**Differential Forms: Theory and Practice–Errata**

The vast majority of these typos were found by Michael Janou Glaser, Bruce Gould, and Richard Louis Rivero, all of whom the author sincerely thanks.

Page 10, lines -2 and -1: Replace by
*In particular, if $n$ is odd, then $(\varphi^*)^* = \varphi$ for any $k$. If $n$ is even, then $(\varphi^*)^* = \varphi$ for $k$ even and $(\varphi^*)^* = -\varphi$ for $k$ odd.*

Page 15, line 6: $\varphi - x^2dx - z^2dy$ should be $\varphi = x^2dx - z^2dy$.

Page 16, lines 11 and 12: $dx_1dx_2$ should be $dx_1dx_2dx_I$, $dx_1dx_3$ should be $dx_1dx_3dx_I$, and $dx_{n-1}dx_n$ should be $dx_{n-1}dx_ndx_I$.

Page 19, line -1: $\psi = x^5y^3z^2$ should be $\psi = x^5y^2z^3$.

Page 20, line 2: $d\psi = 0$ should be $d\varphi = 0$.

Page 21, line 13: $\varphi_1 = df_0$ should be $\varphi^1 = df_0$.

Page 22: The labels on the axes of both graphs should be reversed.

Page 23, line 17: $df_0 = d\varphi^1$ should be $df_0 = \varphi^1$.

Page 23, line -6: $\theta_2(-1-1)$ should be $\theta_2(-1,-1)$.

Page 25, line -7: $f_{x_1}$ should be $f^1$.

Page 25, line -5: $f_1$ should be $f^1$.

Page 26, lines 4 and 5: $g_2$ should be $g^2$.

Page 27, line 15: $x^2z^2$ should be $x^2z^3$.

Page 27, line 16: $x^2$ should be $2xy$.

Page 38, line 10: $R$ should be $\mathcal{R}$.

Page 39, line -4: $\varphi = Adx + bdy + cdz$ should be $\varphi = Adx + Bdy + Cdz$.

Page 44, line -3: $dx_n$ should be $dx_m$.

Page 46, line 7: the second occurrence of $dx_m dx_{I_1}$ should be $dx_m dx_{I_2}$.

Page 47, line 5: $\tilde{\varphi}$ should be $\varphi$.

Page 57, lines 15 and 16: $\mathbf{F}_1$ should be $\mathbf{F}^1$.

Page 64, lines 10 and 11: $S_n$ should be $S_k$.

Page 64, line 11: $\sigma' = \sigma\tau$ should be $\sigma' = \tau\sigma$.

Page 64, line 13: $\mathbf{v}_2^p$ should be $\mathbf{v}_p^2$.

Page 66, lines -6,-5: $\sigma_j$ should be defined by $\sigma_j(1) = j+1$, $\sigma_j(i) = i-1$ for $2 \leq i \leq j+1$, $\sigma_j(i) = i$ for $j+1 < i \leq m$.

Page 67, line -3: $\operatorname{sign}(\sigma_1 \times \sigma_2) = \operatorname{sign}(\sigma_1) + \operatorname{sign}(\sigma_2)$ should be $\operatorname{sign}(\sigma_1 \times \sigma_2) = \operatorname{sign}(\sigma_1)\operatorname{sign}(\sigma_2)$.

Page 67, line -1: $\varphi$ should be $\varphi_1$.

Page 68, line 2: $\sigma$ should be $\sigma_1$ and $\varphi$ should be $\varphi_1$.

Page 68, line 4: $\sigma$ should be $\sigma_2$ and $\varphi$ should be $\varphi_2$.

Page 68, line 6: Insert $\operatorname{sign}(\sigma)$ after the summation sign.

Page 69, line -3 and page 73, line -7: $p = (2, 5, 3)$ should be $p = (2, 5, -3)$.

Page 74, line -4: Insert $D_{\mathbf{v}_p^m}$ after $(-1)^{m-1}$.

Page 81, line 13: *$\mathcal{B}$ gives the standard basis of* $\mathbf{R}^n$ should be *$\mathcal{B}$ gives the standard orientation of* $\mathbf{R}^n$.

Page 89, lines 20, 25: to verify $(*_2)$ should be to verify $(*_1)$.

Page 89, lines -2,-1: should read ... where $H = (h_1, \ldots, h_{k-1})$ ranges over all multi-indices of degree $k-1$, up to permutation. Furthermore ...

Page 90, line 5: should read ... either $H$ is a permutation of $I_m$ for some $m$, in which case we may choose $H = I_m$, or else $H$ has an entry that is not in $I$.

Page 90, line 9: product of terms should be sum of terms.

