Coherent and Non-coherent MIMO Radar Techniques for Target Localization

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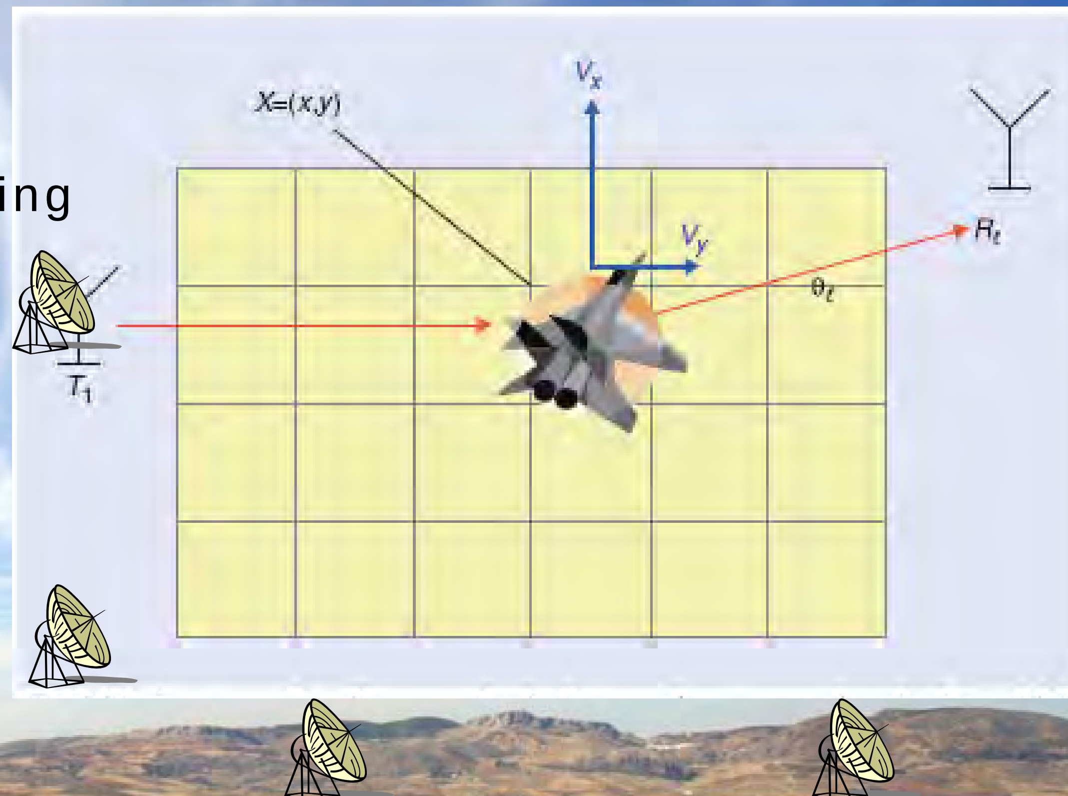
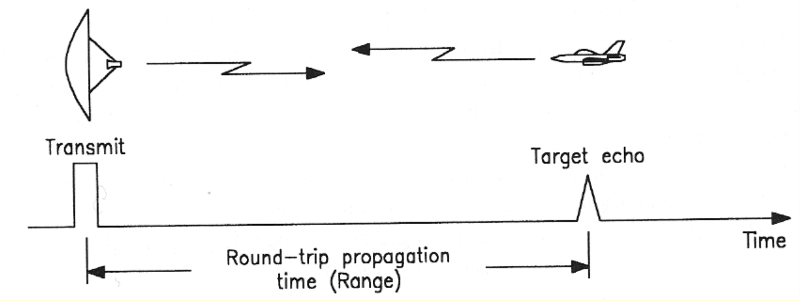
Overview

- Radar problem
- Target/signal models
- Spatial decorrelation
- Non-coherent processing
- Coherent processing
- Concluding remarks



Radar problem

- In its simplest form, the radar problem is: given a transmitted a waveform *s(t)* known to the receiver, and observing a returned signal *r(t)*



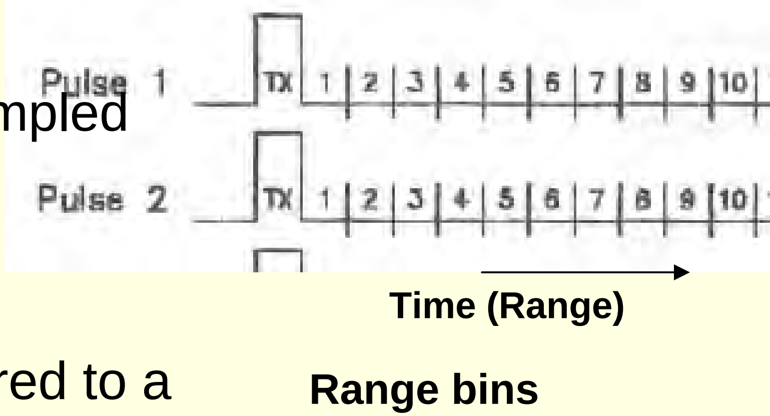
$$r(t) = As(t-\tau_0) + noise,$$

  - Detect the presence of a target
  - Estimate location
    - Range estimate from its relation to the time delay $\tau_0 = 2r_0/c$
    - Bearing
  - Estimate velocity
  - Track
  - Identify

Range resolution cell

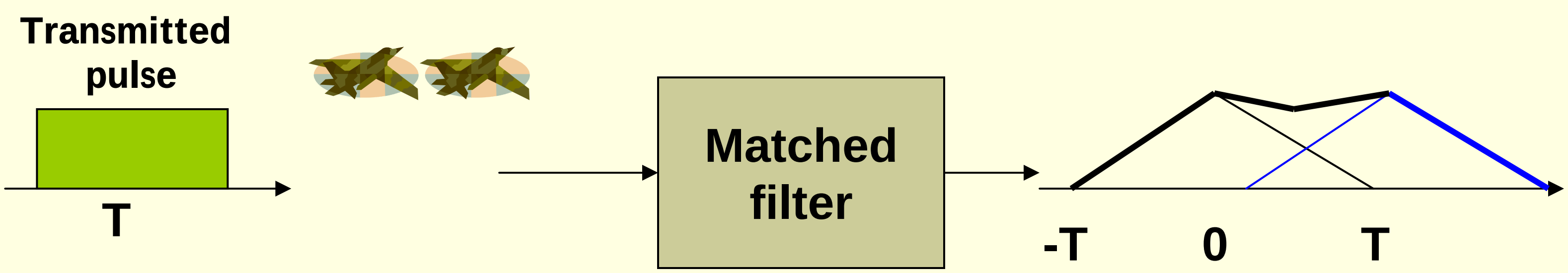
Detection/location estimation

- Receiver output is matched filtered and sampled
- Sampling rate ≈ radar bandwidth
- Each sample marks a *resolution cell*
- Samples in each resolution cell are compared to a threshold to determine target detection
- Detection and estimation are merged

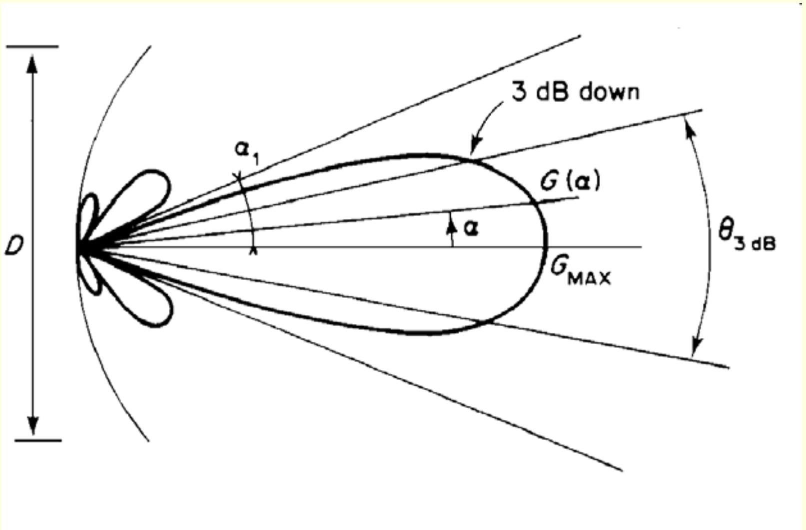


Resolution cell

How close can targets be and still be distinguished?



