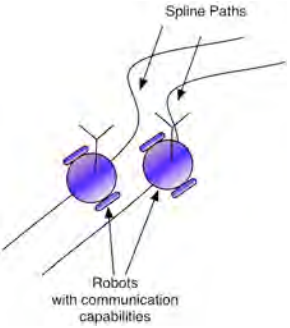
Multi-Vehicle Path Coordination in Support of Communication

Pramod Abichandani, Hande Benson, Moshe Kam

Data Fusion Laboratory, Drexel University

Multi-vehicle Path Coordination

- Coordination of motion of n vehicles in a shared workspace so that they avoid collisions
  - A widely studied problem
- We study this problem along fixed paths and under Communication Connectivity constraints
- **Path Planning**: plan spatial paths around obstacles
  - Obstacles: Stationary or moving
- **Path Coordination**: coordinate motion along fixed paths by changing speeds
  - Avoid collisions if paths overlap



Why Path Coordination?

- Quite often in practical applications
    - Paths are fixed
    - Multiple vehicles travel along different or same paths
    - One cannot choose arbitrary paths
- Examples
    - Motion in urban environments (freeways, lanes) - 2007 DARPA Urban Challenge
    - Search and Rescue
    - Intelligent Vehicle Highway Systems (IVHS)

Our Goals

- Plan the velocities of a group of n mobile vehicles confined to fixed paths and seeking to arrive from a set of initial points to specified final destinations
- Minimize the maximum task/scenario completion time
    - T_{max}
- Remain in communication range of k other co-travelers at all times
    - A set of rich constraints is possible here

Previous Work

- **Kant and Zucker** - "Towards efficient trajectory planning: The path-velocity decomposition," International Journal of Robotics Research, 5(3):72-89, 1986.
- **S. LaValle and S. Hutchinson** - "Optimal motion planning for multiple robots having independent goals," *IEEE Transactions on Robotics and Automation*, vol. 14, no. 6, pp. 912925, 1998.
- **P. O' Donnell and T. Lozana - Perez** -"Deadlock-free and collision-free coordination of two robot manipulators," Proceedings of the IEEE International Conference on Robotics and Automation (ICRA), Scottsdale, AZ, 1989, pp. 484489.
- **Peng and Akella** - "Coordinating multiple robots with kinodynamic constraints along specied paths," The International Journal of Robotics Research, vol. 24, no. 4, pp. 295310, 2005.
- **Abichandani, Benson, and Kam** - "Multi-vehicle path coordination under communication constraints," Proceedings of the American Control Conference (ACC08), Seattle,WA, June 2008.

Outline

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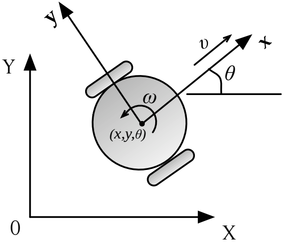
1. Path Planning, Path Coordination, Our Goals
2. Models - Robots, Path, Communication
3. Problem Formulation
4. Numerical Results
5. Decentralized Formulation
6. Conclusion and Future Work

Robot Architecture and Model

- Two wheeled differential drive mobile robots
- Non-holonomic constraints



Equations of Motion

$$\dot{x} = v\cos(\theta); \quad \dot{y} = v\sin(\theta); \quad \dot{\theta} = \omega$$

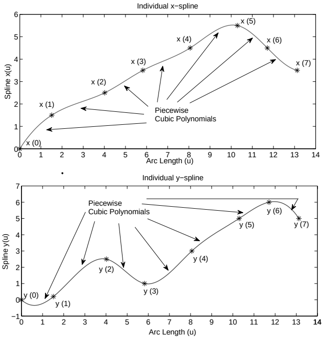
$$\dot{x}\sin(\theta) - \dot{y}\cos(\theta) = 0.$$

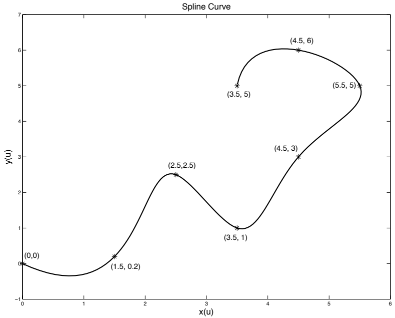
- v and ω are the linear and angular velocities of the robot respectively;
- x, y and θ are the coordinates of the robot with respect to the global (X, Y) coordinate system.

