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★**Factorization: unique and otherwise.**

CMS Treatises in Mathematics.
Canadian Mathematical Society, Ottawa, ON; A K Peters, Ltd., Wellesley, MA, 2008. *x*+260 *pp.* $49.00. *ISBN* 978-1-56881-241-0

Although lines and distances are often not of the same dimension, it is often said that the shortest distance between two points is a straight line. The author of the book under review takes this view quite seriously in presenting a first course in number theory to the student. In essence, the purpose of the book is to start with the Fundamental Theorem of Arithmetic (FTA), and head directly toward Algebraic Number Theory (ANT), with stops along the way only for essential material. The author then covers his tracks by including a series of appendices which cover many of the fundamental topics in this subject, which are not found somewhere along the path from FTA to ANT.

Personally, I do like this novel approach, and with the fact that this book is very nicely presented, with detailed explanations and many examples and exercises, it is safe to say that a first course in number theory following this book closely will be accessible and enjoyed by most second-year undergraduates and above.

The material covered in the five chapters runs sequentially as follows. The first chapter is of an introductory nature, presenting the basic properties of integral domains, which primarily are the sets in which the entire book will concentrate on. The author moves on to unique factorization domains in Chapter 2, covering unique factorization in the integers, and discussing examples of cases where unique factorization fails. The presentation of this phenomenon (non-unique factorization) is explicit, and I would expect that most students, who may simply take unique factorization as a given, will certainly find this chapter most engaging. It is nice to see this topic presented early on in the textbook. The author then further expands on the idea of integer by pursuing the Gaussian integers in considerable detail, which again should broaden any student's perspective considerably. Similarly, in Chapter 4, the author discusses a more general family of integral domains, namely rings of integers of quadratic fields. Considerable attention is paid to the unit group in the real case, which naturally prompts a detailed study of the Pell equation. Chapter 5 is the final destination, where the author now puts the pedal to the floor, and provides as much of the full picture to the student as is reasonably possible. The concepts of algebraic number, ideal theory, prime ideal, and even Dirichlet's Unit Theorem are presented in such a way as not to overwhelm the student, but rather to enlighten and motivate the student to pursue the subject further.

As stated above, this is an excellent book for a first course in number theory, as the novel approach has a goal in mind, and yet covers most of the essential topics covered in any other introductory book on number theory. The only shortcoming is that the student is not provided with any pointers for further reading. Such pointers would be particularly useful, since some of the bigger theorems in this book are not proved (for good reason, as the proof of say Dirichlet's unit theorem is well beyond the scope of this book). Nevertheless, since anyone using this book to teach a course could easily fill this hole, this is not a serious issue.

Reviewed by *P. G. Walsh*