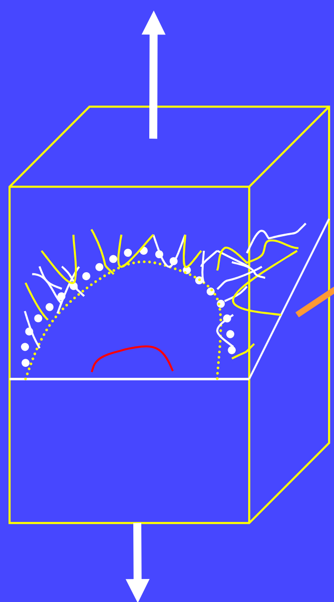
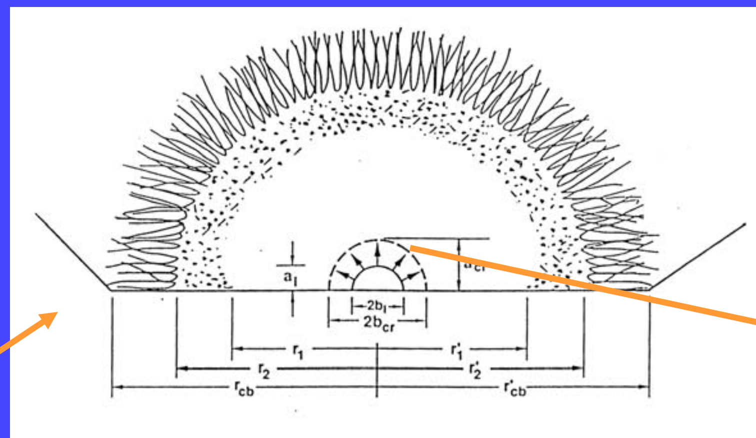
Mechanical Properties of Glass

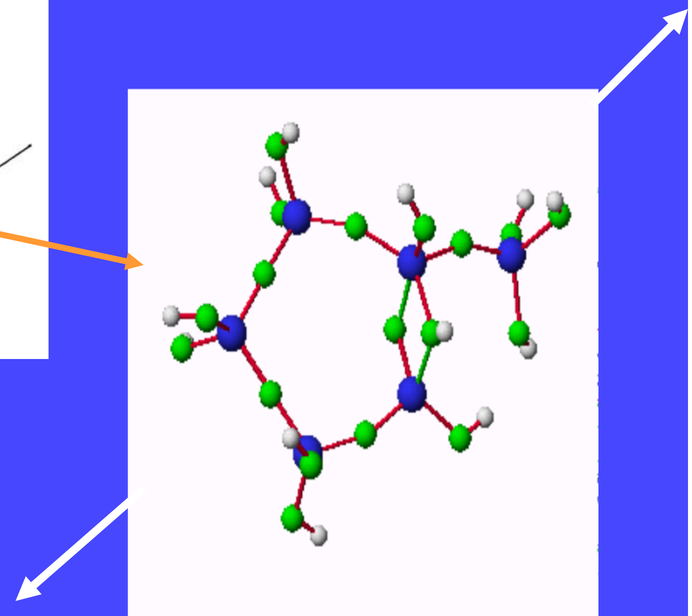
- Elastic Modulus and Microhardness
  [Chapter 8 – The "Good Book"*]
- Strength and Toughness [Chapter 18]
  - Fracture mechanics tests
  - Fractography
  - Stress Corrosion
  - Fracture Statistics

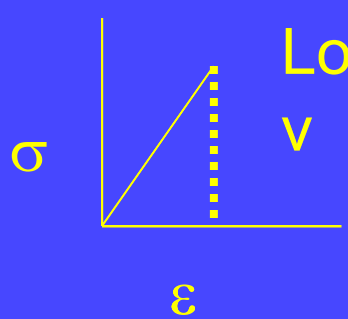
*A. Varshneya, "Fundamentals of Inorganic Glasses", Society of Glass Technology (2006)

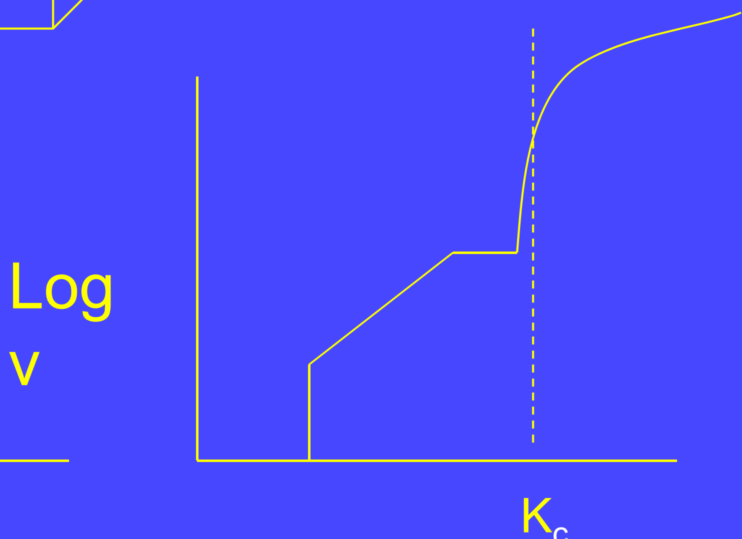
Bond Breaking Leads to Characteristic Features

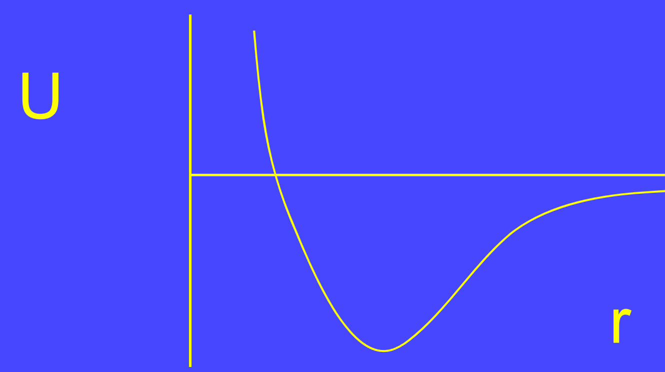






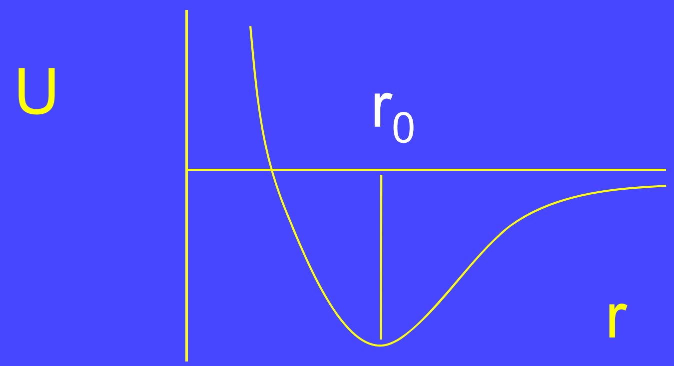


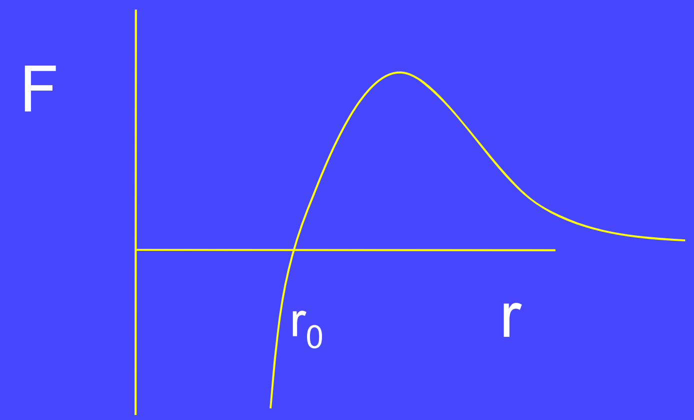




$$\text{Log } K = \text{Log } (Y\sigma\, c^{1/2})$$

Elastic Modulus Is Related To The Strength of Nearest Neighbor Bonds





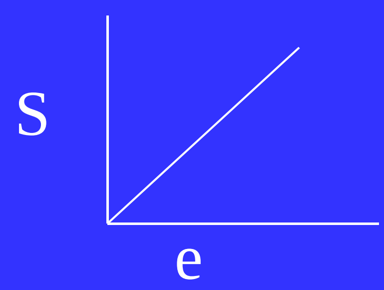
Force = F = - dU/dr

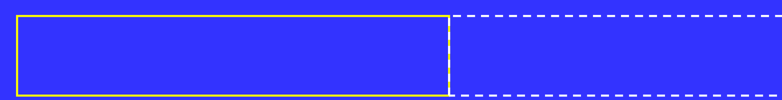
Stiffness = $S_0 = (dU^2/dr^2)_{r = r0}$

Elastic Modulus = E = S / r_0

There Are Several Important Properties in Mechanical Behavior:

Elastic Modulus – Governs Deflection





Hardness Measures Surface Properties

Strength – Governs Load Bearing Capacity

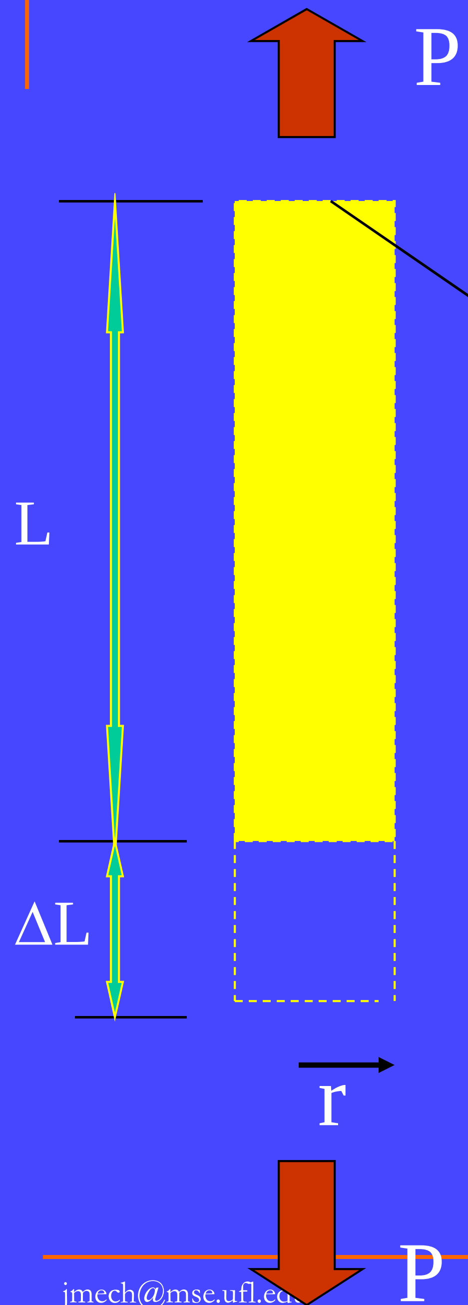
Toughness – Governs Crack Propagation

Stress = P / A

A = Cross-sectional Area = πr^2

P = Load On Sample

Strain = $\Delta L / L$

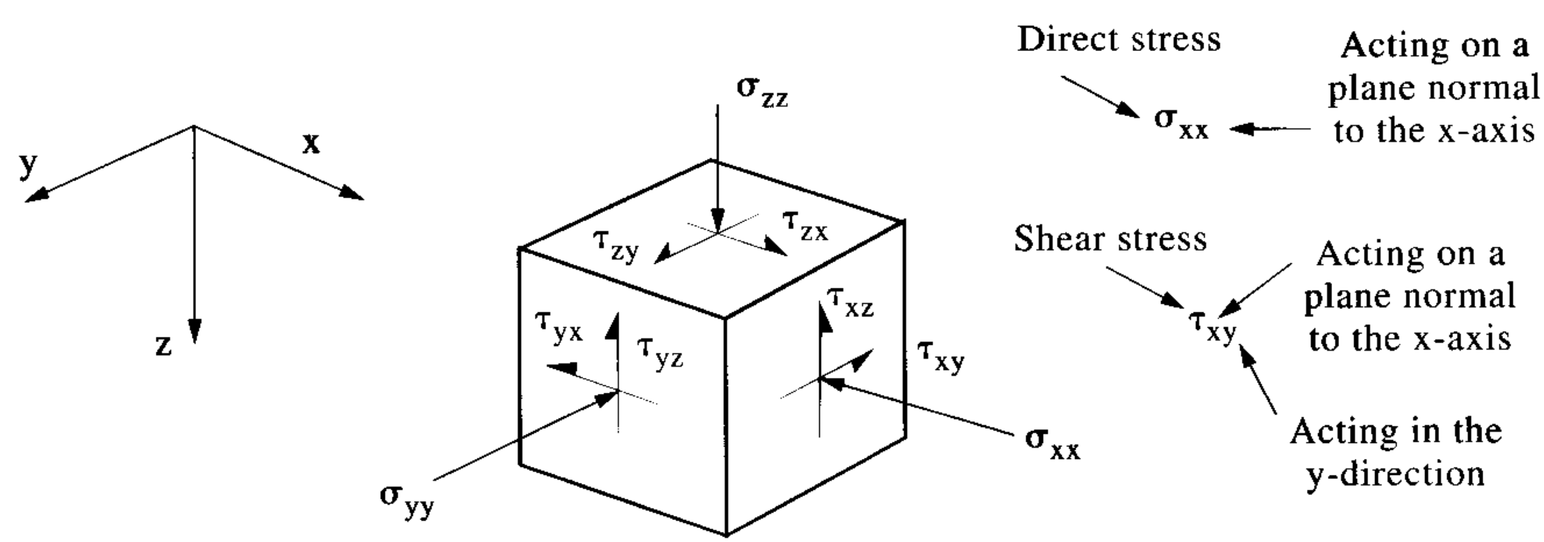


A = Cross-sectional Area = πr^2

L = Length

ΔL = Change In Length

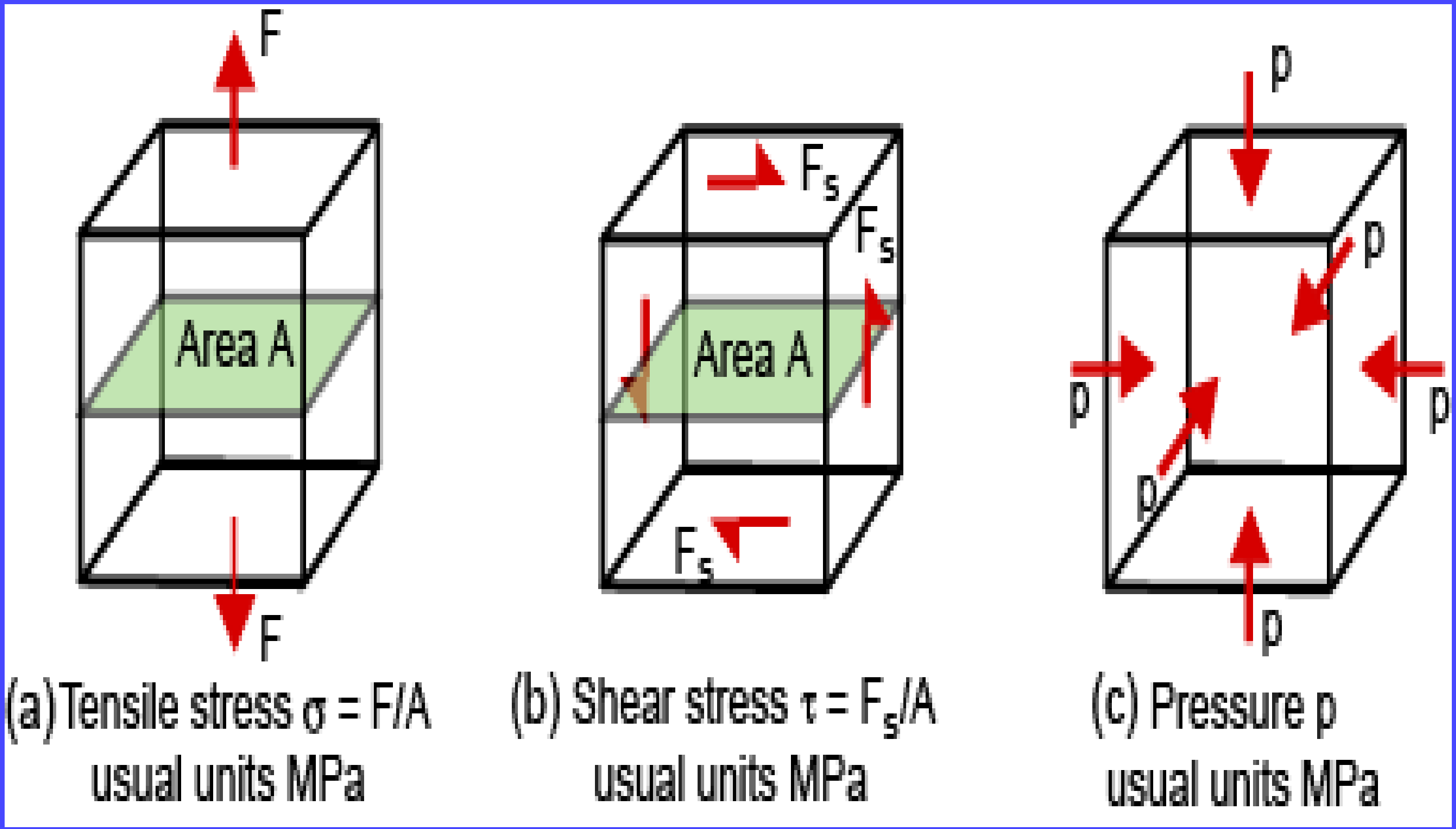
Infinitesimal cube represents triaxial state of stress.



The normal and shear stress components on an infinitesimal cube aligned with the Cartesian axes.

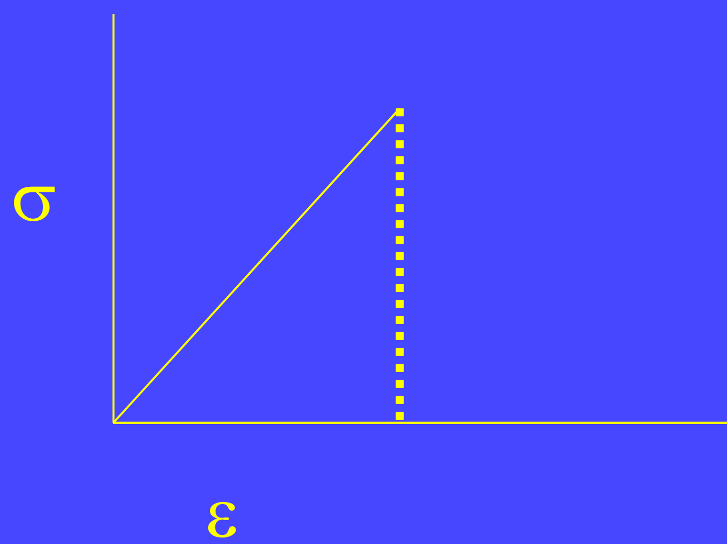
$$\varepsilon_y = (1/E)[\sigma_y - \nu(\sigma_x + \sigma_z)] \qquad \gamma_{xy} = [2(1+\nu)/E](\tau_{xy})$$
$$\varepsilon_x = (1/E)[\sigma_x - \nu(\sigma_y + \sigma_z)] \qquad \gamma_{yz} = [2(1+\nu)/E](\tau_{yz})$$
$$\varepsilon_z = (1/E)[\sigma_z - \nu(\sigma_y + \sigma_x)] \qquad \gamma_{zx} = [2(1+\nu)/E](\tau_{zx})$$

Special Cases of Loading Often Occur



(a) Tensile stress. *(b) Shear stress.* *(c) Hydrostatic pressure.*

In uniaxial loading in the x direction, E (or Y) relates the stress, σ_x, to the strain, ε_x.



$$\sigma_x = E\,\varepsilon_x$$

- $\varepsilon_y = \varepsilon_z = -\nu\,\varepsilon_x$
- $\sigma_{xy} = G\,\gamma$

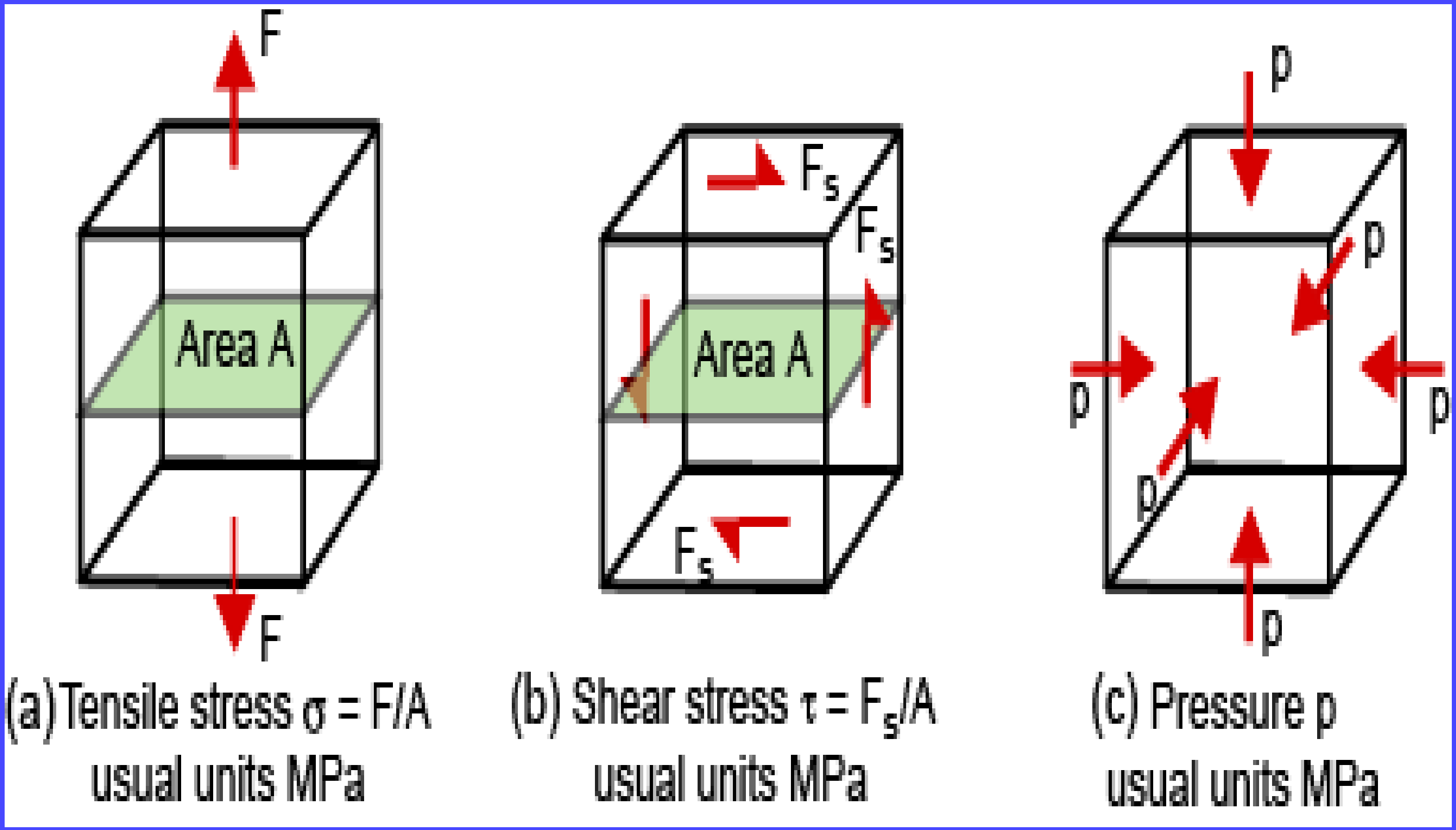
$$p = K\,\Delta V$$

In the case of shear loading, the shear modulus is appropriate

$$G = \frac{\sigma_{xy}}{\varepsilon_{xy} + \varepsilon_{yx}} = \frac{\sigma_{xy}}{2\varepsilon_{xy}} = \frac{\sigma_{xy}}{\gamma_{xy}}$$

$$= \frac{E}{2(1+\nu)}$$

$$\gamma_{xy} = \varepsilon_{xy} + \varepsilon_{yx} = 2\varepsilon_{xy}$$

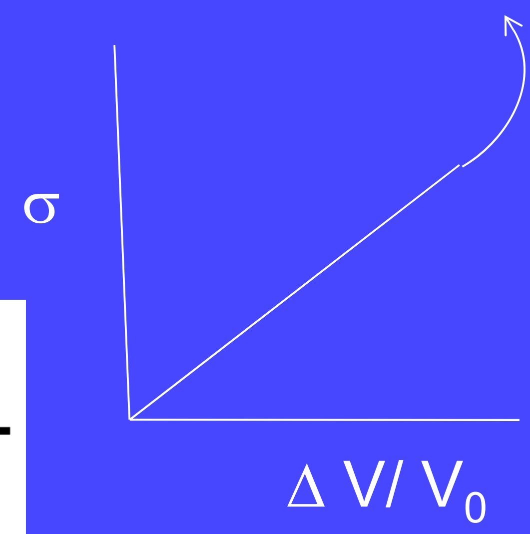


(a) Tensile stress. *(b) Shear stress.* *(c) Hydrostatic pressure.*

In the case of hydrostatic pressure, the bulk modulus is appropriate.

$$K=\frac{\sigma}{\Delta V/V}=\frac{\sigma}{\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}}$$

$$=\frac{E}{3(1-2\nu)}$$



There is a relationship between E, G and K (and of course Poisson's ratio, ν)

$$G = E / [2 (1+\nu)]$$

$$K = E / [3(1-2\nu)]$$

Note: $-1 \leq \nu \leq 0.5$.