Radar measurements



Bearing estimation

- Spatial resolution ~ λ/D, λ is carrier wavelength, D is antenna aperture length

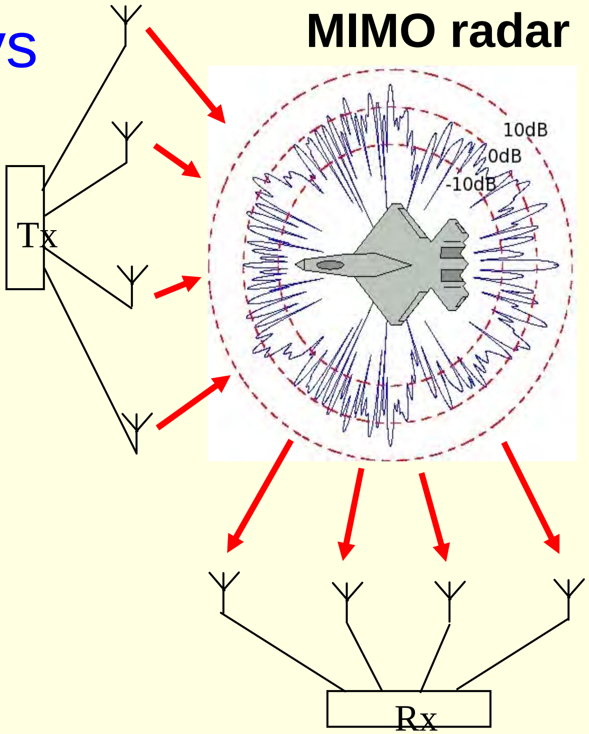
Range rate

- Range rate resolution scales with 1/duration of observation.
- Uncertainty principle: it is not possible to measure both range and range rate with arbitrary resolution

What is MIMO radar?

MIMO radar: a radar system that employs multiple transmit and receive elements, and has the ability to jointly plan transmissions and process received signals.

MIMO radar



Antenna elements of MIMO radar can be co-located or distributed

Key features

- Multiple transmit elements
- Multiple receive elements
- Distributed architecture
- Tight cooperation among system elements

Point scatterer

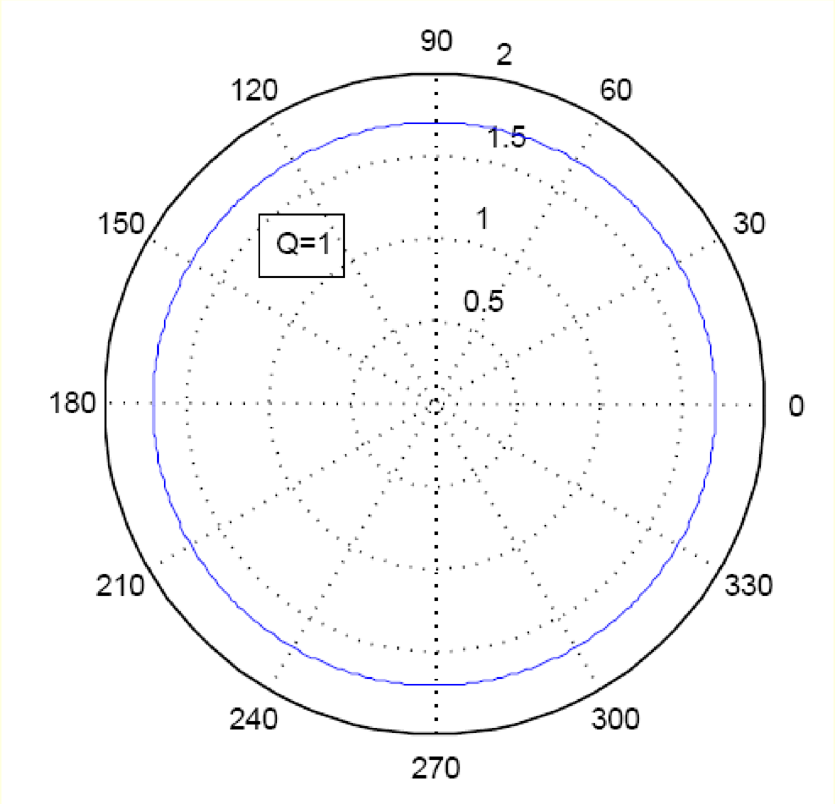
- Transmitted signal $s(t)$
- Received signal from point scatterer

  $r(t) = \varsigma s(t - \tau_0) + n(t)$

  $\varsigma =$ complex reflection coefficient

  $\sigma = |\varsigma|^2$ radar cross section (RCS)
- RCS of sphere = πa^2 surface area

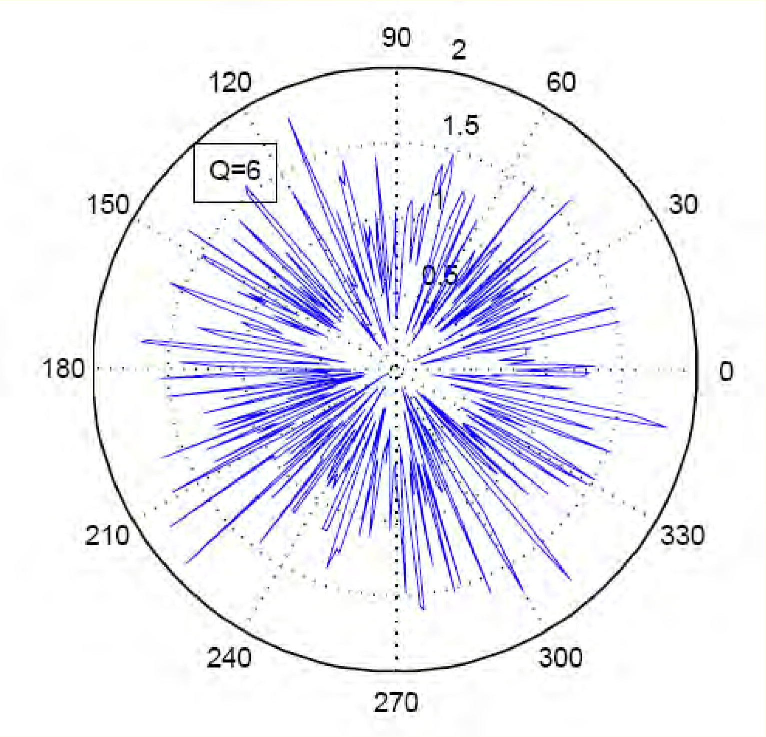
  of cross section through origin



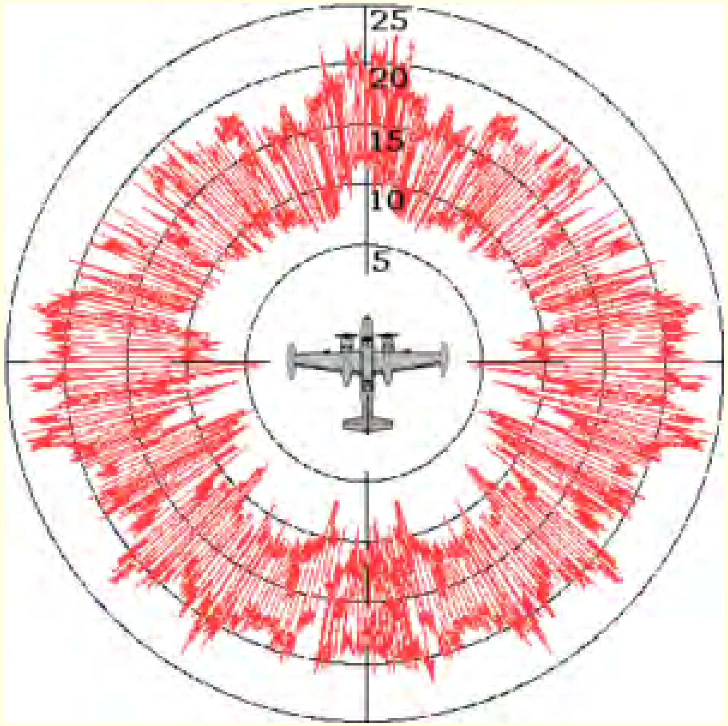
RCS of point scatterer (sphere)

- RCS measures the ability of targets to reflect incident energy.