Paths

- Two dimensional cubic spline curves are used to represent the paths of the robots
- Parametric curves with arc length u along the curve as the parameter
- Obtained by combining two one dimensional piecewise cubic splines $x(u)$, and $y(u)$
- Continuous first and second derivatives
- Result in smooth, kinematically feasible paths

Two Dimensional Cubic Splines





Communication Model

- The mobile robots form a MANET
- Free space path loss : Friis's equation

Path Loss

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d} \right)^{\alpha}$$

  - P_t = Transmitter power
  - P_r = Received power
  - d = Euclidean distance
  - α = Path loss exponent
- Shadowing and fading ignored in this exposition
- Signal to Interference and Noise Ratio (SINR)
  - SINR $>$ Threshold $\Rightarrow$ Communication $C^{ij}(t) = 1$
  - SINR $<$ Threshold $\Rightarrow$ Communication $C^{ij}(t) = 0$
- $C^{ij}(t)$ indicates whether robots i and j are in communication range at time t

Outline

Optimization Model Formulation

- Mobile robots equipped with wireless nodes
- Constraints on
    - Dynamics
    - Kinematics
    - Collision Avoidance
    - Communication Connectivity
    - Network Partition Elimination
- Time is discretized: t represents steps in time
- Mixed Integer Nonlinear model :
    - Continuous variables - Speeds and distances
    - Discrete variable - Communication, Arrival

Objective Function

- Minimize T_{max}: the time of arrival of the last arriving robot

Definition of T_{max}

$$A^i(t) = \begin{cases} 0, & \text{if } (d^i_{goal}(t) \neq 0) \\ 1, & \text{if } (d^i_{goal}(t) = 0) \end{cases}$$

$$T\text{max} = \max_{i=1,\ldots,n} \left(\sum_{t=0,\ldots,T} (1 - A^i(t)) \right)$$

- $d^i_{goal}(t)$ is the distance from the goal for i^{th} robot at time step t and n is the total number of robots

Dynamic Constraints

- Upper and lower bounds on speeds, accelerations
- $s^i(t)$ = speed of robot i at time t

Bounds on speeds and accelerations

$$s_{min} \leq s^i(t) \leq s_{max}$$

$$\dot{s}_{min} \leq \dot{s}^i(t) \leq \dot{s}_{max}$$

- Assumption
    - maximum curvature of the path is within the achievable bounds of the angular velocity and radial acceleration of the robots
    - the angular velocity $\omega^i(u)$ corresponding to the optimal speed $s^i(t)$ will always be achievable

$$\omega^i(u) = s^i(u)\kappa^i(u)$$

  - $\kappa^i(u)$ is the curvature at u

Kinematic Constraints

- Continuous paths in the form of splines $ps^i(t)$

$$(x^i(t), y^i(t)) = ps^i(u^i(t))$$

- Fixed start point o^i and end points e^i

$$\begin{aligned}(x^i(0), y^i(0)) &= o^i \\ (x^i(T), y^i(T)) &= e^i\end{aligned}$$

- Arc length $u^i(t)$ increased based on speed

$$\begin{aligned}u^i(0) &= 0 \\ u^i(t) &= u^i(t-1) + s^i(t)\Delta t\end{aligned}$$

Collision and Communication Jamming Avoidance Constraints

- Distance between robots should be greater than d_{safe} to avoid collisions at all times

Collision Avoidance

$$d^{ij}(t) \geq d_{safe}$$

- Remain outside the jamming range of the jammer robots

Communication Jamming Avoidance

$$d^{im}(t) \geq d_{jam}$$

One Hop Communication Connectivity Constraints

Each robot is in communication with at least k other robots at all time steps

One Hop Communication Connectivity Constraint

$$C^{ij}(t) = \begin{cases} 0, & \text{if } (SINR_r^{ij}(t) \leq \tau) \\ 1, & \text{otherwise} \end{cases}$$

$$\sum_{j:j \neq i} C^{ij}(t) \geq k$$

where the robots remain in communication with at least $k = 1, \ldots, n$ other robots at all time steps

Network Partitions and Elimination Constraints

- One hop communication constraint may result in situations (for $k \leq n/2$) where the network will be partitioned into subgroups of robots whose members can communicate within each group, but not across groups
- A constraint of the following form for every possible subgroup I of size $k+1, \ldots, n-k-1$ at each time period

Partition Elimination constraints

$$\sum_{i \in I, j \notin I} C^{ij}(t) \geq 1$$

- Ensures that the communication network is connected

Network Partition Elimination Algorithm

- Adding P.E. constraints for every possible subgroup for each time period to the problem would increase its size exponentially
- Instead, *Network Partition Elimination Algorithm* is used
    - Solves the problem without these constraints first and then detects any partitions in the solution
    - For each partition detected, add one P.E. constraint
    - Resolve the problem until no more partitions are detected