(When $\nu = 0.5$, $K \rightarrow \infty$ and $E \rightarrow 3G$. Such a material is called incompressible.).

There is a relationship between E, G and K (and of course Poisson's ratio, ν)

$$G = E / [2 (1+\nu)]$$

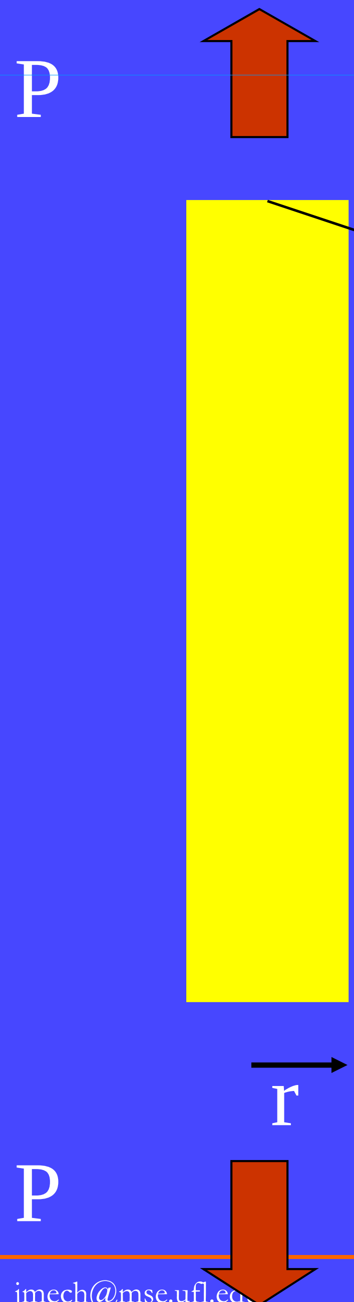
$$K = E / [3(1-2\nu)]$$

So, when we determine any two parameters, (for isotropic materials) we can calculate the others.

There are several techniques used to measure the elastic modulus:

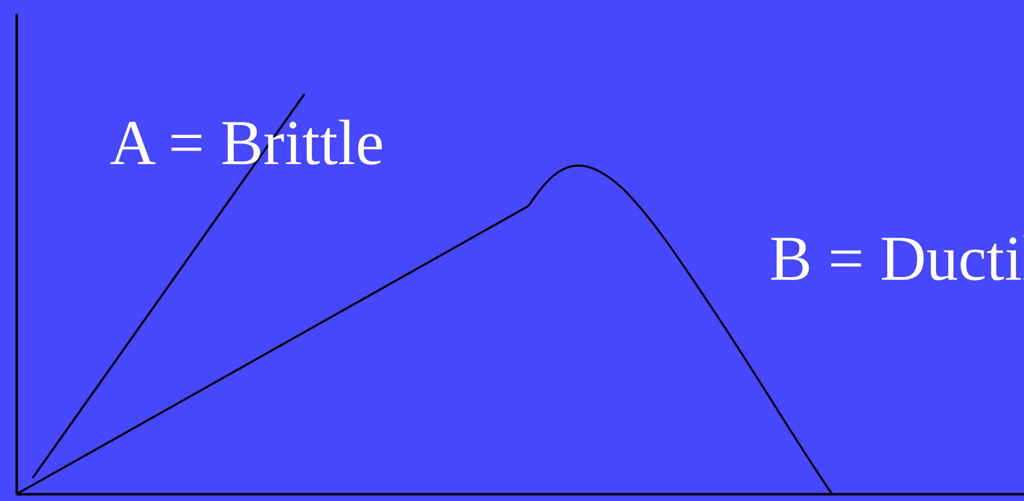
A. Stress-strain directly (load-displcament)
    1. tension
    2. 3-pt flexure
    3. 4-pt flexure
    4. Hydrostatic pressure
    5. Torque on rod
B. Ultrasonic wave velocity
    1. Pulse echo
    2. Direct wave
C. Beam Vibration

Elastic Modulus = Stress / Strain



$$A = \text{Area} = \pi r^2$$

S or σ

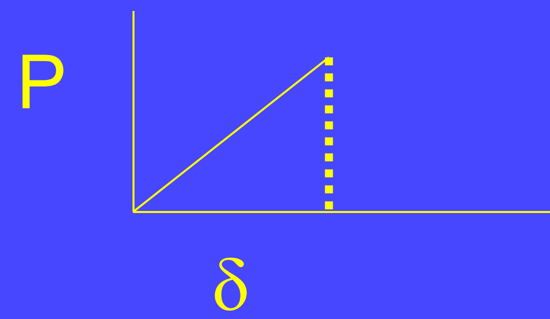


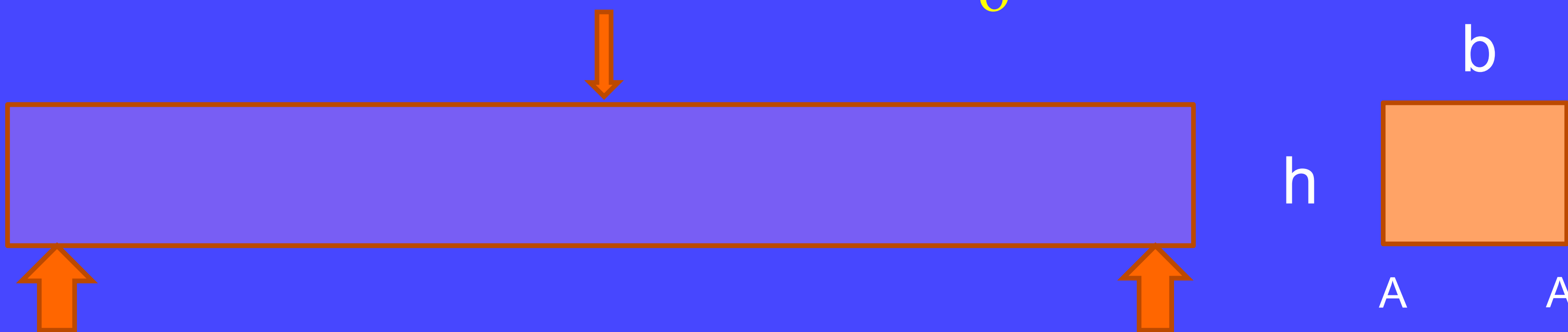
Strain = e or ε

S =Stress = P / A

Strain = ΔL / L

To measure E from flexure, need to calculate the stress and strain.





$$\sigma = 3PL / (2\, b\, h^2) \qquad \varepsilon = \delta / L$$

Pulse echo technique is often used to measure modulus

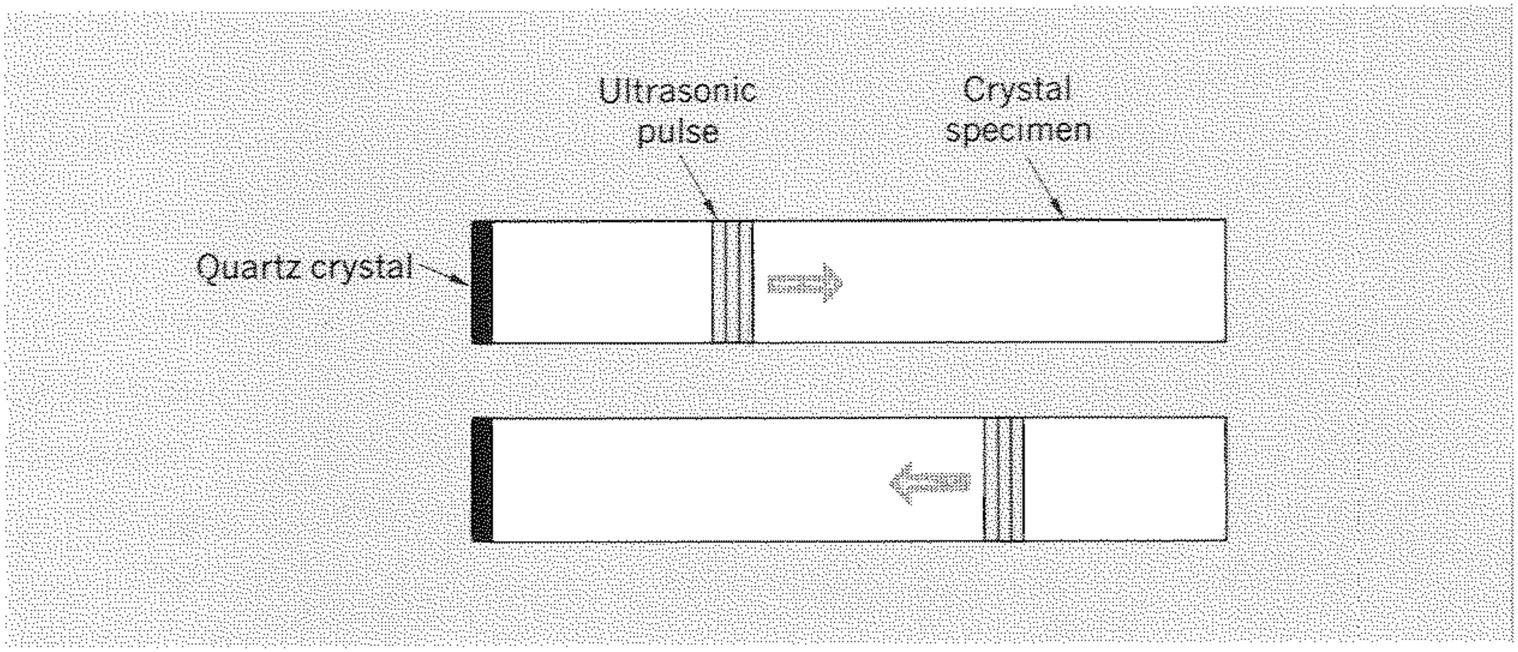


Figure 12 In the ultrasonic pulse method for the determination of elastic wave velocities a pulse of sound is generated by a piezoelectric transducer. The pulse makes successive reflections and is detected each time it reaches the transducer.

C. Kittel, Intro. To Solid State Physics, J. Wiley & Sons

Pulse Echo technique is one of the most reliable.