Multi-scatterer, complex targets



RCS of a collection of closely spaced 6 scatterers



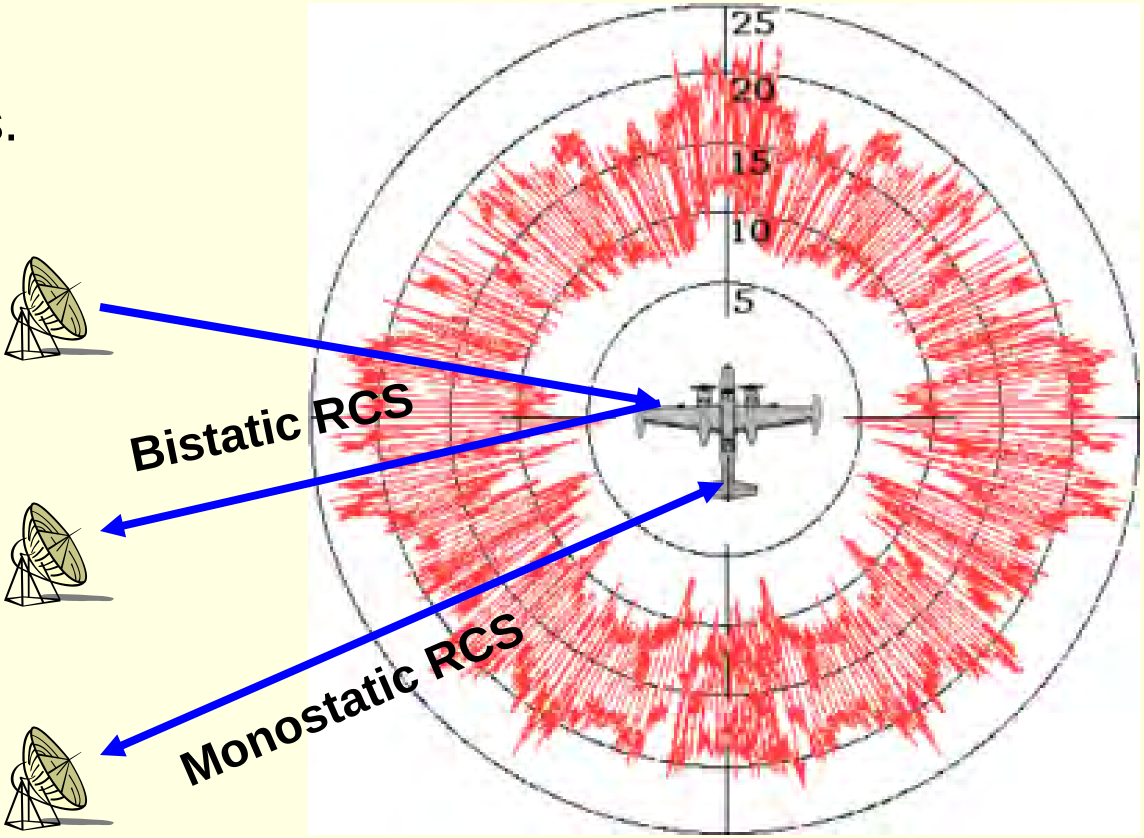
RCS of an airplane

Backscatter as a function of azimuth angle, 10-cm wavelength [Skolnik 2003].

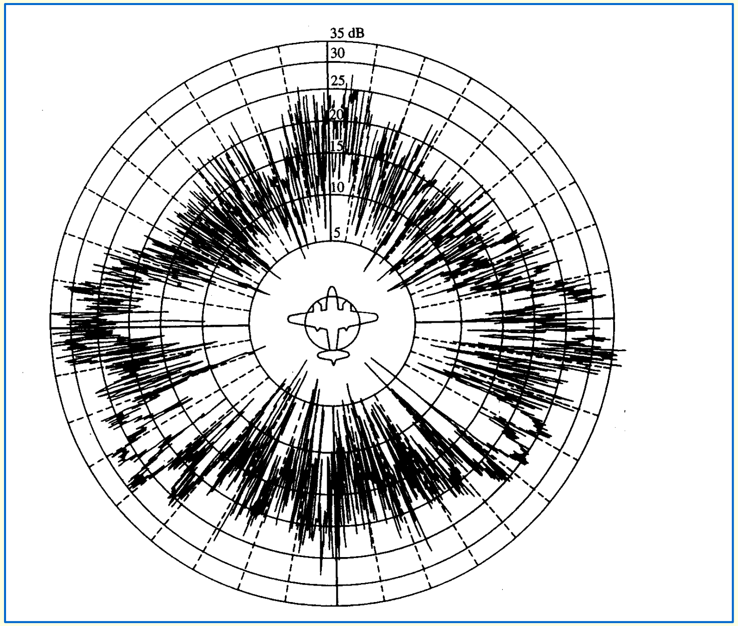
- Complex targets exhibit RCS with many lobes

Monostatic/Bistatic/Multistatic

- Monostatic RCS: transmitter and receiver are collocated.
- Bistatic RCS: transmitter and receiver are separated
- Multistatic RCS: multiple transmitters and receivers.



Advantages of MIMO radar



Distributed transmit and receive elements

Detection/localization

- RCS diversity
- Localization by multilateration

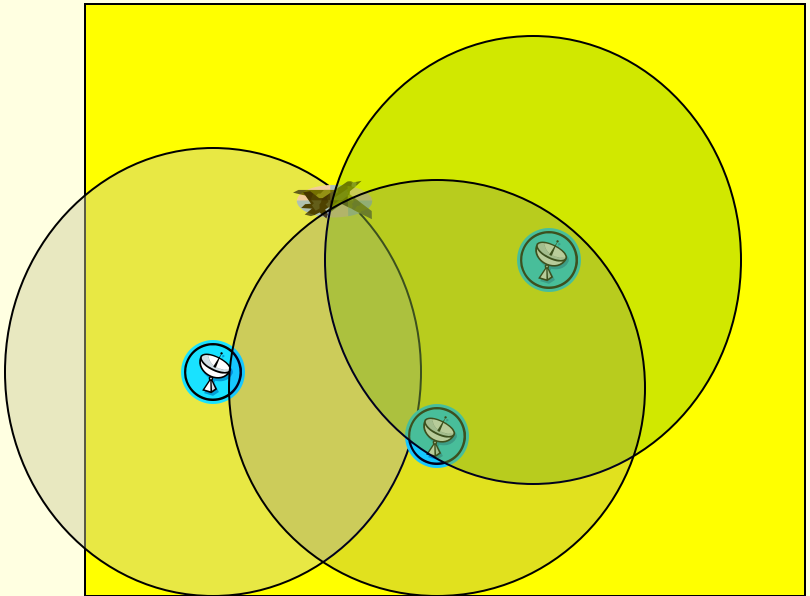
Velocity estimation

- Improved Doppler processing through diversity of look angles and mitigation of the problem of low radial velocities

Identification/high resolution localization

"Large aperture"

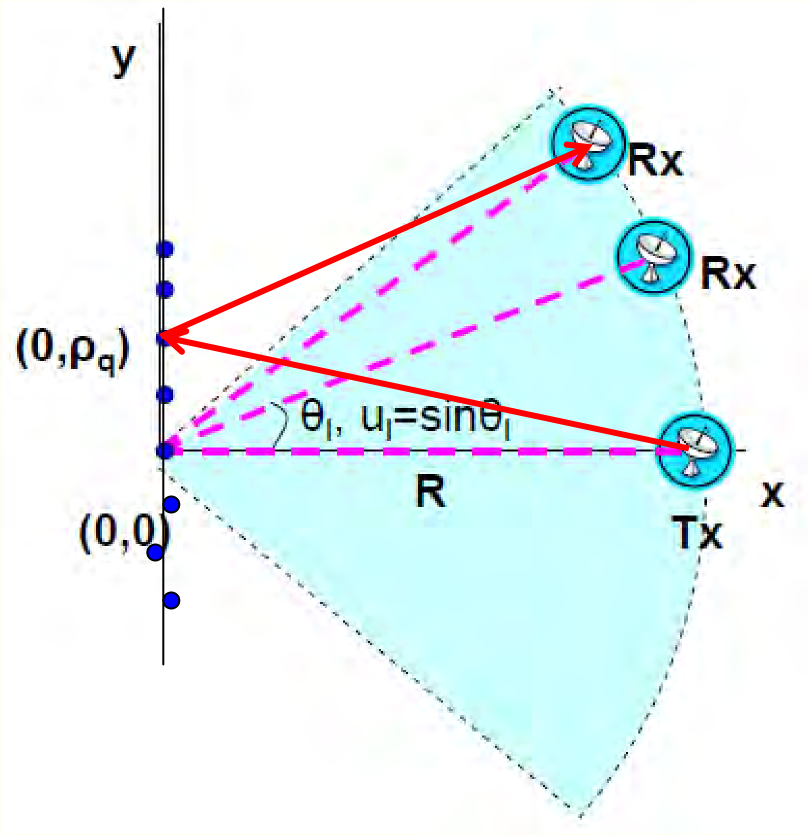
- High resolution localization with coherent mode