Page 93, lines 8,9: should read ... is determined by its values $\xi(d\varphi)(\mathbf{e}_p^J)$ on all $k$-tuples $\mathbf{e}^J$ up to permutation. Thus ...

Page 93 line, 13 through Page 94, line 6: Replace by

$$\begin{aligned} dx_j dx_I(\mathbf{p}_{tp}, \mathbf{e}_{tp}^J) &= dx_j(\mathbf{p}_{tp}) dx_I(\mathbf{e}_{tp}^J) \\ &\quad - dx_j(\mathbf{e}_{tp}^{j_1}) dx_I(\mathbf{p}_{tp}, \mathbf{e}_{tp}^{j_2}, \mathbf{e}_{tp}^{j_3}, \ldots, \mathbf{e}_{tp}^{j_k}) \\ &\quad + dx_j(\mathbf{e}_{tp}^{j_2}) dx_I(\mathbf{p}_{tp}, \mathbf{e}_{tp}^{j_1}, \mathbf{e}_{tp}^{j_3}, \ldots, \mathbf{e}_{tp}^{j_k}) \\ &\quad \ldots + (-1)^k dx_j(\mathbf{e}_{tp}^{j_k}) dx_I(\mathbf{p}_{tp}, \mathbf{e}_{tp}^{j_1}, \mathbf{e}_{tp}^{j_2}, \ldots, \mathbf{e}_{tp}^{j_{k-1}}). \end{aligned}$$

Note that the first term is $x_j$ if $J = I$ and is 0 if $J$ is not a permutation of $I$.

As for the remaining terms in the summation, if there is no value of $\ell$ for which $j_\ell = j$, then all of them are equal to 0, as $dx_j(\mathbf{e}_{tp}^{j_\ell}) = 0$ for every $j_\ell$.

If $j_\ell = j$ for some $\ell$, then every term except for possibly that one is 0, and that one has the value $(-1)^\ell dx_I(\mathbf{p}_{tp}, \mathbf{e}_{tp}^{j_1}, \ldots, \mathbf{e}_{tp}^{j_{\ell-1}}, \mathbf{e}_{tp}^{j_{\ell+1}}, \ldots, \mathbf{e}_{tp}^{j_1})$. If $J_\ell$ is not a permutation of $I_m$ for any $m$ then this is equal to 0, as when this is expanded out there will be a $j$-index not equal to an $i$-index in every term. Otherwise we may assume, after reordering $J$, that $J_m = I_m$, and then this term has the value $(-1)^m dx_{i_m}(\mathbf{p}_{tp}) dx_{I_m}(\mathbf{e}_{tp}^{J_m}) = (-1)^m x_{i_m}$.

Page 95, line 10 and page 97, line 11: eliminate second occurrence of $\mathcal{R}$.

Page 95, line 14: $A(tp)$ should be $A^I(tp)$.

Page 95, line -3: from $(*_3)$ should be from $(*_4)$.

Page 103, line 1: topmost entry of $\mathbf{r}'(2)$ should be 1.

Page 103, line 11: $r(1)$ should be $r(0)$.

Page 103, line 12: $\mathbf{r}'(1)$ should be $\mathbf{r}'(0)$.

Page 107, line 12: the point $p$ should be the point $t_0$.

Page 110, line 6: function $\mathcal{S}$ defined on $N$ should be function $f$ defined on $\mathcal{S}$.

Page 111, line 5: $k$-forms should be $j$-forms.

Page 111, lines 6 and 7: $\mathcal{F}(S)$ should be $\mathcal{F}(\mathcal{S})$ and $\mathcal{F}(R)$ should be $\mathcal{F}(\mathcal{R})$.

Page 113, line 14: $k^*(f\psi + g\rho)$ should be $k^*(a\psi + b\rho)$.

Page 114, line 1: $M$ should be $\mathcal{R}$.

Page 114, line 7: $\varphi$ is linear should be $\varphi$ is multilinear.

Page 114, line -2: $\mathbf{v}_p^1$ should be $k_*(\mathbf{v}_p^1)$ and $\mathbf{v}_p^j$ should be $k_*(\mathbf{v}_p^j)$.

Page 116, line 5: $k(f(p))$ should be $f(k(p))$.

Page 117, line -6: Lemma 3.3.8 should be Lemma 3.2.7.

Page 118, line 7: $k(u_1, \ldots, u_n)$ should be $k(u_1, \ldots, u_m)$.

Page 119, lines 2 and 3, and page 120, line 8: $v^3 + 2v$ should be $2v^3 + v$.

Page 119, lines 4 and 5, and page 120, lines 10 and 11 should both be: $= (2u^3 + 4v^3 + 5u^2 + 2u + 3v)du + (3u^2 + v^3 + u^2 + 2u - v + 1)dv$.