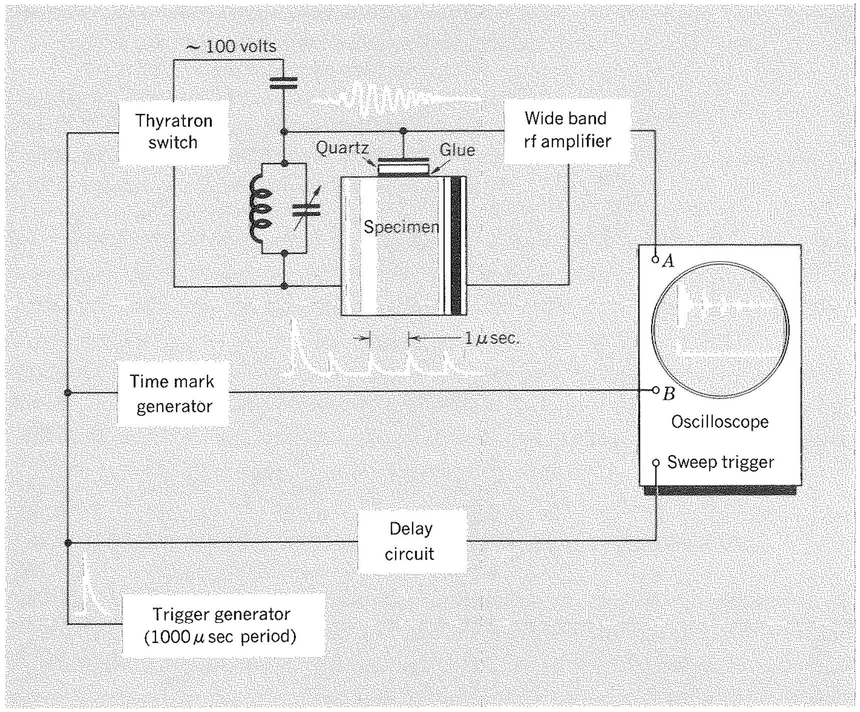


Figure 13 Block diagram of an electronic apparatus used to measure the velocity of sound by the ultrasonic pulse-echo technique. The trigger generator starts the process by closing the thyratron switch; this discharges a condenser through the LC network and imposes the indicated voltage waveform on the quartz transducer glued to the specimen. The waveform becomes a sound pulse which bounces back and forth within the specimen to produce the train of echoes shown on the oscilloscope face. The time mark generator and sweep delay circuit allow the time of arrival of some later echo to be measured accurately. [After G. A. Alers and J. R. Neighbours, J. Phys. Chem. Solids **7**, 58, (1958).]

In the simplest case for isotropic materials there are direct relationships.

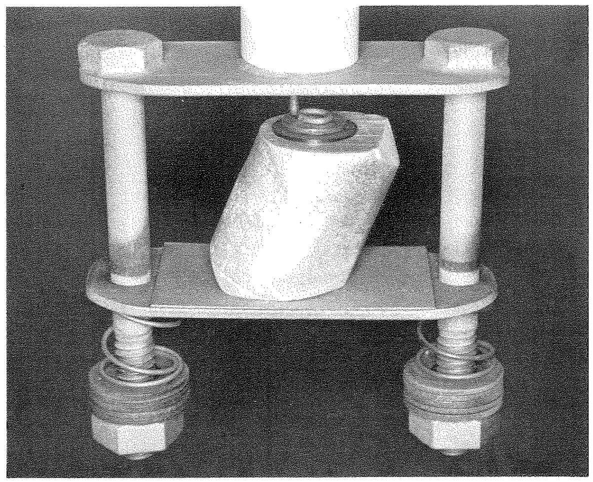


Figure 14 Photograph of a single crystal of aluminum mounted in a spring-loaded holder for making velocity of sound measurements by the ultrasonic pulse-echo technique. The top face of the crystal is a (110) crystal plane on which is glued a quartz transducer with a metal electrode and rf lead in place. (Courtesy of the Scientific Laboratory, Ford Motor Company.)

$$v_L = [\,E / \rho]^{1/2}$$

(Longitudinal waves)

$$v_S = [\,G / \rho]^{1/2}$$

(Shear waves)

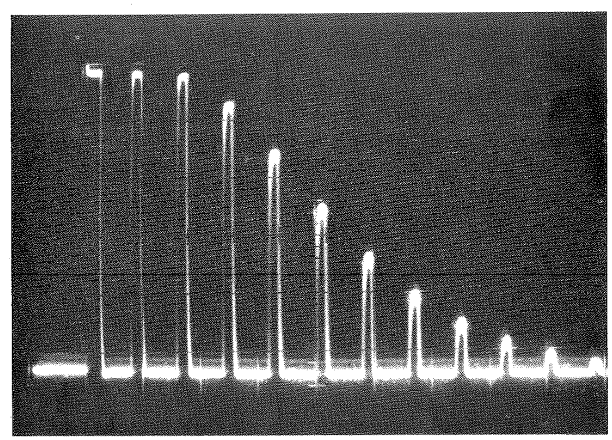
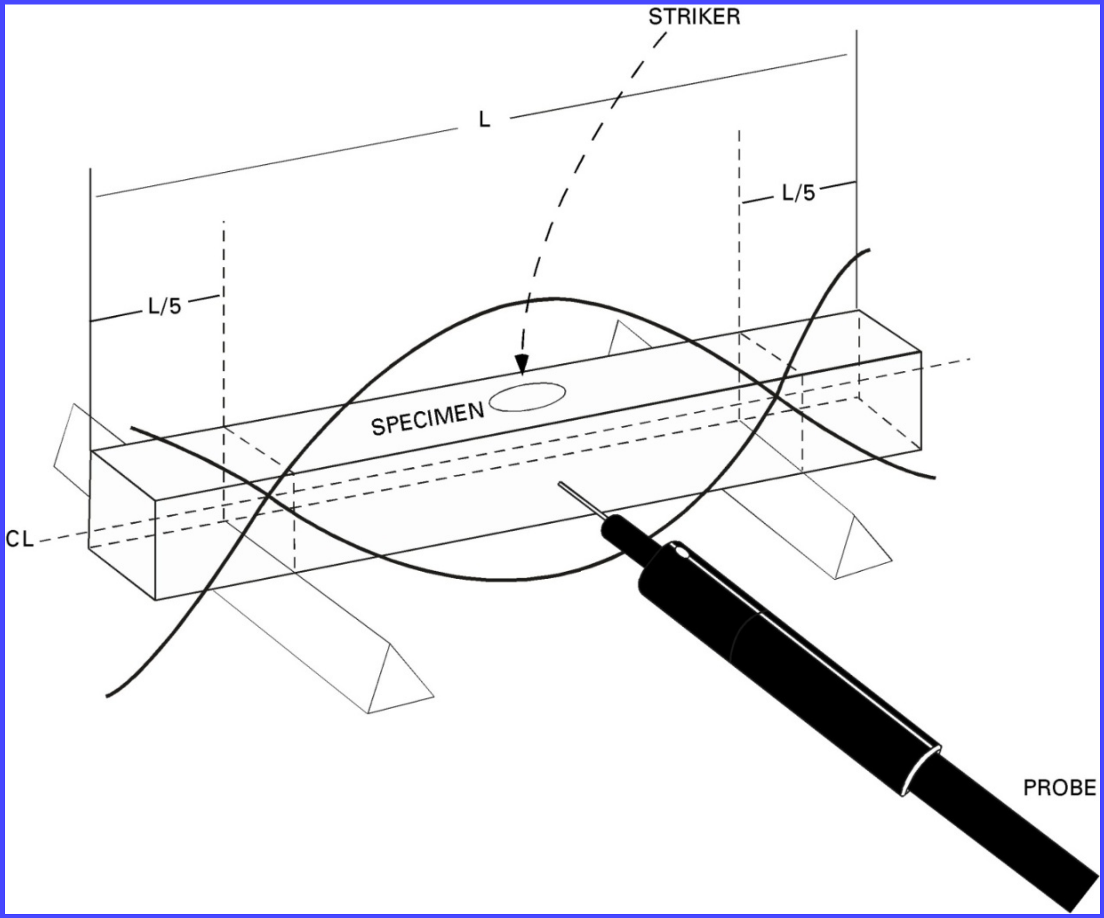


Figure 15 Successive ultrasonic pulse echoes across a crystal. The time interval between successive pulses is measured on the oscilloscope. This interval is the round-trip travel time of the ultrasonic pulse. The attenuation of the wave may be found from the decrease in pulse height of successive pulses, with allowance for losses on reflection at the ends. The voltage across the transducer is directly proportional to the stress in the wave. (Courtesy of H. J. McSkimin.)

For the beam vibration technique, we stimulate the flexural modes.



For beam bending:

$E = (0.946\ L^4\ f^2\ \rho\ S) / h^2$

f = frequency
S = shape factor
H = width and height
L = length
ρ = density

Fig 8-5

In general, E decreases as the size and concentration of the alkali cations increases