Signal model

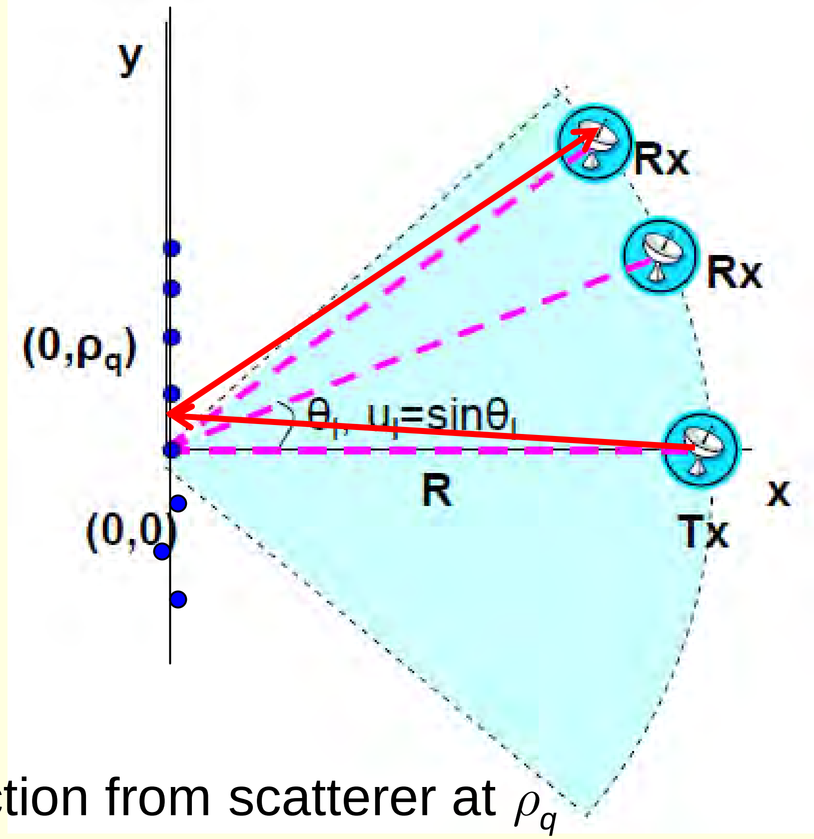
- System elements are arrayed on arc of radius R
- Target much smaller than distance to system elements
- Target constituted of point scatterers along y-axis

- Each scatterer has complex reflection coefficient ς_q
- Coefficient ς_q modeled random



Signal model

- Time delay from element k to sensor l through origin = 0
- Time delay from element k to sensor l through ρ_q



$$\tau_{kl}\left(\rho_q\right) = 2\pi\rho_q\left(u_k + u_l\right)$$

where $u_k = \sin\theta_k$

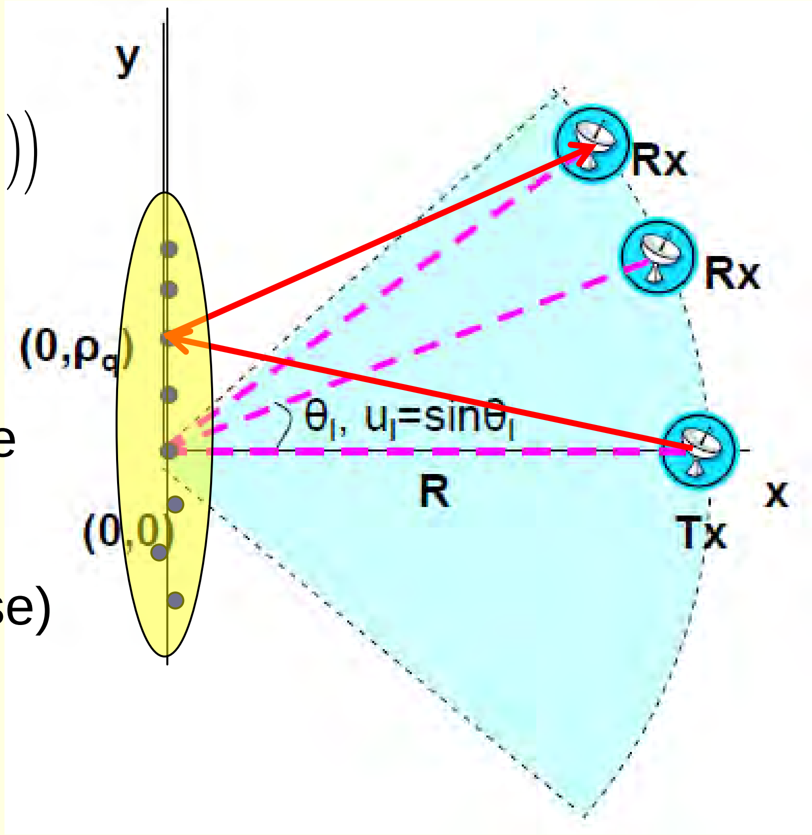
- Signal observed at sensor ℓ due to to transmission from sensor k and reflection from scatterer at ρ_q

$$r_{kl}\left(t\right) = \varsigma_q e^{-j2\pi\rho_q\left(u_k + u_l\right)} s_k\left(t - \tau_{kl}\left(\rho_q\right)\right)$$

Signal model

- Target constituted of Q point scatterers
- Target response

$$r_{kl}(t) = \sum_{q=1}^{Q} \varsigma_q e^{-j2\pi\rho_q(u_k+u_l)} s_k\left(t - \tau_{kl}\left(\rho_q\right)\right)$$



Narrowband assumption:

change of $s_k(t)$ across target is negligible

Signal observed at sensor l (ignoring noise)

$$r_{kl}(t) \approx h_{kl}(\rho_0) s_k\left(t - \tau_{kl}(\rho_0)\right)$$

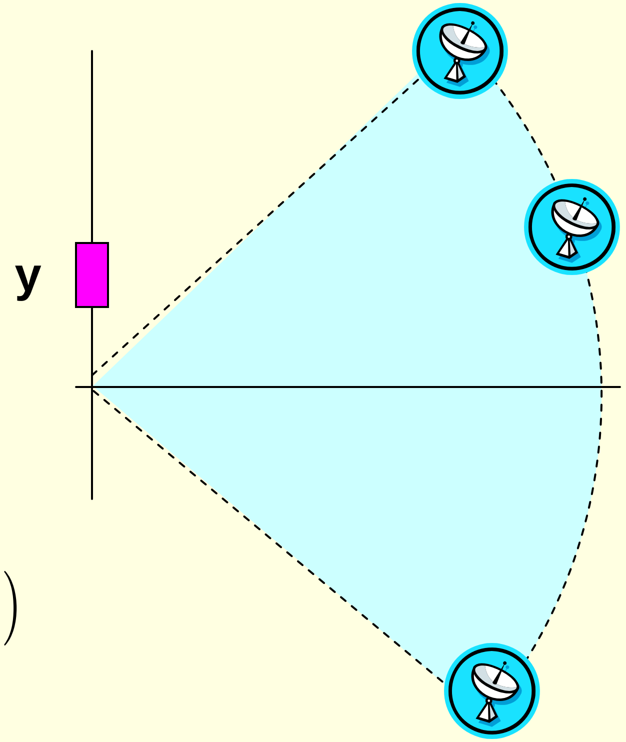
Target reflection coefficient

$$h_{kl}(\rho_0) = \sum_{q=1}^{Q} \varsigma_q e^{-j2\pi\rho_q(u_k+u_l)}$$

Non-coherent processing (1/2)

[Fishler, Haimovich, Blum,..2004, 2006]

- MIMO radar with M transmit elements and N receive elements
- Elements are time synchronized, but not phase synchronized
- Test for a target presence at y



- Match filter received signal with $s_k^*\left(t-\tau_{kl}\left(y\right)\right)$