Algorithm 1 Partition Elimination (P.E.)

```
Require: $n$ robots with fixed paths
Ensure: Eliminate partitions
  repeat
    Solve the model.
    Let Done = FALSE
    for $t \in \{1, \dots, T\}$ do
      Let $\mathbb{I} = \{1\}$.
      for $i \in \mathbb{I}, j \notin \mathbb{I}$ do
        if $C^{ij}(t) > 0$ then
          $\mathbb{I} = \mathbb{I} \cup \{j\}$
        end if
      end for
      if $\mathbb{I} = \{1, \dots, n\}$ then
        Done = TRUE
      else
        add the constraint $\sum_{i \in \mathbb{I}, j \notin \mathbb{I}} C^{ij}(t) \geq 1$ to the model
      end if
    end for
  until Done = FALSE
```

Special Case: Presence of a Jammer Robot

- Jammer robots with similar mathematical model as the other robots
- Known initial and final location, fixed path
- Fixed jamming range d_{jam}
- Once a robot (other than the jammer robots) is within the jamming radius, it loses communication with all other robots
- **Assumption**: Known velocity profile
- In reality, an additional step to estimate the vehicle dynamics and velocity might be required

$$
\begin{array}{lll}
\text{minimize} & T_{\max} + \sigma \sum_{i,t} d^{i}_{goal}(t) & (5) \\
 & \forall i \in \{1, 2, \ldots n\}, \quad \forall t \in \{1, 2, \ldots T\}, & \\
 & \forall j \in \{1, 2, \ldots n\}, j \neq i & \\
 & \forall m \in \{1, 2, \ldots n_{jam}\} & \\
\text{subject to} & (x^{i}(0), y^{i}(0)) = o^{i} & (6) \\
 & (x^{i}(T), y^{i}(T)) = e^{i} & (7) \\
 & u^{i}(0) = 0 & (8) \\
 & u^{i}(t) = u^{i}(t-1) + s^{i}(t)\Delta t & (9) \\
 & (x^{i}(t), y^{i}(t)) = ps^{i}(u^{i}(t)) & (10) \\
 & & \\
 & s_{min} \leq s^{i}(t) \leq s_{max} & (11) \\
 & \dot{s}_{min} \leq \dot{s}^{i}(t) \leq \dot{s}_{max} & (12) \\
 & & \\
 & d^{ij}(t) \geq d_{safe} & (13) \\
 & d^{im}(t) \geq d_{jam} & (14) \\
 & & \\
 & 0 \leq A^{i}(t) \leq 1 & (15) \\
 & A^{i}(t) d^{i}_{\text{goal}}(t) = 0 & (16) \\
 & \forall i, T_{\max} \geq \left(\sum_{t=0}^{T} \left(1 - A^{i}(t) \right) \right) & (17) \\
 & 0 \leq C^{ij}(t) \leq 1 & (18) \\
 & l^{ij}(t) = \text{SNR}^{ij}_{r}(t) - \tau & (19) \\
 & C^{ij}(t) l^{ij}(t) \geq 0 & (20) \\
 & \sum_{j: j \neq i} C^{ij}(t) \geq k & (21)
\end{array}
$$

Outline

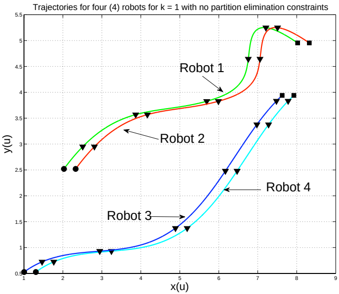
Simulation Setup

- Numerical testing performed in MATLAB + AMPL
- The solver LOQO was used to perform the optimization
- Results demonstrate
    - Trade off between T_{max} and the communication connectivity requirements in scenarios with and without jamming
    - Dependence of computation time on the number of robots
    - Effect of the P.E. algorithm on the problem size
- We consider the following cases
    - Active partition elimination constraints, no jammer
    - Active partition elimination constraints, jammer present

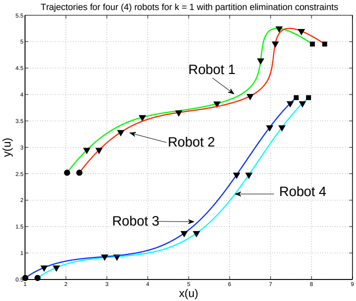
Parameter values for simulation

d_{safe}	0.01 m	s_{min}	0	s_{max}	2 m/s
$\dot{s}_{min}$	-1 m/s^2	$\dot{s}_{max}$	0.5 m/s^2	σ	100
d_{jam}	0.45 m	Δt	1 s	$\dot{s}_{jam}$	0 m/s^2
τ	4.5 x 10^{-3}	T	10	α	2