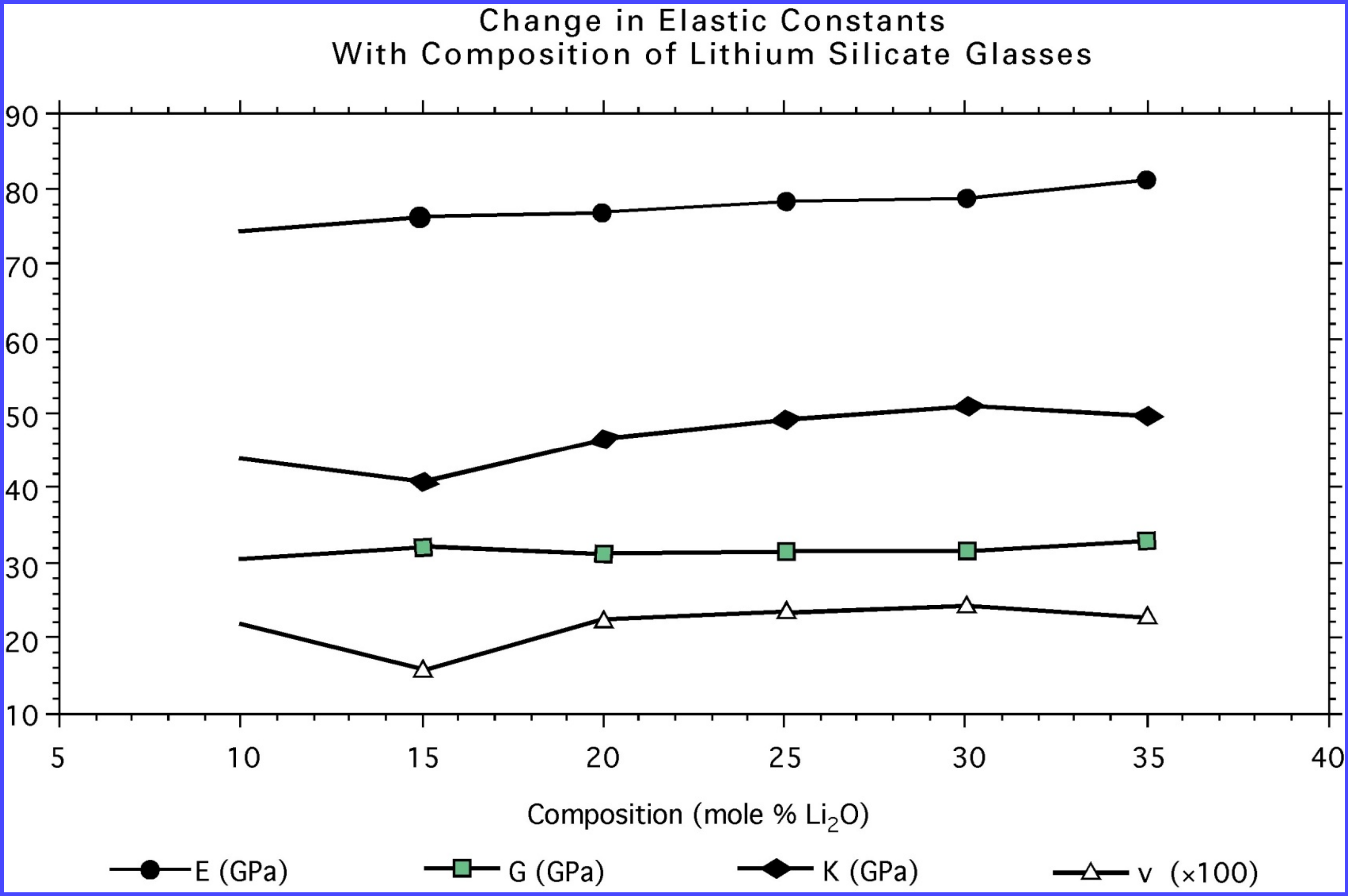


Fig 8-6a

E decreases as the size and concentration of the alkali cations increase

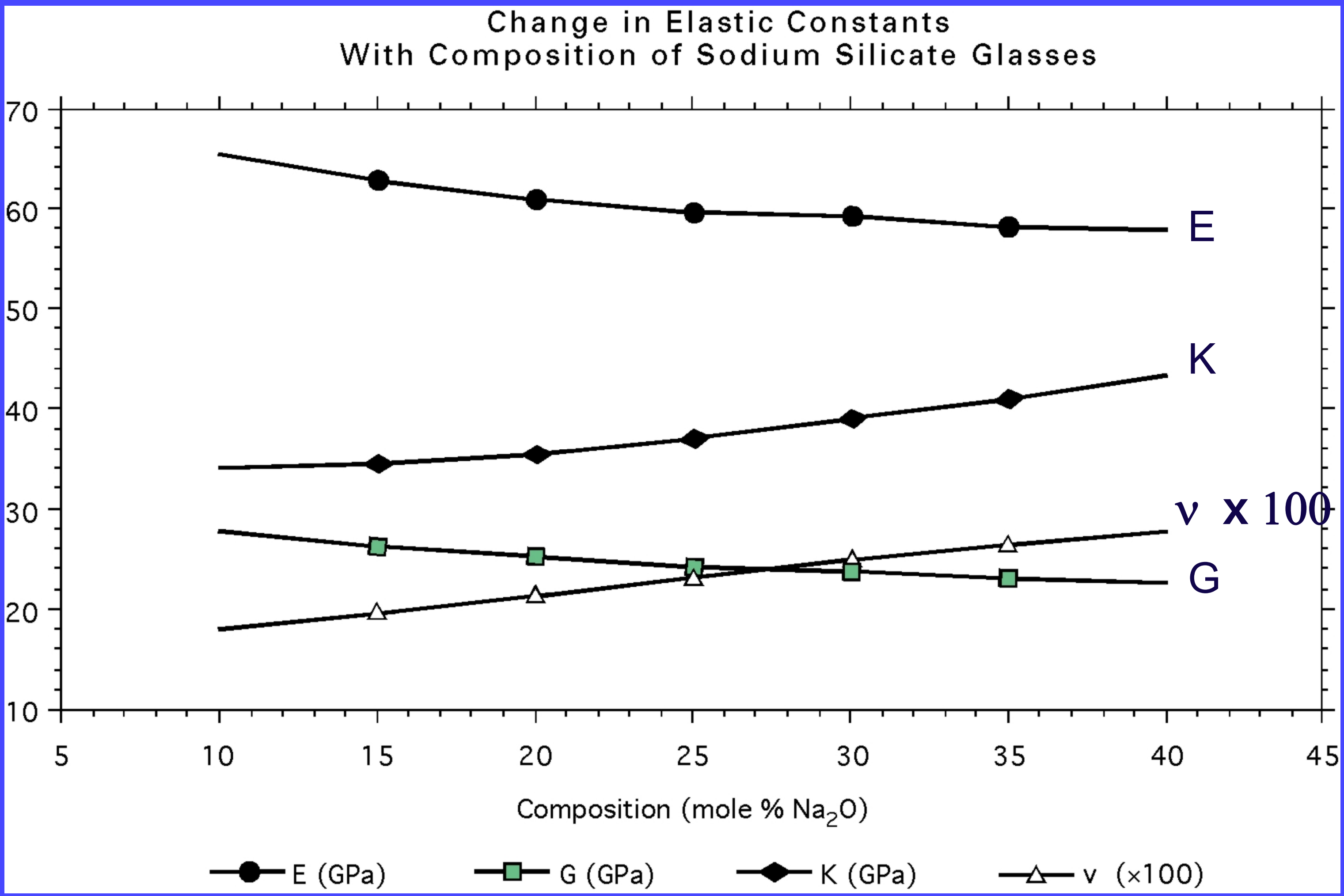


Fig 8-6b

E decreases as the size and concentration of the alkali cations increases

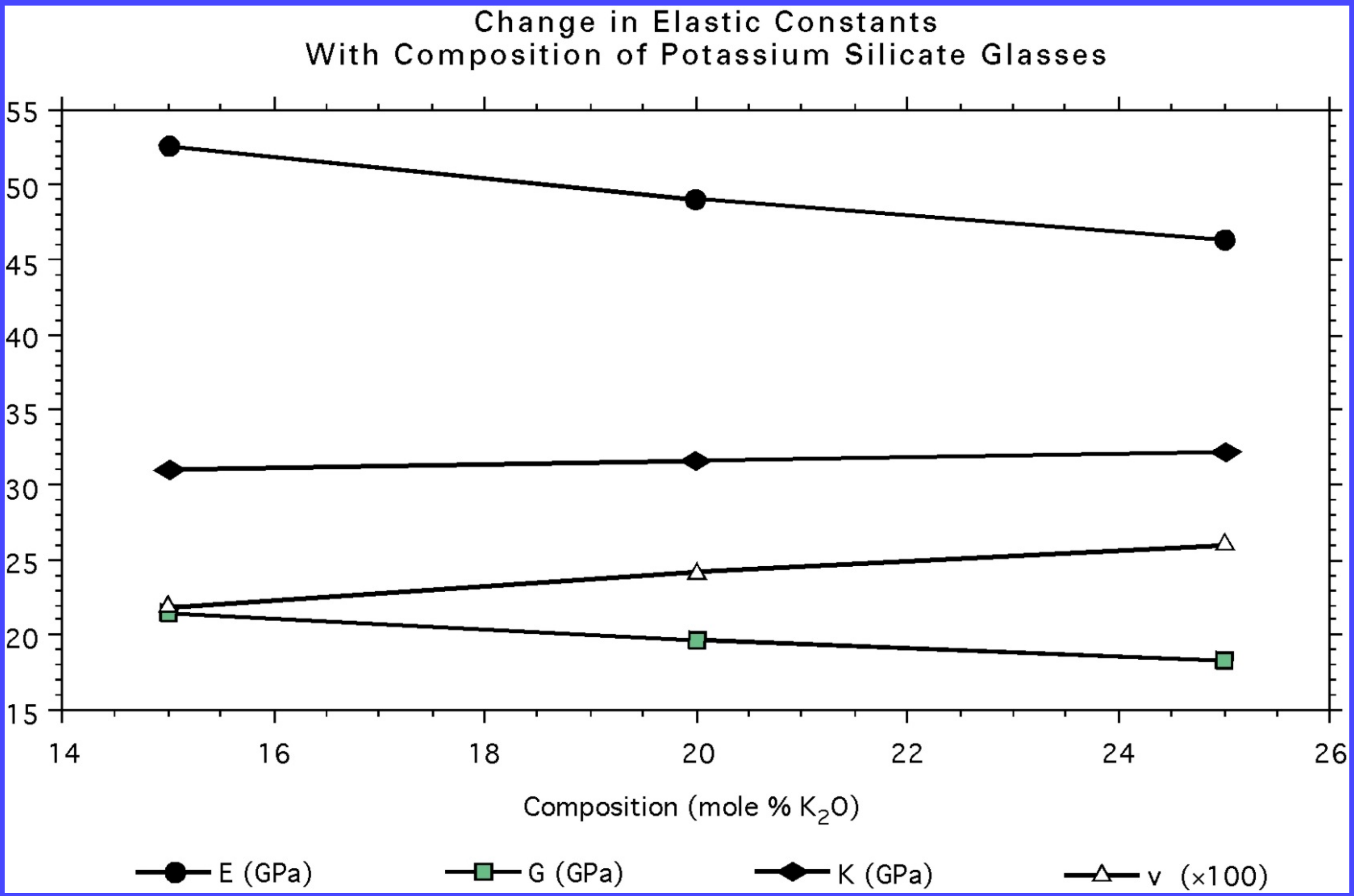


Fig 8-6c

E increases with addition of metal oxide (MO) [except PbO]

$Na_2O \bullet x\ MO \bullet 5SiO_2$

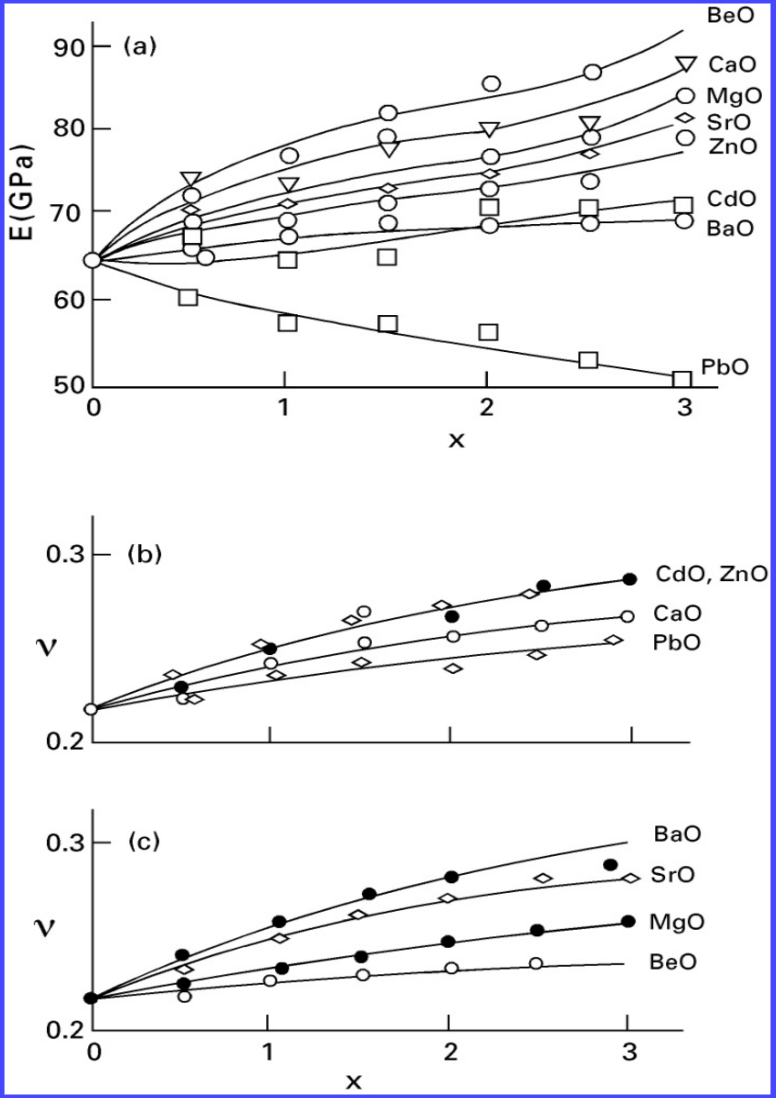
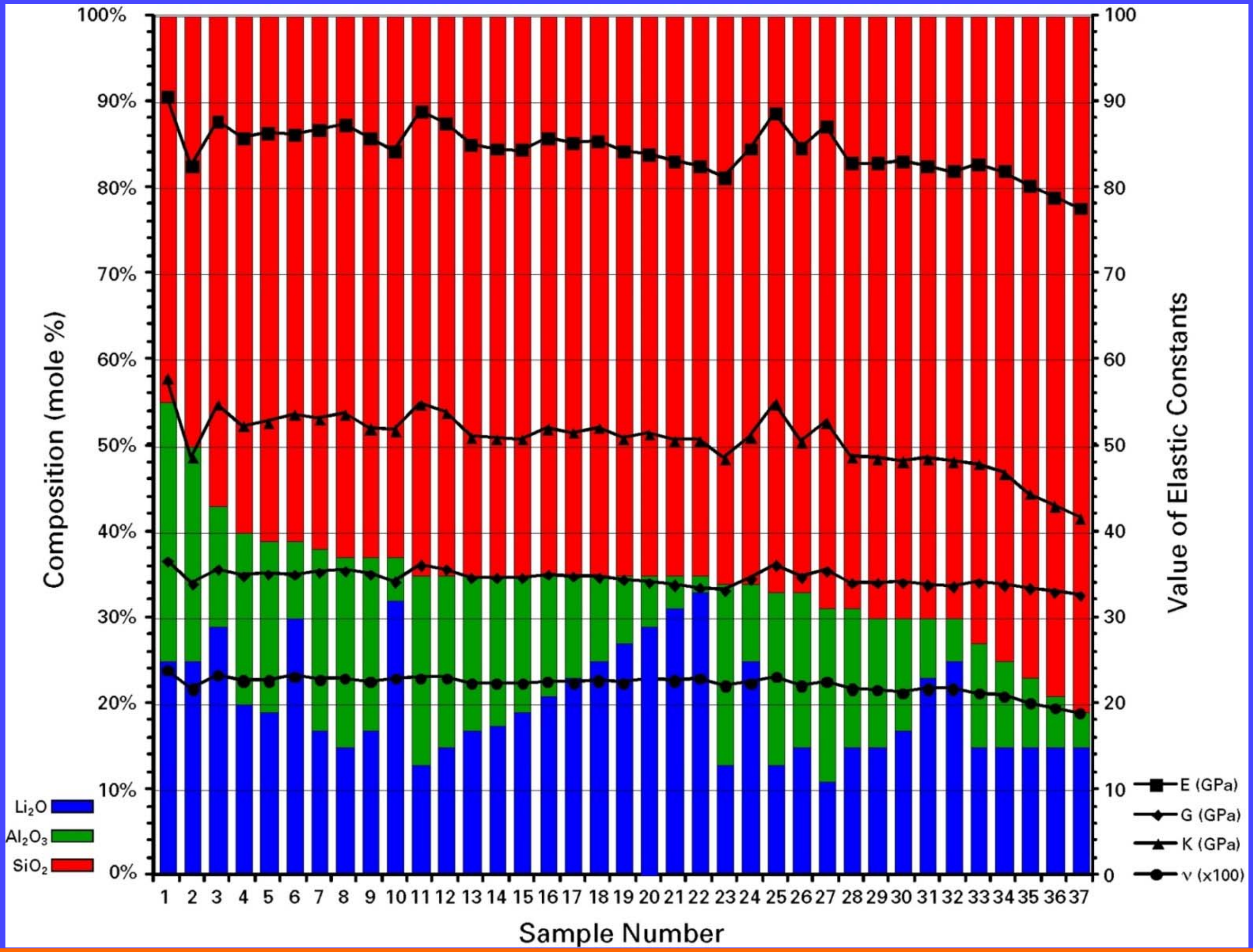