Detection metric based on log-likelihood

$$T\left(y\right)=\sum_{l=1}^{N}\sum_{k=1}^{M}\left|\int r_l\left(t\right)s_k^*\left(t-\tau_{kl}\left(y\right)\right)dy\right|^2$$

$$=\sum_{l=1}^{N}\sum_{k=1}^{M}\left|h_{kl}\left(\rho_o\right)\right|^2\left|\sum_{k'=1}^{M}\int s_{k'}\left(t-\tau_{kl}\left(\rho_o\right)\right)s_k^*\left(t-\tau_{kl}\left(y\right)\right)dy\right|^2+\text{noise}$$

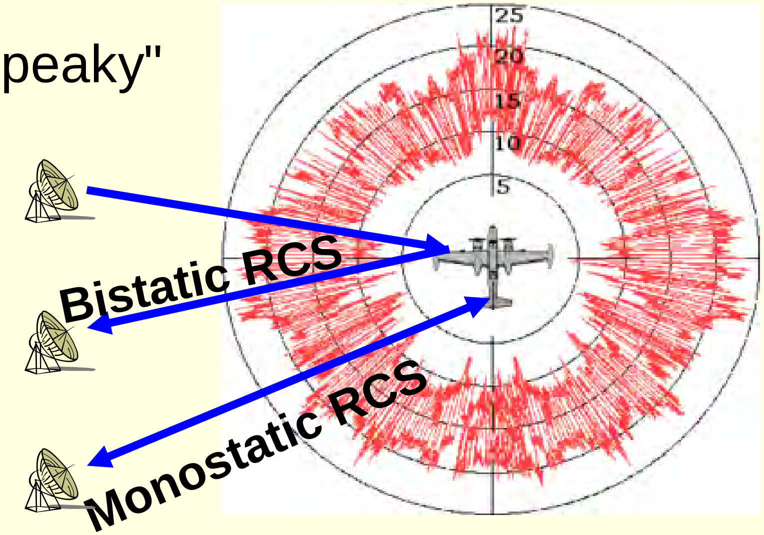
Non-coherent processing (2/2)

- Detection metric can be written

$$T(y) = \sum_{l=1}^{N} \sum_{k=1}^{M} \left| h_{kl}(\rho_o) \right|^2 A\left(\tau_{kl}(\rho_o) - \tau_{kl}(y)\right) + \text{noise}$$

$$\left| h_{kl}(\rho_o) \right|^2 = \text{ bistatic RCS for pair of elements k,l}$$

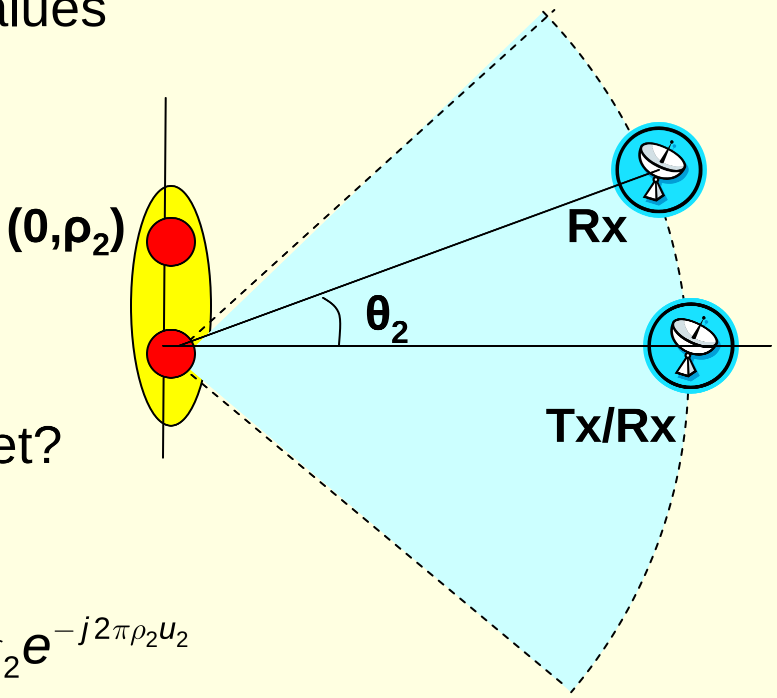
- If a target is detected, its location is known to within a resolution cell
- To improve resolution $\rightarrow$ make $A(\cdot)$ "peaky" (wideband waveforms $s_k(t)$)
- To improve detection probability $\rightarrow$ diversity of RCS values $\left| h_{kl}(\rho_o) \right|^2$



Spatial decorrelation (1/2)

- The condition for RCS diversity, is that values of $h_{kl}\left(\rho_o\right)$ are uncorrelated,

$$E\left[h_{kl}h_{kl'}^{*}\right] \approx 0$$



Simple example

- What are the conditions for uncorrelated target coefficients of a two-scatterer target?

$$u_1 = 0, \quad u_2 = \sin\theta$$

$$h_{11} = \sum_{q=1}^{Q} \varsigma_q e^{-j4\pi\rho_q u_1} = \varsigma_1 + \varsigma_2, \quad h_{12} = \varsigma_1 + \varsigma_2 e^{-j2\pi\rho_2 u_2}$$

$$E\left[h_{11}h_{12}^{*}\right] = E\left[\left|\varsigma_1\right|^2\right]\left(1 + e^{j2\pi\rho_2 u_2}\right)$$

$$\left|E\left[h_{11}h_{12}^{*}\right]\right| = \left|\cos\left(\pi\rho_2 u_2\right)\right|$$

- For a two $-$ scatterer target, no separation among sensors can ensure uncorrelarted target coefficients.

Spatial decorrelation (2/2)

- Target comprising many scatterers

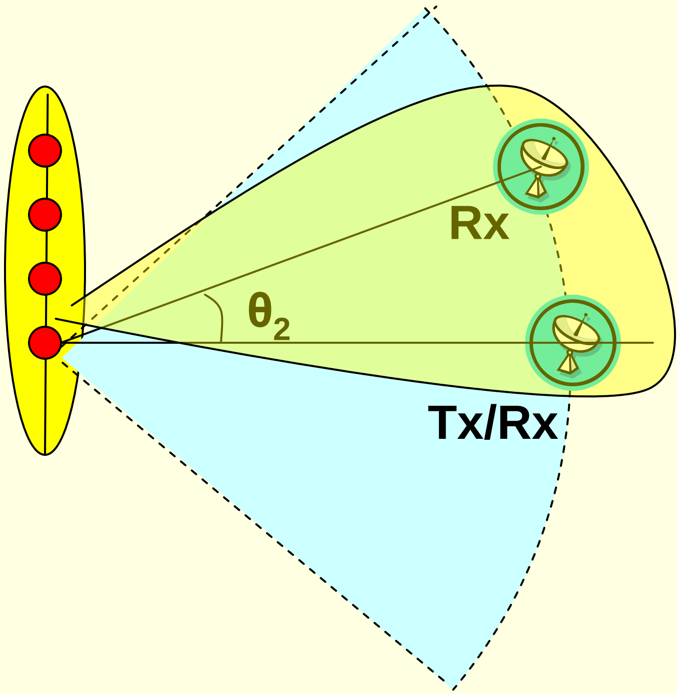
Assume scatterers uniformly placed $\rho_q = p(q-1)$

$$h_{11} = \sum_{q=1}^{Q} \varsigma_q e^{-j4\pi\rho_q u_1} = \sum_{q=1}^{Q} \varsigma_q,$$

$$h_{12} = \sum_{q=1}^{Q} \varsigma_q e^{-j2\pi p(q-1)u_2}$$

$$E\left[h_{11}h_{12}^*\right] = E\left[\left|\varsigma_1\right|^2\right]\frac{1}{Q}\sum_{q=1}^{Q} e^{-j2\pi p(q-1)u}$$

$$\left|E\left[h_{11}h_{12}^*\right]\right| = \left|\frac{\sin(\pi pQu)}{Q\sin(\pi pu)}\right|$$



The target coefficients decorrelate if the two sensors do not fall within the beam of the antenna whose baseline is formed by the scatterers.

