This data refers to generalized continued fractions (see Maxwell Anselm and Steven H. Weintraub, A generalization of continued fractions, J. Number Theory 131 (2011), 2442-2460). We adopt the terminology of that paper, and refer to it for results cited here. But we call the reader's attention to several items.

A cf_N expansion is a continued fraction expansion with "numerator" N.

Roughly speaking, the "best" cf_N expansion of a positive real number is the cf_N expansion that provides the best approximation at every stage.

The analysis of these expansions shows there is a difference between the cases N small (for E) and N large (for E). If $D = [\sqrt{E}]$, then N is small (for E) if $N \leq 2D$, and N is large (for E) otherwise.

We present a table of all nonsquare values of $E \leq 200$ and $N \leq 200$ for which $[[\sqrt{E}]]_N$, the best cf_N expansion of $\sqrt{E}$, has period ≤ 100.

Here are some highlights/statistics of this data.

There are a total of 4361 such pairs (E, N). Of these, 2114 pairs have N small and 2247 pairs have N large. There are:
214 pairs with period length 1.
2817 pairs with period length 2.
15 pairs with period length 3.
843 pairs with period length 4.
14 pairs with period length 5.
196 pairs with period length 6.
2 pairs with period length 7.
124 pairs with period length 8.
4 pairs with period length 9.
112 pairs with period length $10 - 19$.
14 pairs with period length $20 - 29$.
5 pairs with period length $30 - 39$.
1 pair with period length 88.

Here are the values of (E, N) which have the longest periods:
$E = 166$, $N = 2$: period 88
$E = 172$, $N = 2$: period 38
$E = 151$, $N = 2$: period 36
$E = 163$, $N = 2$: period 32
$E = 190$, $N = 4$: period 32
$E = 157$, $N = 2$: period 30

Odd periods are relatively rare. Within the range of this table, there are 214 pairs with period length 1 and 41 pairs with odd period length > 1, and there are only 6 cases of odd period > 10. They are:
$E = 181$, $N = 1$: period 21
$E = 157$, $N = 1$: period 17
$E = 109$, $N = 1$: period 15
$E = 193$, $N = 1$: period 13
$E = 61$, $N = 1$: period 11
$E = 97$, $N = 1$: period 11
There are only 5 cases for $N > 1$ with odd period ≥ 5. They are:
$E = 118$, $N = 2$: period 5
$E = 139$, $N = 3$: period 5
$E = 162$, $N = 2$: period 5
$E = 166$, $N = 5$: period 5
$E = 181$, $N = 4$: period 5
There is only 1 case of N large with odd period > 1:
$E = 53$, $N = 112$: period 3

We say that a sequence $b_1, \ldots, b_i$ is palindromic if it reads the same left-to-right as right-to-left. We say that a sequence $b_1, \ldots, b_i, c_1, \ldots, c_j$ is semi-palindromic of type (i, j), abbreviated as $\mathrm{sp}(i, j)$, if it consists of a palindrome of length i followed by a palindrome of length j.

If we write $[[\sqrt{E}]]_N = [[a_0, a_1, a_2, ...]]_N$, then $a_0 = D$. As shown in that paper, if N is small and $[[\sqrt{E}]]_N$ is periodic of period k, then the period begins with a_1 if N is small and a_2 if N is large. In case N small, the periodic part is always $\mathrm{sp}(k-1, 1)$, and $a_k = 2D$. (Of course, in the case $N = 1$ it is known that the continued fraction expansion of $\sqrt{E}$ is always periodic and always of this form.) Non-semipalindromic cases seem to be extremely rare. There are only two cases in this range where $[[\sqrt{E}]]_N$ is not semipalindromic:
$E = 31$, $N = 13$: period 4
$E = 187$, $N = 58$: period 6

The accompanying files are two tables, each in three formats:
periodic_table is a table with one line for each periodic case, giving the values of E and N, the length of the period, whether or not the expansion is semipalindromic, and if so, of what type.
periodic_table-long is a table with the above line for each periodic case followed by line(s) giving the cf_N expansion up until the end of the first period.

The files are:
periodic_table.html an html file suitable for viewing onscreen,
periodic_table a UNIX text file suitable for download,
periodic_table.txt a DOS text file suitable for download,
periodic_table-long.html an html file suitable for viewing onscreen,
periodic_table-long a UNIX text file suitable for download,
periodic_table-long.txt a DOS text file suitable for download.
(With my operating system and browser, both of the files *periodic_table* and *periodic_table-long* are suitable for viewing onscreen. But since not all browsers handle files the same way, I am including the html versions just in case.)