Partition elimination constraints, no jammer - 4 Robots

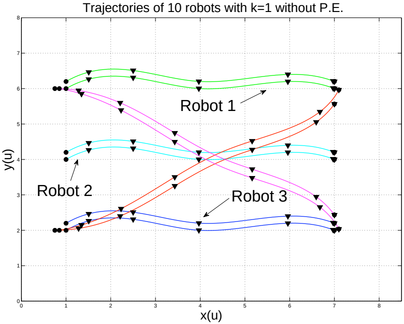

Trajectories for four (4) robots for k = 1 with no partition elimination constraints

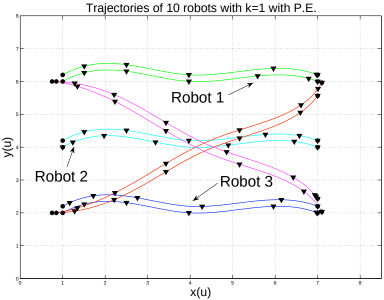
- No P.E. constraints
- $T_{\max} = 8$


Trajectories for four (4) robots for k = 1 with partition elimination constraints

- P.E. Algorithm added
- $T_{\max} = 8$

Partition elimination constraints, no jammer - 10 Robots



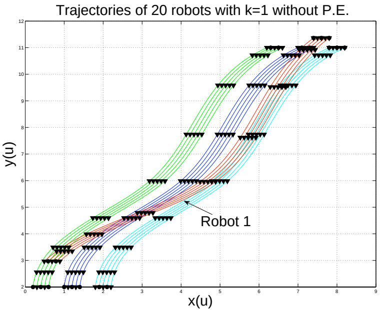


- No P.E. constraints
- $T_{\max} = 5$

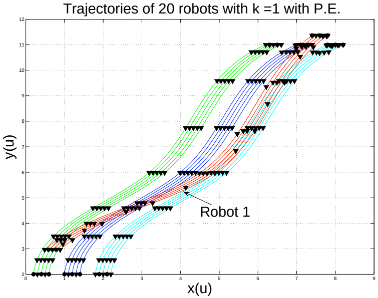
- P.E. Algorithm added
- $T_{\max} = 7$

Partition elimination constraints, no jammer - 20 Robots

Trajectories of 20 robots with k=1 without P.E.



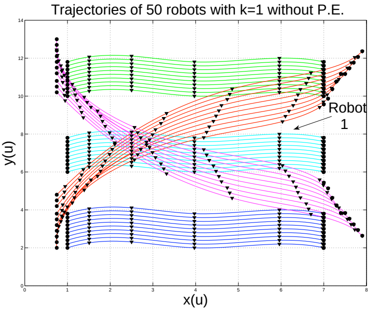
Trajectories of 20 robots with k =1 with P.E.

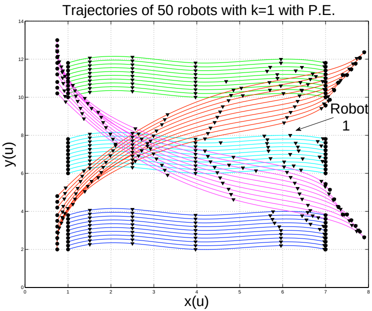


- No P.E. constraints
- $T_{\max} = 7$

- P.E. Algorithm added
- $T_{\max} = 8$

Partition elimination constraints, no jammer - 50 Robots

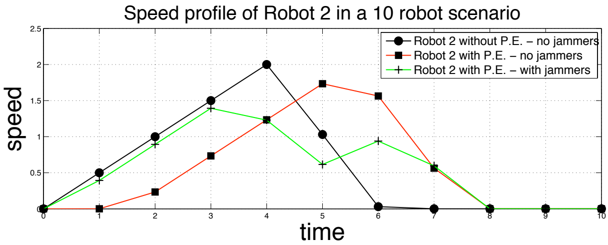

Trajectories of 50 robots with k=1 without P.E.

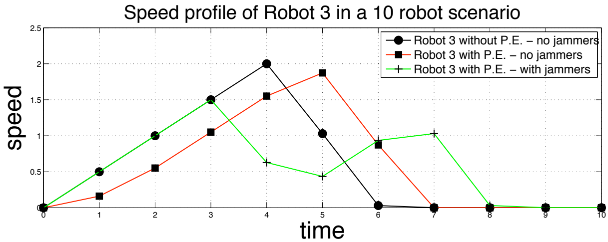

Trajectories of 50 robots with k=1 with P.E.