- Decorrelation of target coefficients when:
  - pQu >> 1 or u >> 1/pQ
  - Q >> 1

Example: Non-coherent localization

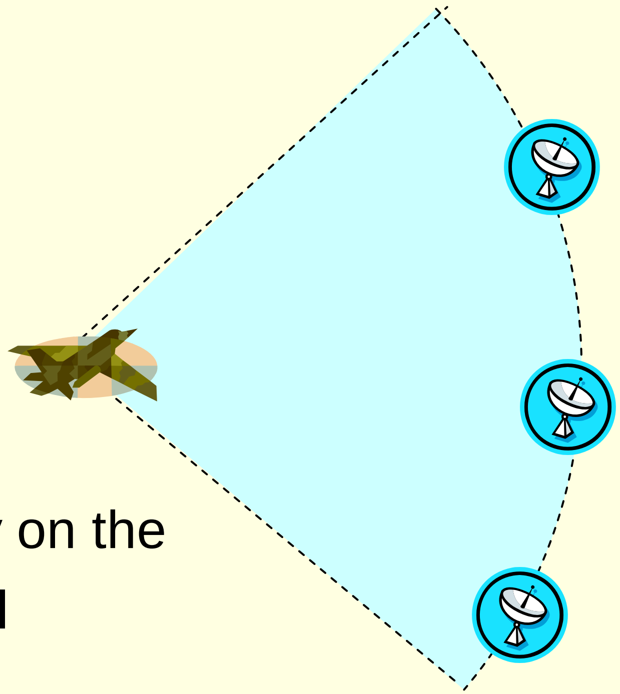
- Multi-scatterer target, two-sensor system

$$T(y) = \left|h_{12}(\rho_o)\right|^2 A\left(\tau_{12}(\rho_o) - \tau_{12}(y)\right)$$

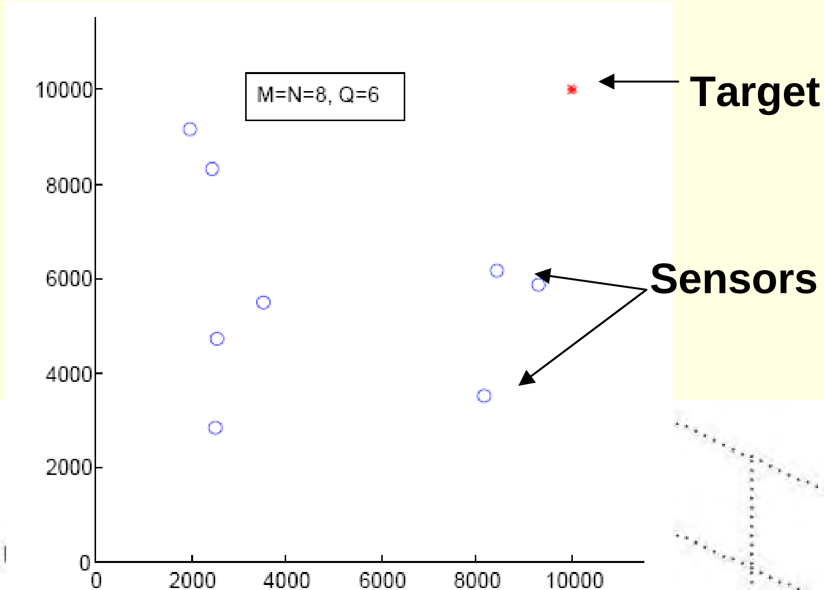
$$h_{12}(\rho_o) = \sum_{q=1}^{Q} \varsigma_q e^{-j2\pi p(q-1)u_2}$$

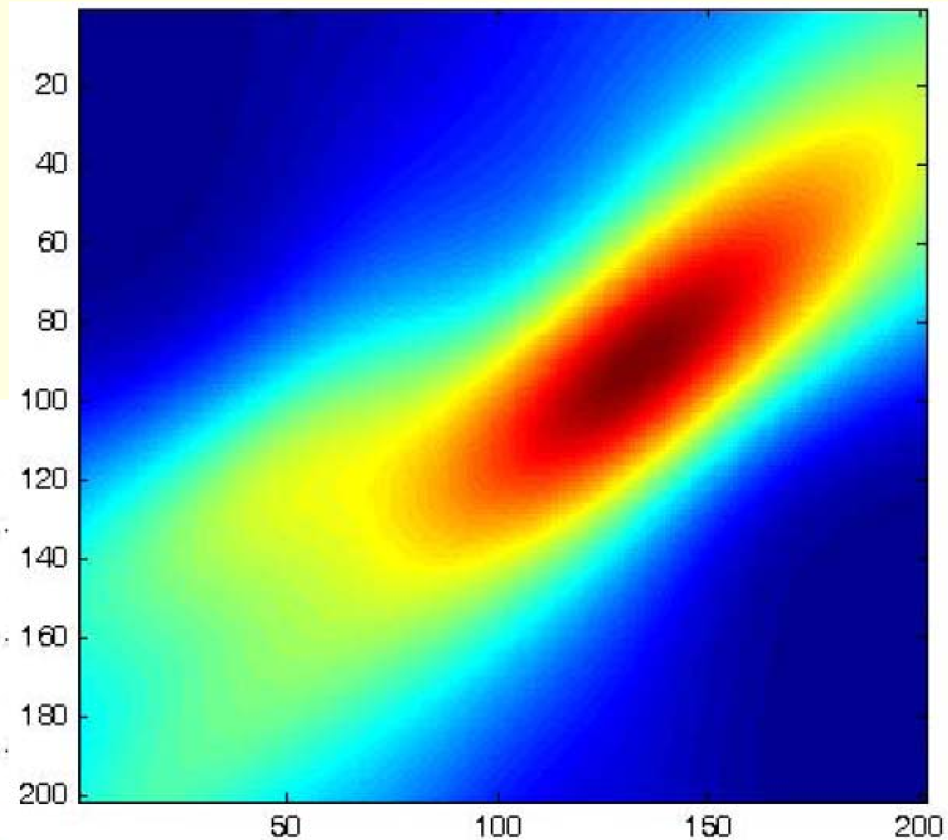
$$\left|h_{12}(\rho_o)\right|^2 = \text{ bistatic RCS not function of y}$$

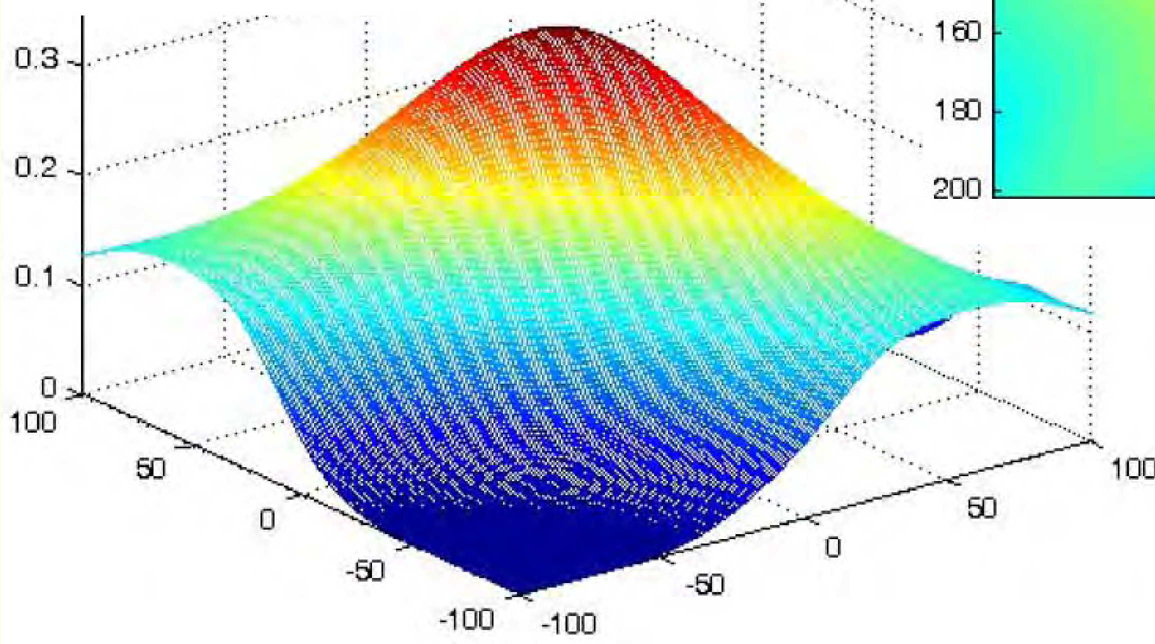
- Non-coherent detection metric depends only on the autocorrelation function of transmitted signal



