underlying full-model simulations. Although the long-term dynamical predictions of the developed LOMs may be sometimes inaccurate, causing unsatisfactory prediction of the attractors, their short-term prediction capabilities are useful especially in real-time flow control applications.

The flow exhibits self-sustained time-periodic oscillations for $400 \leq Re \leq 1200$. The developed GP-based LOMs also predict the same transitional behavior within this $Re$ range. Short-term predictions of the GP based-LOMs developed at $Re = 1050$ with $M_1 = 4$ are shown in Figure 2b. These predictions are in good agreement with the expansion coefficients obtained from direct projection of the numerical data (Figure 2a).

4.5 ANN Predictions

ANN-based LOM reproduces successfully the dynamical behavior of the velocity field for $380 \leq Re \leq 1500$.

4.6 Intelligent Flow Control using Artificial Neural Networks

Controlling transitional and turbulent flow systems is highly desirable in engineering applications [18-20]. Most recent investigations concentrate on the control of convectively unstable flows, such as boundary-layer flows which are noise amplifiers and have been controlled successfully using mostly linear control methods. Control of flows such as wakes [3] is a more challenging task. Thus, implementation of linear control schemes based on point actuation and sensing devices may not be effective enough due to the complex spatial structure of these flow systems [3]. Attention needs to be given to nonlinear intelligent control strategies [21] using ANNs.

As mentioned in Sec. 2.2, spatio-temporal (coherent) structures play an important role in the representation of transitional or turbulent flow systems. The dynamical behavior of flow systems can be explained in terms of the evolution of a relatively small set of spatial modes. POD extracts an appropriate set of modes in which the velocity field is represented by a finite-dimensional basis eigenfunctions.

For the control of transitional flow systems, a neural network can be trained to emulate the modal dynamics of the flow. Then, an adjoined neural network may be designed to control the flow and to minimize flow fluctuations [3]. The main objective of the flow control strategy is to provide an external, time-dependent, non-linear control input, $u_\text{ext}(t)$, to the flow, such that the future fluid state, described by a finite number of mode amplitudes, corresponds to some desired state. The neural emulator of the flow dynamics provides a prediction of the fluid state, given initial mode amplitude conditions and values for external
control parameters (see Figure 4). The predicted response of the fluid is used by the neural controller, such that the predicted response to an applied control minimizes the control system error and the flow is driven towards a desired state.

5 CONCLUSIONS

In this study, low-order dynamical models of transitional flow in a grooved channel are developed. The four most energetic eigenfunctions extracted by applying POD are used in the development of GP-based low-order models. These reduced models are then used to generate the training data for different Re and to construct ANN-based LOMs of the flow system. GP-based LOMs easily generate training data with less computational time compared to the solution of full model Navier-Stokes equations. Thus, they are used here to reduce the computational effort.

Considerable information about the dynamical behavior of the flow system is represented by the small set of ODEs (GP-based LOMs). The use of ANNs in order to emulate the dynamical behavior of the flow system combines all the benefits of GP-based LOMs with the parallel and fast processing capabilities of neural networks. Neural networks do not need to know any physical relationship between input and output data including the flow governing equations. However, the use of physically meaningful training data produced using GP-based LOMs carries considerable amount of dynamical information about the flow system into the neural network emulator. Once trained successfully, the neural network emulator acts as another low-order dynamical model of the flow system, providing the solution of the reduced set of ODEs, and its incorporation into a nonlinear intelligent adaptive flow control scheme becomes possible.

6 References


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[14] NeuralWare Professional II/Plus, Version 5.0, NeuralWare Inc., 1995.


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<table>
<thead>
<tr>
<th>m</th>
<th>Re = 430 Cumulative Energy, %</th>
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