Fig.8-7 (Varshneya)

Lithia-aluminosilicates have greater E values than SiO_2

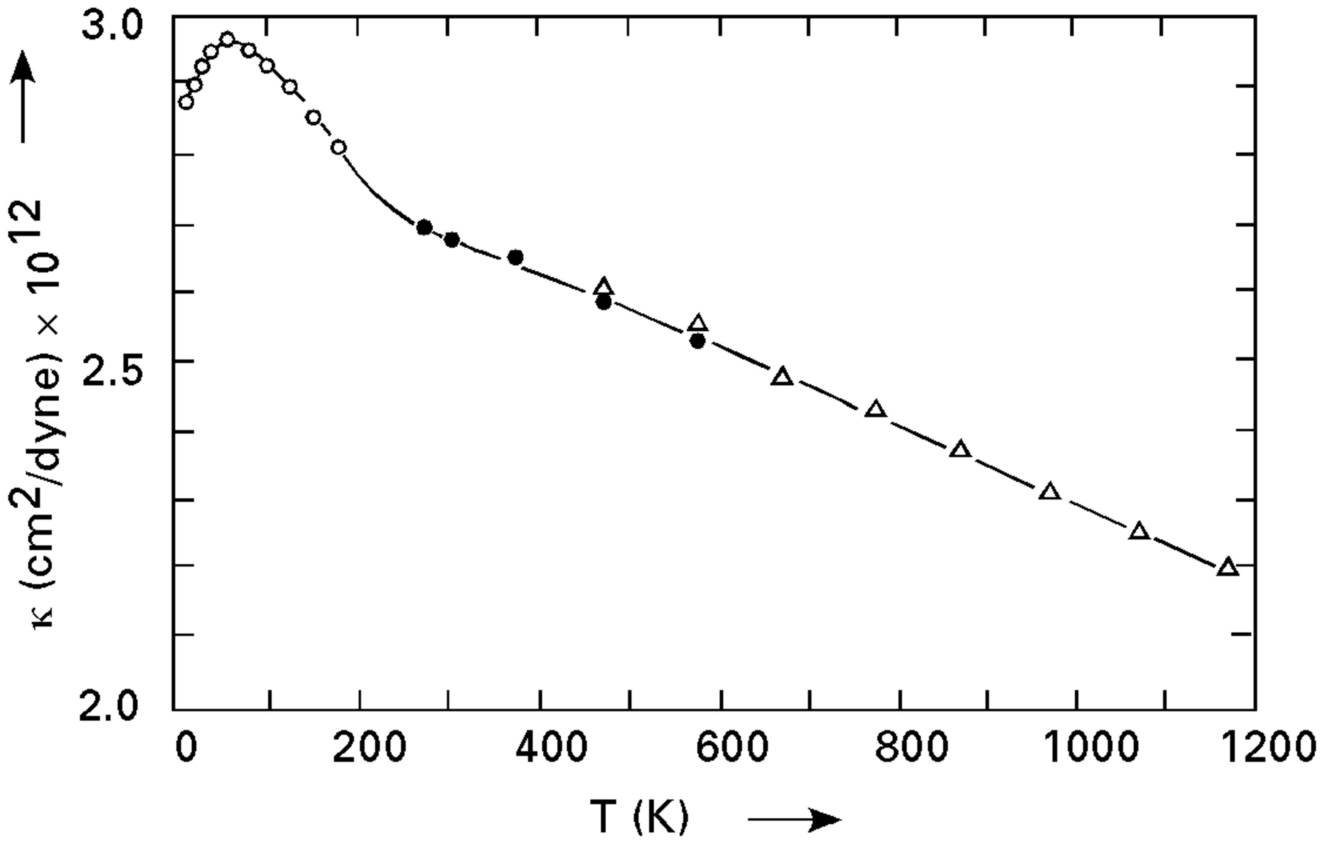
Fig.8-8



In general, bulk moduli of silicate glasses increase with temperature (except at low temperatures [0 - 60K])

N.B. - the compressibility, κ, is being graphed in the figure (Fig. 8-9).

(The compressibility is the reciprocal of the bulk modulus.)



Composition and structure affect the values of elastic moduli.

N.B.: at low (< 10mol%) alkali content, E⬇ with B_2O_3 addition.

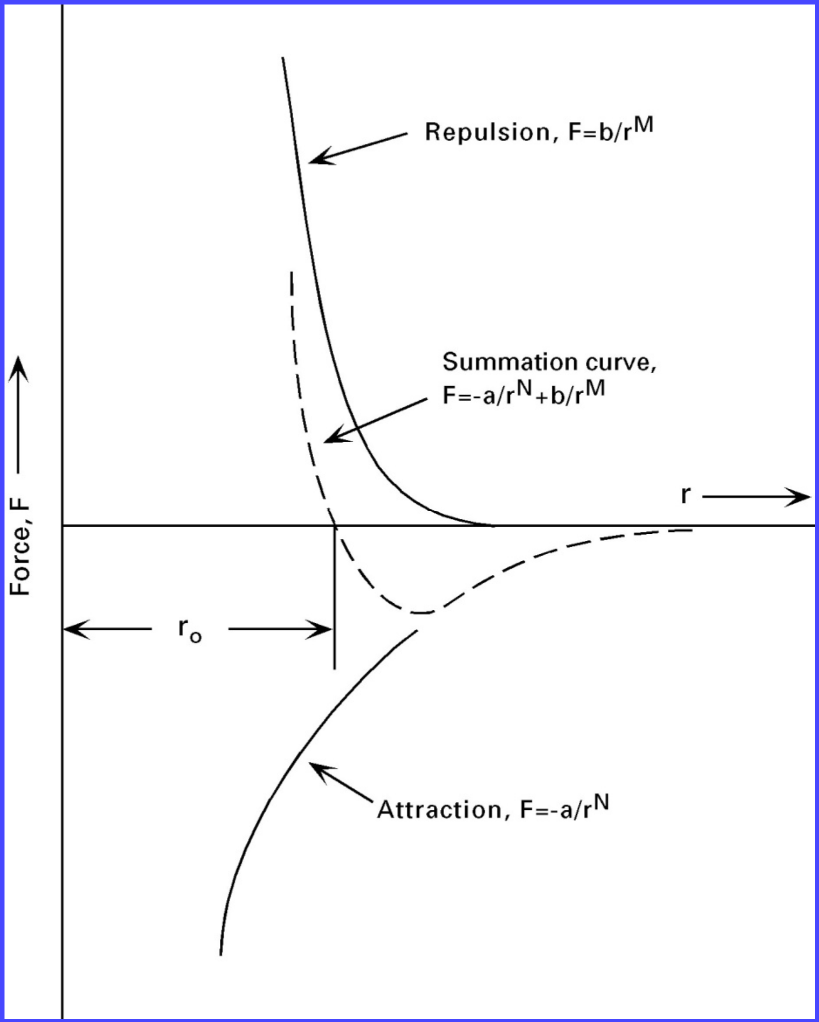
However, with greater alkali content glasses addition of B_2O_3 leads to a maximum in E.

Table 8-1. Young's Moduli of Na_2O–B_2O_3–SiO_2 Glasses[a]

Composition (mol %)				Composition (mol %)			
Na_2O	B_2O_3	SiO_2	*E* (GPa)	Na_2O	B_2O_3	SiO_2	*E* (GPa)
10	10	80	81.5	20	60	20	61.1
10	15	75	78.6	20	65	15	56.2
10	20	70	76.4	20	70	10	50.9
10	25	65	73.9	20	75	5	51.3
10	30	60	63.1	25	50	25	71.8
10	35	55	60.3	25	55	20	73.2
10	40	50	54.1	25	60	15	68.8
10	45	45	50.3	25	65	10	65.1
10	50	40	47.1	25	70	5	67.4
10	55	35	47.4	30	5	65	73.5
10	60	30	46.3	30	10	60	75.1
10	65	25	44.9	30	15	55	75.8
10	70	20	38.6	30	20	50	77.2
10	75	15	37.8	30	25	45	82.4
10	80	10	36.5	30	30	40	84.5
10	85	5	32.8	30	35	35	78.9
20	5	75	76.2	30	40	30	73.5
20	10	70	78.0	30	45	25	79.9
20	15	65	82.8	30	50	20	75.1
20	20	60	83.8	30	55	15	72.4
20	25	55	83.5	30	60	10	70.9
20	30	50	84.2	30	65	5	70.5
20	35	45	76.1				
20	40	40	75.5				
20	45	35	73.3				
20	50	30	70.0				
20	55	25	67.2				

[a] After M. Imaoka, H. Hasegawa, Y. Hamaguchi, and Y. Kurotaki, *Yogyo Kyokai shi*, 79, 164 (1971).

Complications of silicate glasses makes predictions difficult



$F = [-a / r^n] + b / r^m$

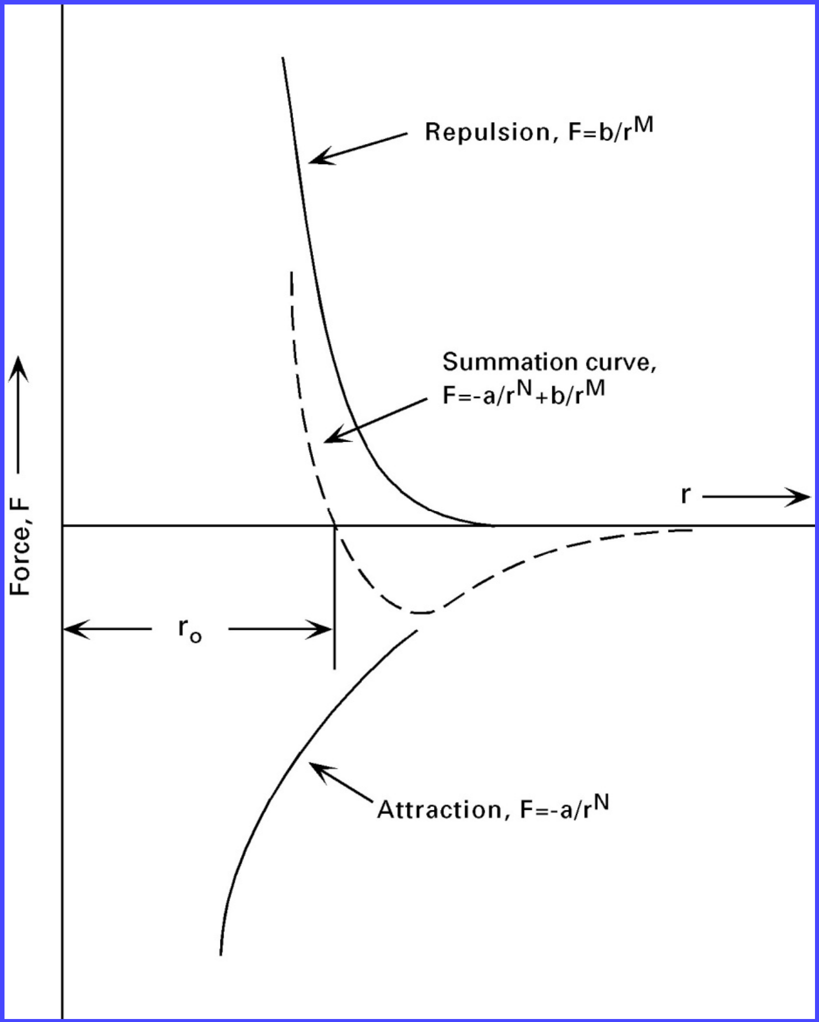
(Condon-Morse)