Example

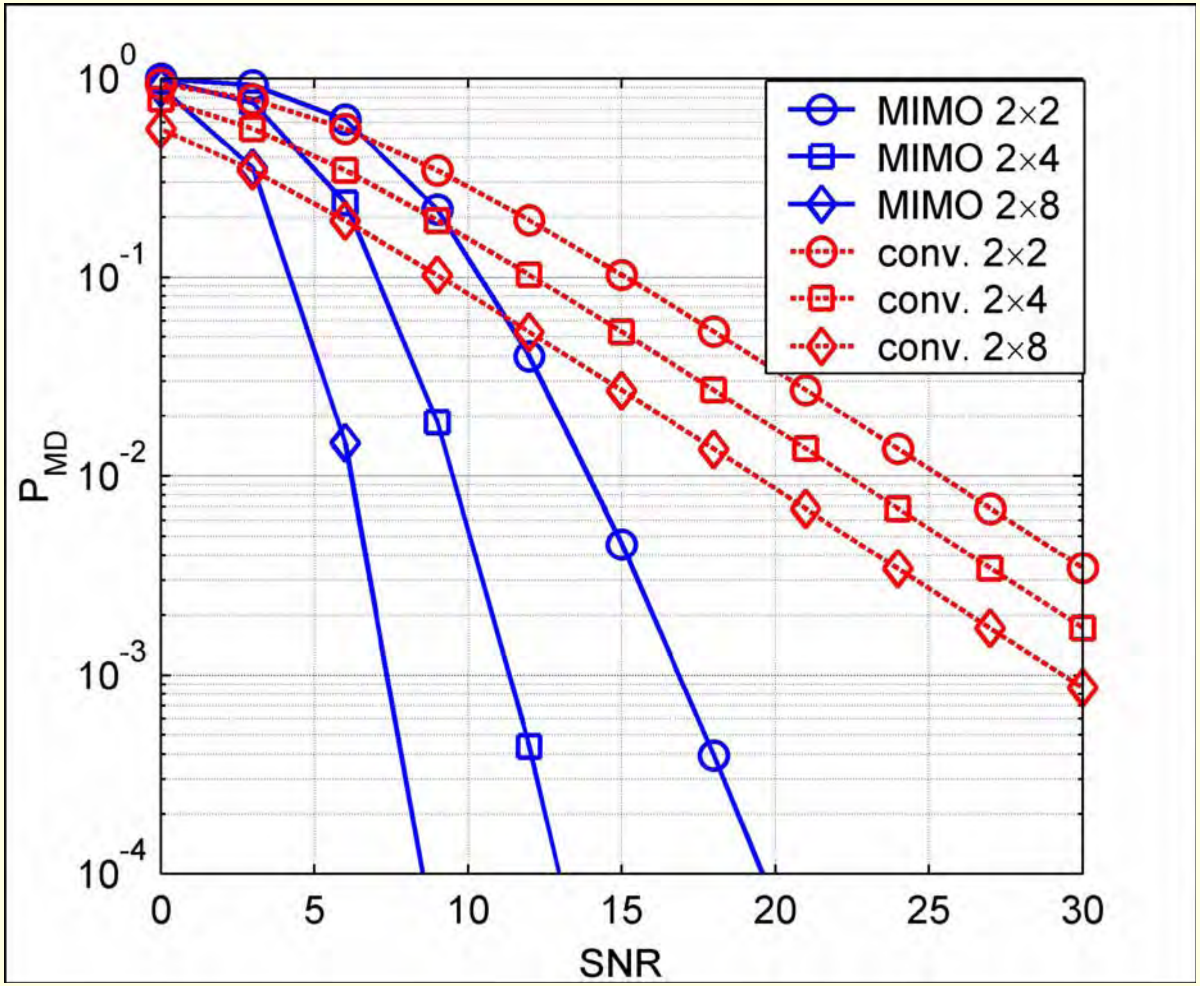






Non-coherent processing
8 transmit x 8 receive
Target consists of 6 scatterers

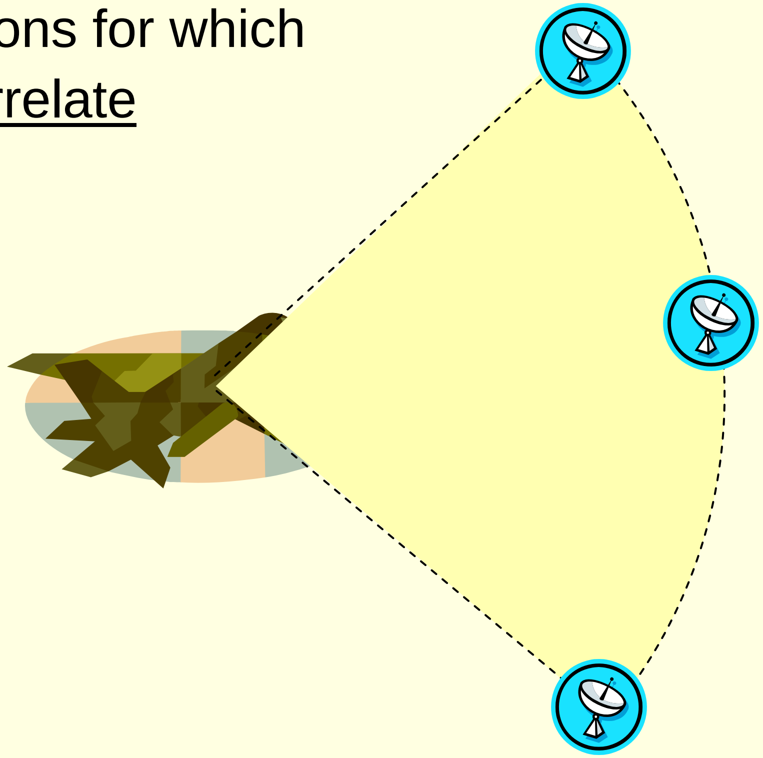
Spatial diversity gain in radar



Miss probability of MIMO radar compared to conventional phased-array.
Miss probability is plotted versus SNR for a fixed false alarm probability of 10^{-6}.

Coherent MIMO radar scenario

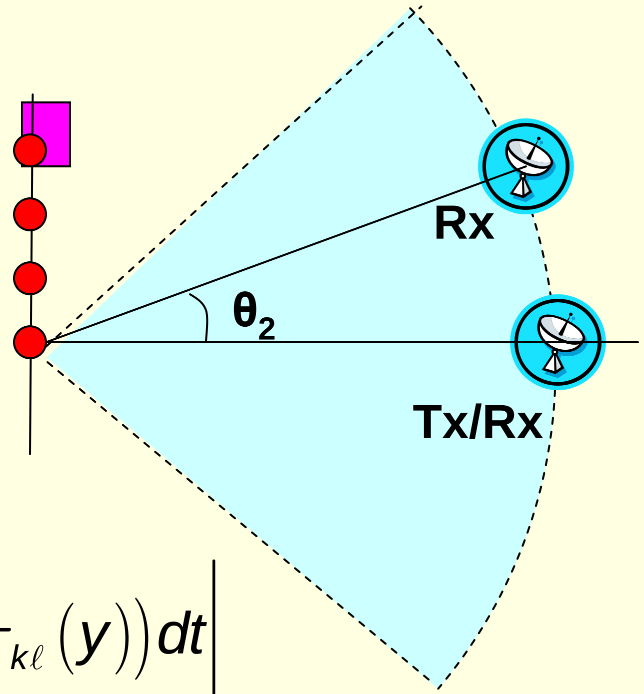
- In the noncoherent mode, localization resolution is limited by bandwidth.
- The coherent mode overcomes the bandwidth limitation. It seeks to measure parts of the target over which coherency is preserved.
- Coherency is preserved over sections for which the target coefficients <u>do not decorrelate</u>



Coherent localization

[Lehmann, Haimovich, Blum...2006]

- For a hypothetical target at position y, coherent processing consists of:
  - Matched filtering with $s_k^*\left(t-\tau_{k\ell}(y)\right)$
  - Compensate phase with $\omega_c \tau_{k\ell}(y)$



