

This data refers to spreads of nonsingular pairs (nsp-spreads, for short) in symplectic vector spaces (see Steven H. Weintraub, Spreads of nonsingular pairs in symplectic vector spaces, J. Geom. 86 (2006), 165-180). We adopt the terminology of that paper, and refer to it for results cited here. But we call the reader's attention to several items. (We are simplifying and specializing the discussion here to the items most relevant to the material here.)

We let \mathbb{F} be a field, and we let V be the vector space $V = \mathbb{F}^{2n}$, which we regard as consisting of row vectors, and we let V be equipped with the symplectic form $\langle v, w \rangle = vJ^t w$ where J is the $2n$ -by- $2n$ matrix $J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$ with I the n -by- n identity matrix. V has isometry group $\text{Sp}(V) = \text{Sp}_{2n}(\mathbb{F})$, which acts on the left on V by $g(v) = vg^{-1}$.

DEFINITION 1. A *nonsingular pair* in V is $\Delta = \{\delta, \delta^\perp\}$ where δ is an n -dimensional subspace of V such that the restriction of \langle, \rangle to δ is non-singular, and δ^\perp is its orthogonal complement. In this situation the restriction of \langle, \rangle to δ^\perp is also non-singular and $V = \delta \oplus \delta^\perp$.

We regard $\Delta = \{\delta, \delta^\perp\}$ as the union of the two subspaces δ and δ^\perp . We call two subsets A and B of V almost disjoint if $A \cap B = \{0\}$.

DEFINITION 2. A *spread of nonsingular pairs* or *nsp-spread* in V is $\sigma = \{\Delta_i\}_{i \in I}$, a set of nonsingular pairs that are pairwise almost disjoint and whose union is V . Equivalently, $\sigma = \{\Delta_i\}_{i \in I}$ is an nsp-spread if $\{\Delta_i - \{0\}\}$ partitions $V - \{0\}$.

Our main result is then:

THEOREM 3. Let n be an arbitrary even integer and let \mathbb{F} be any finite field of odd characteristic or any algebraic number field. Then there exist nsp-spreads in $V = \mathbb{F}^{2n}$.

We observe that the action of $\text{Sp}(V)$ on V descends to an action of $G = \text{PSp}(V) = \text{Sp}(V)/\{\pm 1\}$ on $\{\text{subspaces of } V\}$ and on $\{\text{nsp-spreads in } V\}$.

DEFINITION 4. Let n be an even integer and q an odd prime power. Then $j(n, q)$ is the number of nsp-spreads in $V = \mathbb{F}_q^{2n}$ and $b(n, q)$ is the number of orbits of $\{\text{spreads in } V\}$ under the action of G .

Henceforth we restrict our attention to $n = 2$. As is well-known, G is a group of order $q^4(q^2 - 1)(q^4 - 1)/2$. G acts transitively on $\{\text{nonsingular pairs in } V\}$ and using this it is easy to see that there are $q^2(q^2 + 1)/2$ nonsingular pairs in V .

Since V has $q^4 - 1$ nonzero elements and a nonsingular pair has $2(q^2 - 1)$ nonzero elements we immediately observe that any nsp-spread in V consists of $(q^2 + 1)/2$ nonsingular pairs. Then an easy argument shows that every nonsingular pair is contained in $j(n, q)/q^2$ nsp-spreads.

Henceforth we further restrict our attention to $n = 2$ and $q = p$ an odd prime. We have computed:

Theorem 5. The following values are correct:

n	p	$j(n, p)$	$b(n, p)$
2	3	27	1
2	5	14625	2
2	7	16311022	17

For each of $p = 3, 5, 7$ we have files nspairsp and spreadsp.

The file nspairsp is a file with all the nonsingular pairs, in the following format: Each line begins with a sequence number for the nonsingular pair Δ , followed by sequence numbers for each of the nonsingular planes δ and δ^\perp it contains. (These sequence numbers are arbitrary.) It then continues with $p + 1$ vectors in δ and $p + 1$ vectors in δ^\perp . Note that δ and δ^\perp each contain $p^2 - 1$ vectors. The remaining vectors can be obtained by multiplying the given vectors by the nonzero scalars in \mathbb{F}_p .

The file spreadsp is a file with all the spreads, in the following format: Each line begins with a sequence number for the spread σ , followed by the sequence numbers for the $(p^2 + 1)/2$ nonsingular pairs it contains. (Again, the sequence numbers for the spreads are arbitrary.) For $p = 5, 7$, these are followed by a letter which denotes the G -orbit in which the spread lies, this letter being A or B in case $p = 5$, or A, ..., Q in case $p = 7$.

Actually, the files spreads3 and spreads5 exist as stated. Because of the length of the file spreads7 (16311022 lines, with a total size of approximately 2.25 gigabytes), this file is in 7 parts, sprds7pi, $i = 1, \dots, 7$. However, the total size of 2.25 gigabytes exceeds the capacity of this website, so in fact these files are not available here, but can be obtained from the author upon request.