Force = $F = - dU/dr$

Stiffness = $S_0 = (dU^2/dr^2)_{r = r0}$

Elastic Modulus = $E = S / r_0$

Complications of silicate glasses makes predictions difficult



$F = [-a / r^n] + b / r^m$ (Condon-Morse)

$\text{Force} = F = - dU/dr$

$\text{Stiffness} = S_0 = (dU^2/dr^2)_{r = r0}$

$\text{Elastic Modulus} = E = S / r_0$

General rules:

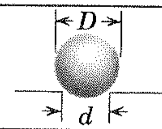
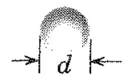
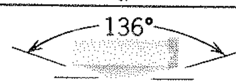
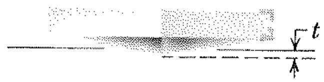
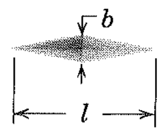
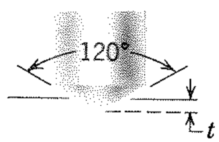
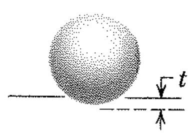
1. E increases as r_0^x decreases
2. E increases as valence, i.e., $q_a \times q_c$
3. E affected by bond type (covalent, ionic, metallic).
4. E affected by structure (density, electron configuration, etc.)

Microhardness is a measure of surface properties and can be related to elastic modulus, toughness and surface tension.

Hardness = Force / Area

Many hardness tests are available

Table 1.1 Hardness Tests

Test	Indenter	Shape of Indentation: Side View	Shape of Indentation: Top View	Load	Formula for Hardness Number
Brinell	10 mm sphere of steel or tungsten carbide			P	$\text{BHN} = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers	Diamond pyramid			P	$\text{VHN} = 1.72P/d_1^2$
Knoop microhardness	Diamond pyramid	 $l/b = 7.11$ $b/t = 4.00$		P	$\text{KHN} = 14.2P/l^2$
Rockwell					
A	Diamond cone			60 kg	$R_A =$ } 100–500t
C				150 kg	$R_C =$
D				100 kg	$R_D =$
B	$\frac{1}{16}$ in. diameter steel sphere			100 kg	$R_B =$ } 130–500t
F				60 kg	$R_F =$
G				150 kg	$R_G =$
E	$\frac{1}{8}$ in. diameter steel sphere			100 kg	$R_E =$

The most common microhardness diamond tips for glasses are Vickers and Knoop

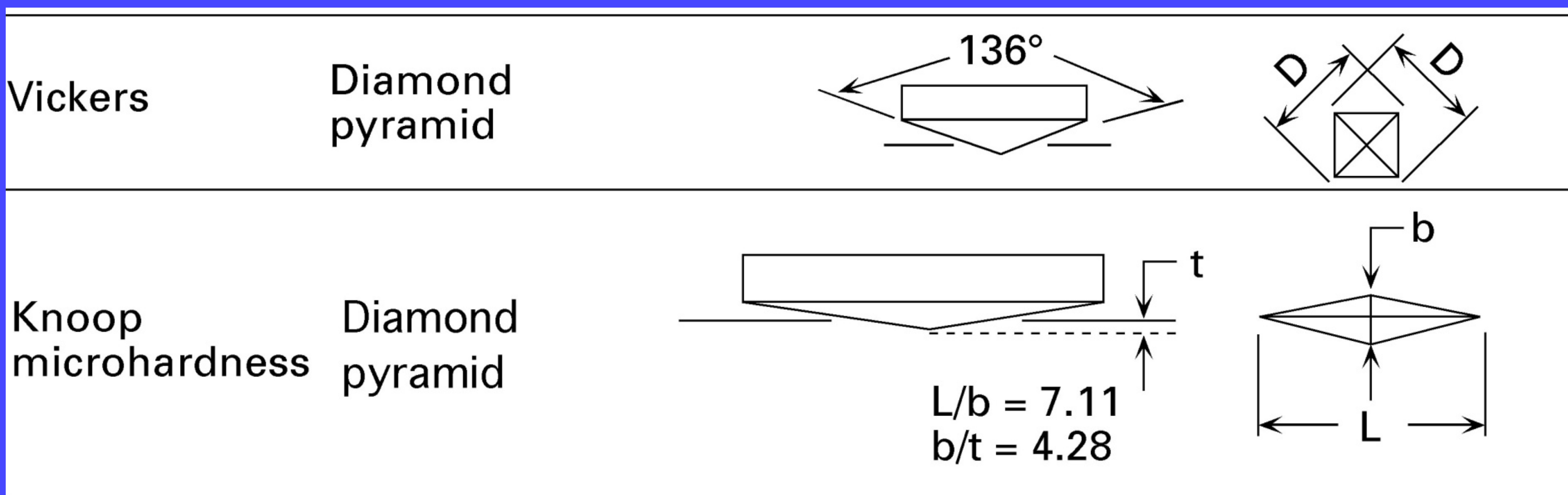
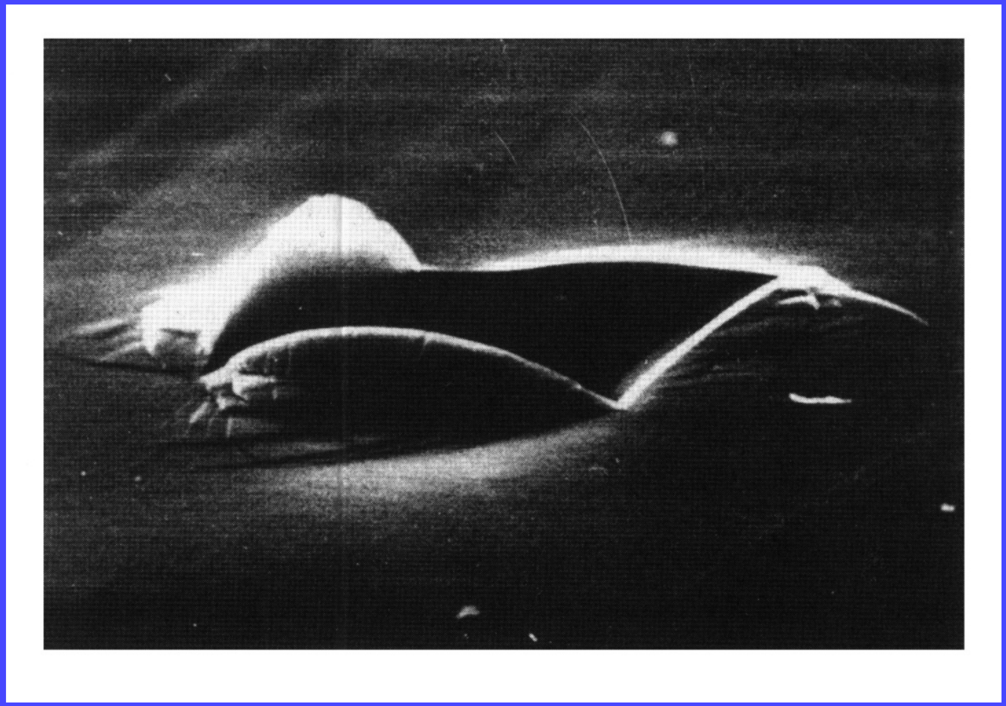


Fig. 8-12

Hardness = Force / Area

$Hv = 1.854\ F / D^2$ (Actual area) $KHN = 14.23\ F / L^2$ (Projected area)

Note plastic flow in silicate glass using a Vickers microhardness indenter.



Plastic flow in Se glass using a Brinell microhardness indentation.

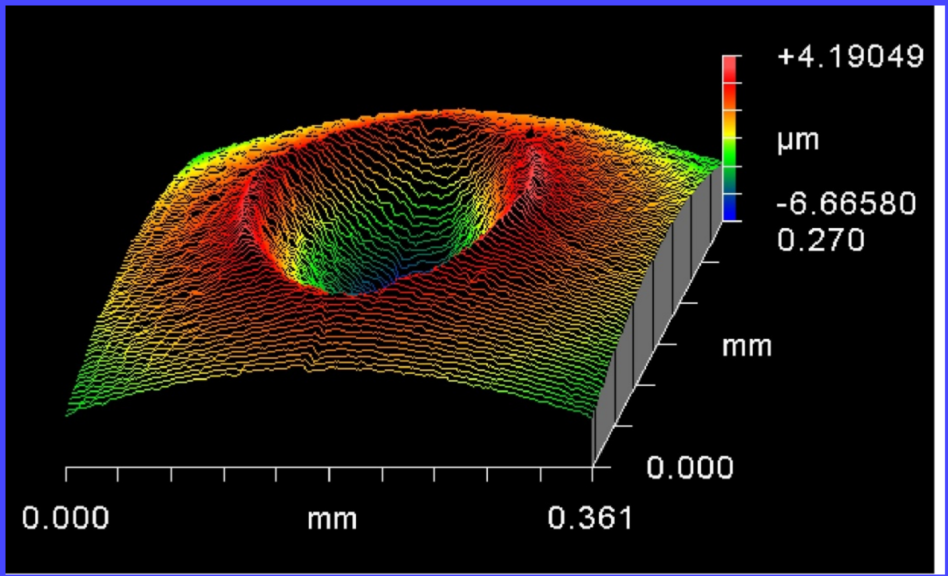
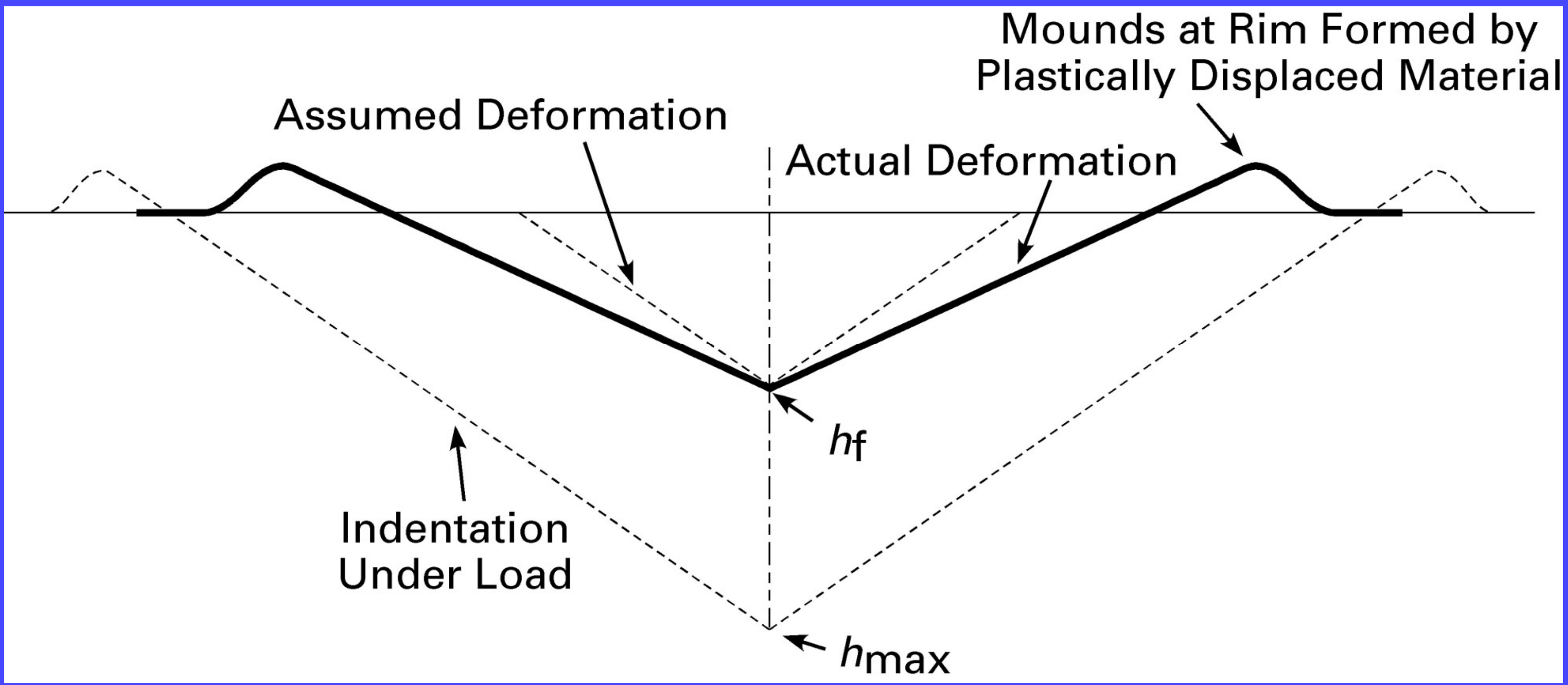


Fig. 8-13 a & b

Diamond hardness indentations can result in elastic and plastic deformation.