- The coherent MIMO radar metric is given by

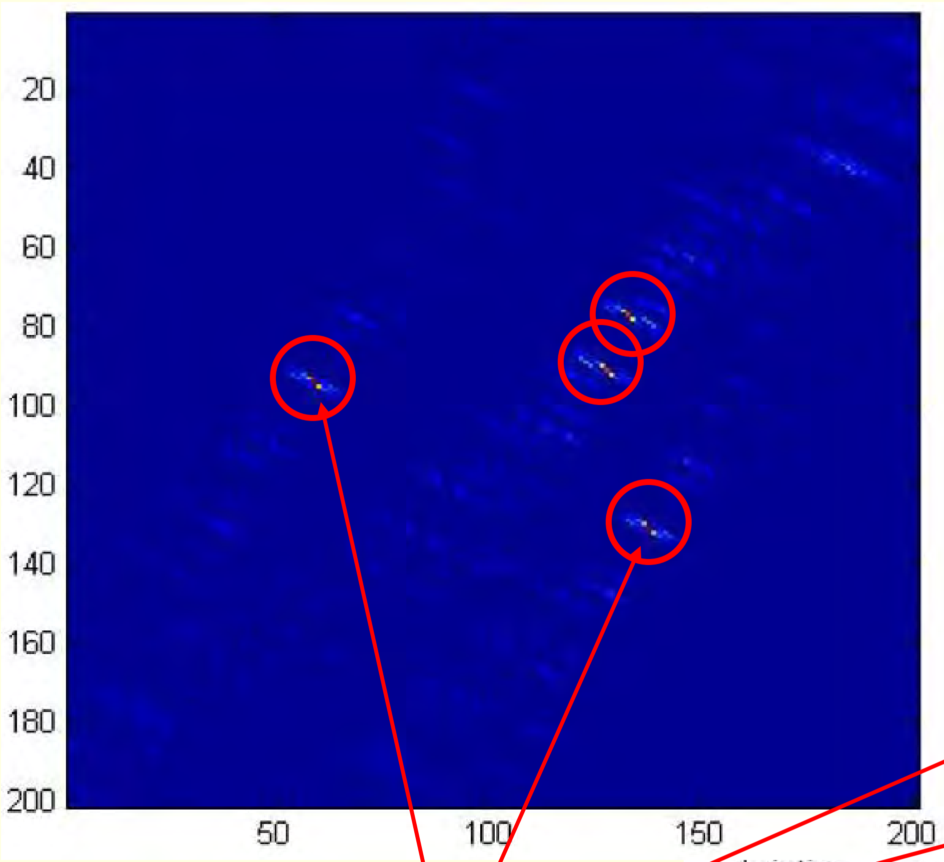
$$T_c(y)=\left|\sum_{\ell=1}^{N}\sum_{k=1}^{M} e^{-j\omega_c\left(\tau_{k\ell}(\rho_0)-\tau_{k\ell}(y)\right)}\int r_\ell(t)s_k^*\left(t-\tau_{k\ell}(y)\right)dt\right|$$

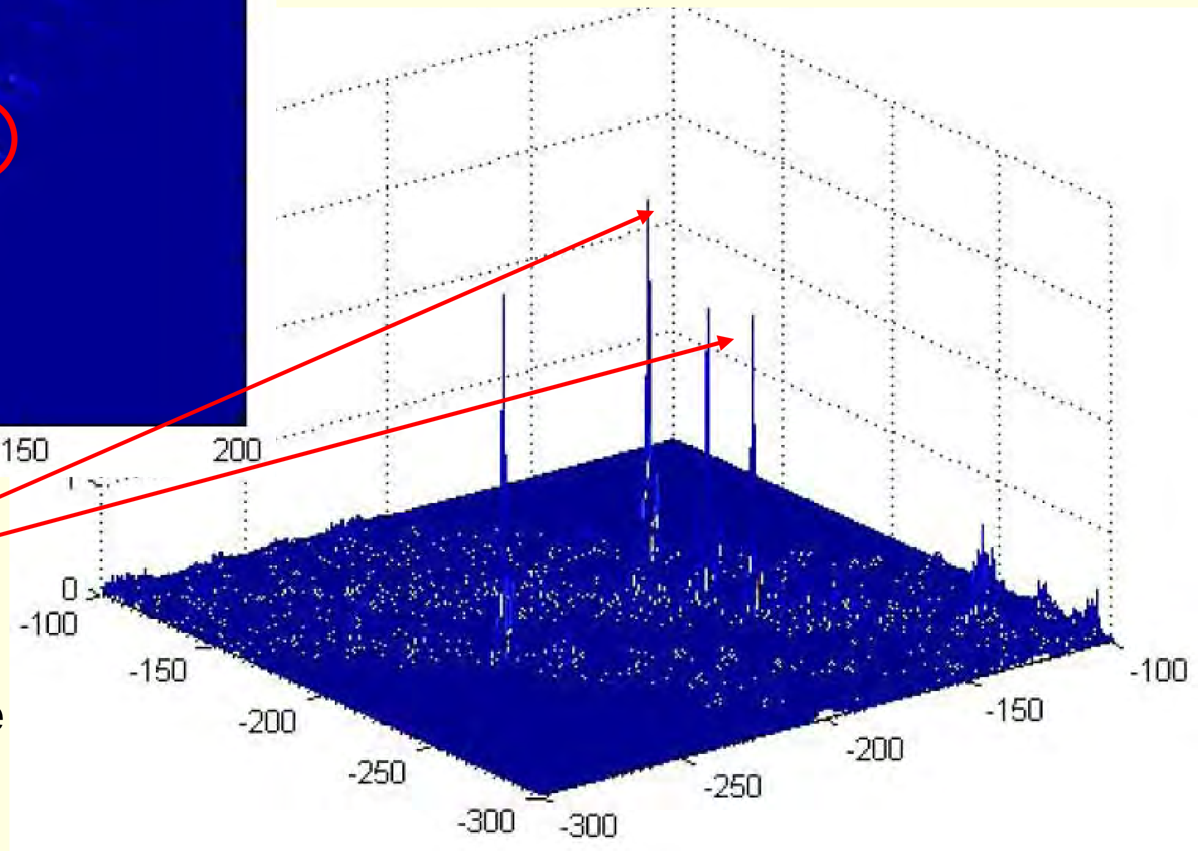
$$\underset{\text{noiseless}}{=}\left|\sum_{\ell=1}^{N}\sum_{k=1}^{M} e^{-j\omega_c\left(\tau_{k\ell}(\rho_0)-\tau_{k\ell}(y)\right)}A_k\left(\tau_{k\ell}(\rho_0)-\tau_{k\ell}(y)\right)\right|$$

- High resolution localization: y and ρ_0 closer than can be distinguished by bandwidth, $A_k\left(\tau_{k\ell}(\rho_0)-\tau_{k\ell}(y)\right)\approx 1$

Example

- Features of coherent MIMO radar:
  - High resolution of the order of carrier frequency
  - Ambiguity peaks





Targets

Beamforming example
8 transmit x 8 receive

Cramer-Rao lower bound

- The Cramer-Rao lower bound on localization accuracy of single target [Godrich, Haimovich, Blum…2008]:

$$\sigma_{CRLB} = \frac{c}{\sqrt{8\pi^2 \,\mathrm{SNR}\left(f_c^2 + \beta^2\right)}} \cdot g$$

$$\approx \frac{c}{\sqrt{8\pi^2 \,\mathrm{SNR}} f_c} \cdot g$$

- The geometric dilution of position (GDOP) term *g* incorporates the effect of sensor locations on the localization error
- Improvement due to MIMO sensors

$$GDOP = \sqrt{\frac{2}{MN}}$$

- The term β is the effective bandwidth
- Localization error is approximately proportional to 1/ f_c
- The effective bandwidth β has little impact
- CRLB multiple targets [Godrich, Haimovich, Blum 2008]

Concluding remarks

- Non-coherent MIMO radar seeks to exploit target RCS diversity to improve detection and estimation performance.
- Coherent MIMO radar supports high resolution, albeit ambiguous target localization.
- Ambiguities can be controlled through the number of sensors.
- Localization with coherent MIMO radar exhibits an error that scales with carrier wavelength
- The use of multiple sensors improves localization accuracy by a factor as low as $\sqrt{2/MN}$

Open questions

- MIMO radar signal optimization for range and range rate estimation
- Signals with low cross correlations over a range of delays
- Signal design for reducing localization ambiguities
- Study the statistics of ambiguities and relations to the various parameters: carrier frequency, bandwidth, number of sensors
- Characterizing the performance of MIMO radar at low SNR (in the presence of noise ambiguities)
- Handling multiple targets