- No P.E. constraints
- $T_{\max} = 7$

- P.E. Algorithm added
- $T_{\max} = 9$

Speed Profiles of Robots 2 and 3 in a 10 robot case


Speed profile of Robot 2 in a 10 robot scenario


Speed profile of Robot 3 in a 10 robot scenario

Observations

- The partition elimination constraints make a difference; they make the robots change their speeds in order to maintain connectivity
- Typically, the "fast" robots whose times of arrival at their respective destinations are less than $T_{\max}$ change their velocity profiles to comply with the new communication connectivity constraint

Problem Size Reduction

- Use of the P.E. Algorithm in the simulated scenarios, reduces the number of partition elimination constraints p added to the problem
    - 4 vehicles, $k = 1$, and $T = 10$
    - $\binom{4}{2}$ possible subgroups of 2 robots per subgroup at each time period.
    - Total 60 constraints
    - By using P.E. algorithm only 2 of these constraints are added, one at time step 2 and the other at time step 3
    - Constraints added at the same time
    - Problem was resolved only once (hot start)

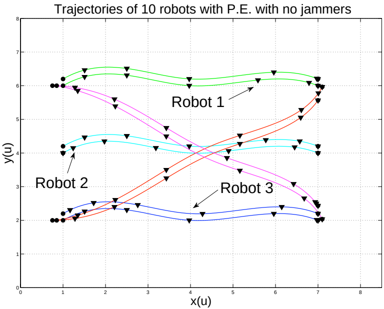
Scalability with respect to the n and Problem Size Reduction

n	$T_{\max}$ no PE	$T_{\max}$ with PE	T_{comp}	p no PE algorithm	p with PE algorithm
4	8	8	7.785s	60	2
10	5	7	46.869s	6270	3
20	7	8	123.217s	6166450	1
50	7	9	1314.897s	6.2616×10^{15}	1

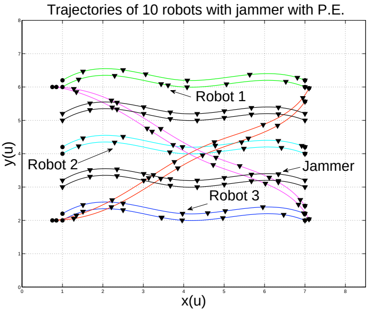
Observations

- T_{comp} increases with the increase in the n
- Number of Partition Elimination Constraints is greatly reduced in the scenarios studied here

Active partition elimination constraints, jammer present



Trajectories of 10 robots with P.E. with no jammers

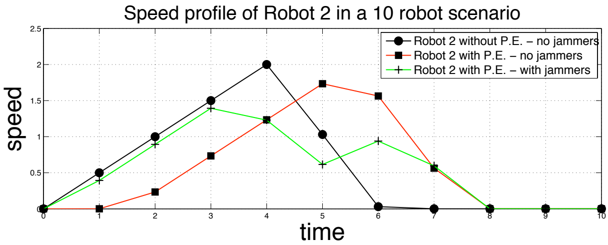


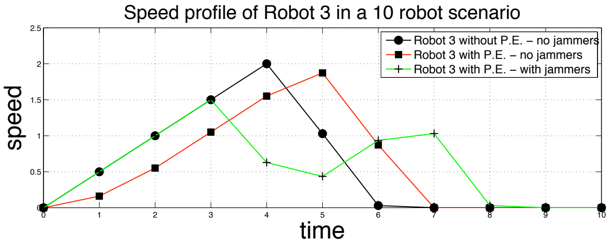
Trajectories of 10 robots with jammer with P.E.

- Jammer Absent
- $T_{\max} = 8$

- Jammer Present
- $T_{\max} = 8$

Speed Profiles of Robots 2 and 3 in a 10 robot case


Speed profile of Robot 2 in a 10 robot scenario


Speed profile of Robot 3 in a 10 robot scenario

Outline

Decentralized Formulation

- At their starting positions, we assume that all robots are in feasible communication range.
- Robots are put in decision making order for each time step.
- A planning horizon is used.
- At each time step t, let each robot i implement the following algorithm:

1. Fixing the other robots' trajectories for the planning horizon, solve the MINLP.
2. If a feasible solution is found, update the robot's trajectory.
3. Broadcast this plan to the other robots.

Outline

Conclusion and Future Work

- Generated time optimal velocity profiles for a group of path constrained mobile robots to support communication requirements
- Effect of additional jamming constraints was demonstrated
- Scalability study of the proposed approach by investigating scenarios involving up to 50 robots
- Problem size reduction was demonstrated
- Future efforts will focus on developing distributed algorithms for solving the problem

Thank You!