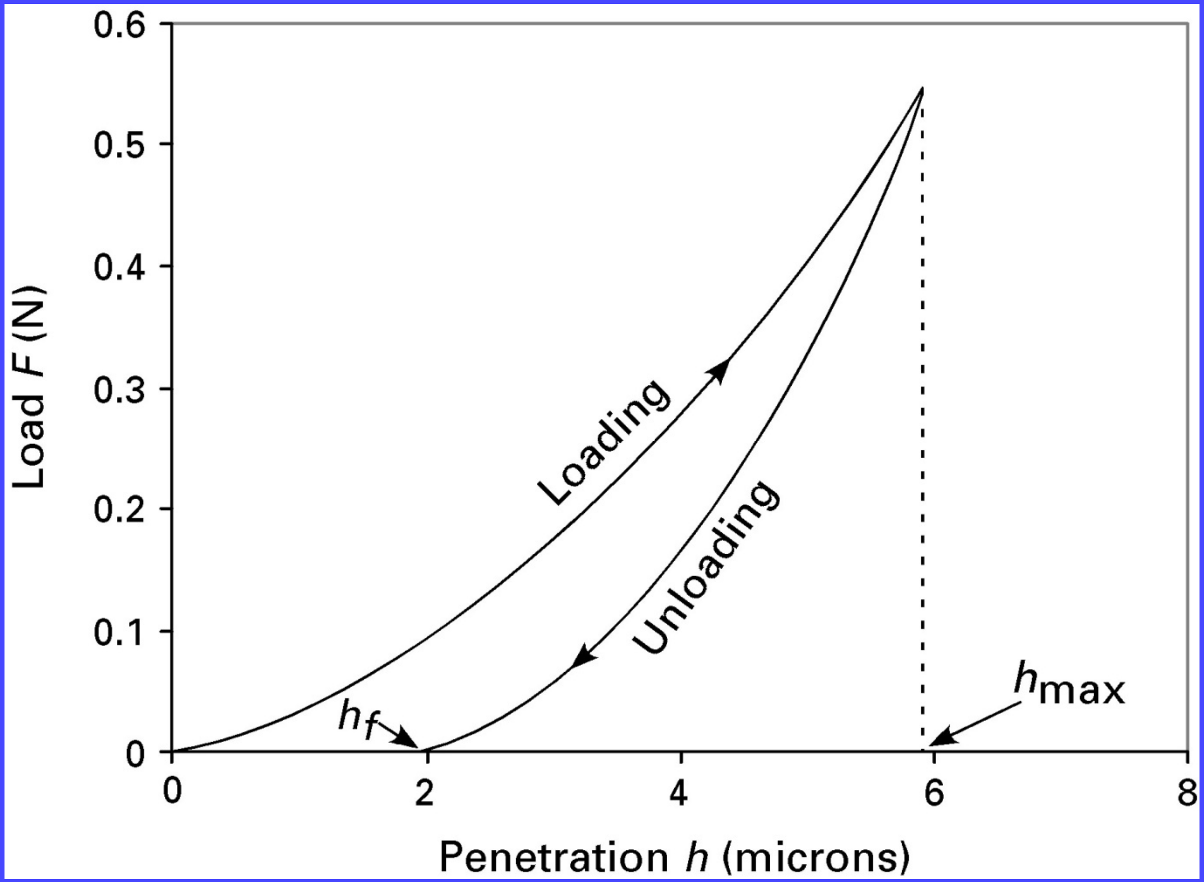
Microhardness can be measured dynamically

$H_{vL} = 37.84\ F / h^2_{max}$

(from loaded depth, h_{max})

$H_{vf} = 37.84\ F / h^2_f$

(from unloaded depth, h_f)



Refs. 34 and 35 in Chapter 8.

$F = a_1 h + a_2\ h^2$ (equation fit to curve)

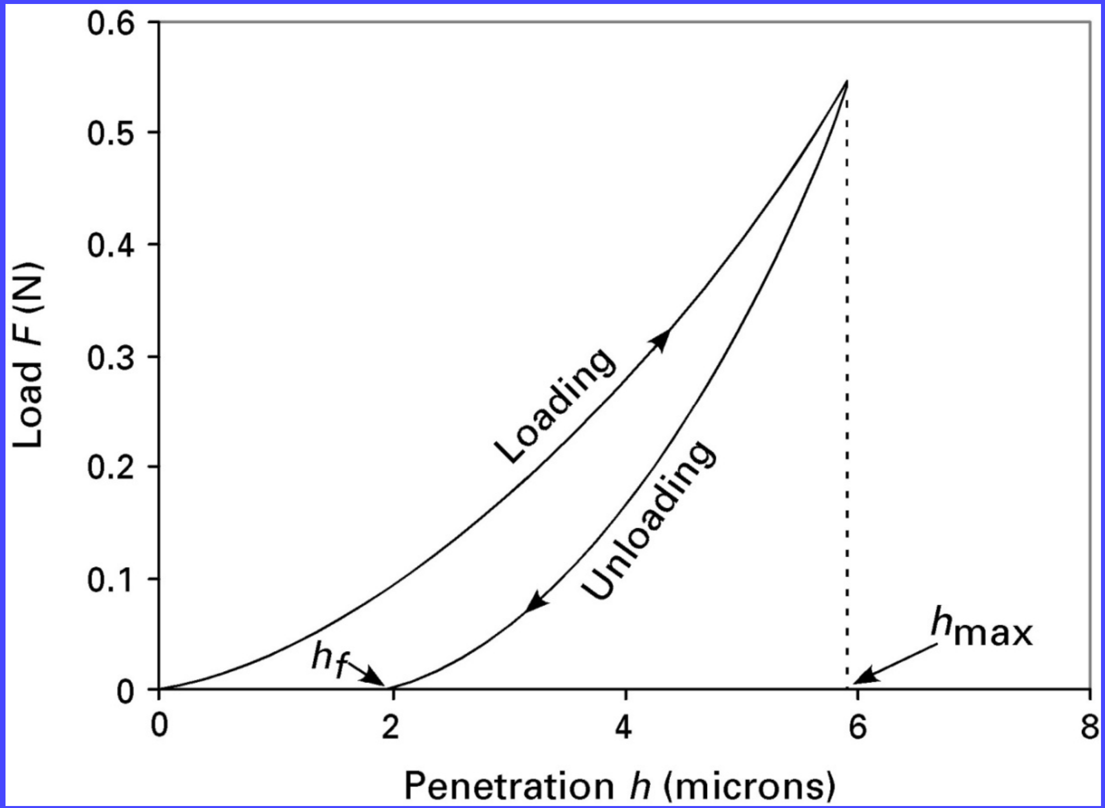
H_{vL2} (GPa)= 37.84 a_2 { load independent hardness; $a_2 = N/\mu m^2$}

Microhardness can be measured dynamically

Measure dF/dh on initial unloading

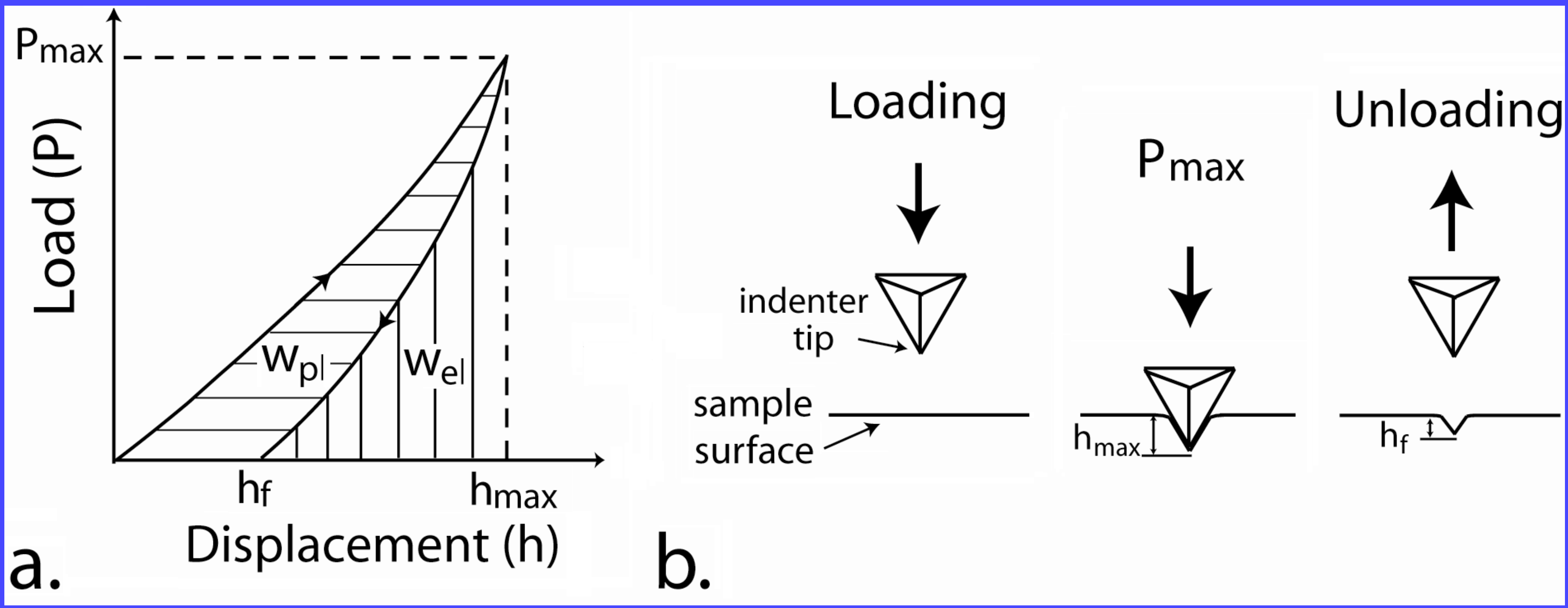
$E_r = (\sqrt{\pi} / 2 \ \sqrt{A}) [dF/dh]$

$E_r = [(1-\nu^2)/E] + [(1-\nu_i^2)/ E_i$



Materials & Methods

- The energy spent during the nanoindentation process can be categorized as plastic energy (W_{pl}) and elastic energy (W_{el}). The indenter penetrates the sample and reaches the maximum penetration (h_{max}) at P_{max}. During the unloading process, the compressed zone recovers and the final depth of the indent (h_f) is often much less than h_{max}.



Elastic Moduli and microhardness are two important mechanical properties.

Elastic modulus is a macroscopic measure of the strength of bonds at the atomic scale.

Hooke's law (stress proportional to strain) defines the moduli of linear elastic solids.

For isotropic glasses only two constants are required – others can be calculated.
Note: $-1 \le \nu \le 0.5$. (When $\nu = 0.5$, $K \longrightarrow \infty$ and $E \longrightarrow 3G$).

Elastic modulus is best measured using the "pulse echo" or similar technique.
For silicate glasses, E 70≈ GPa and $\nu \approx 0.22$.

Hardness is a measure of the resistance to penetration. Both densification and material pile-up are observed in glasses.

Vickers indentation is the most common diamond indenter for glasses.
For a silicate glass, $H_v \approx 5.5$ GPa