Page 120, line -4: The sum should be over $H$, not over $I$.

Page 121, line 4: Theorem 2.2.19 should be Theorem 2.2.20.

Page 123, lines -5 through -2: All occurrences of $\mathbf{v}$ should be $\mathbf{v}_p$.

Page 125, line -10: $(2uv + 1)$ should be $(2u - v + 1)$.

Page 127, line 7: should read ... so $k^*(dA) = k^*(dx_i)$ is closed.

Page 128, line 13: $d(k^*(A^{I_0})$ should be $d(k^*(A^{I_0}))$.

Page 130, line 1 (twice) and lines 5 and 6: $A_I(x_1, \ldots, x_n, t)$ should be $A^I(x_1, \ldots, x_n, t)$.

Page 131, lines -10 and -6: $\tilde{\xi}(\theta)$ should be $\tilde{\xi}(\varphi)$.

Page 132, line 8: by linearity should be by multilinearity.

Page 132, line -4: $dx_I$ should be $dx_I dt$ (twice).

Page 133, line -5: $\mathbf{e}_p^H$ should be $dx_I(\mathbf{e}_p^H)$.

Page 134, lines -8 to -1: replace by

$$\begin{aligned}\big(\tilde{\xi}(d\varphi)\big)(\mathbf{e}_p^H) &= (-1)^{((\ell+1)-1)} \sum_{i=1}^{n} \int_0^1 \big[A_{x_i}(p,t) dx_i dx_I dt(\tilde{\mathbf{e}}^H_{(p,t)}, \tilde{\mathbf{f}}_{(p,t)})\big] dt \\ &= (-1)^{\ell} \sum_{i=1}^{n} \int_0^1 \big[A_{x_i}(p,t) dx_i dx_I(\tilde{\mathbf{e}}^H_{(p,t)})\big] dt \\ &= (-1)^{\ell} \sum_{i=1}^{n} \left[\int_0^1 A_{x_i}(p,t) dt\right] dx_i dx_I(\mathbf{e}_p^H).\end{aligned}$$

For $T_2$: First note that for any $(\mathbf{v}_p^1, \ldots, \mathbf{v}_p^{\ell-1})$,

$$\begin{aligned}\tilde{\xi}(\varphi)(\mathbf{v}_p^1, \ldots, \mathbf{v}_p^{\ell-1}) &= (-1)^{\ell-1} \int_0^1 \left[A(p,t)dx_I dt(\tilde{\mathbf{v}}_{(p,t)}^1, \ldots, \tilde{\mathbf{v}}_{(p,t)}^{\ell-1}, \tilde{\mathbf{f}}_{(p,t)})\right] dt \\ &= (-1)^{\ell-1} \int_0^1 \left[A(p,t)dx_I(\tilde{\mathbf{v}}_{(p,t)}^1, \ldots, \tilde{\mathbf{v}}_{(p,t)}^{\ell-1})\right] dt \\ &= (-1)^{\ell-1} \left[\int_0^1 A(p,t)dt\right] dx_I(\mathbf{v}_p^1, \ldots, \mathbf{v}_p^{\ell-1}).\end{aligned}$$

Thus

$$\tilde{\xi}(\varphi) = (-1)^{\ell-1} \left[\int_0^1 A(p,t)dt\right] dx_I$$

and hence

$$\begin{aligned}d(\tilde{\xi}(\varphi)) &= (-1)^{\ell-1} \sum_{i=1}^n \left[\int_0^1 A(p,t)dt\right]_{x_i} dx_i dx_I \\ &= (-1)^{\ell-1} \sum_{i=1}^n \left[\int_0^1 A_{x_i}(p,t)dt\right] dx_i dx_I,\end{aligned}$$

so, for any $(\mathbf{e}_p^H)$,

$$d(\tilde{\xi}(\varphi))(\mathbf{e}_p^H) = (-1)^{\ell-1} \sum_{i=1}^n \left[\int_0^1 A_{x_i}(p,t)dt\right] dx_i dx_I(\mathbf{e}_p^H).$$

Page 135, line 9: $S = \mathrm{supp}(\tilde{\xi}(\varphi))$ should be $S = \mathrm{supp}(\varphi)$.

Page 137, lines 10, 11, 12: $d(\xi(\varphi))$ should be $d(\Xi(\varphi))$.

Page 157, line -13: the complement of $\mathbb{R}^{n-1}_{--}$ in $\mathbb{R}^{n-1}$ should be the complement of $\mathbb{R}^n_{--}$ in $\mathbb{R}^n_-$.

Page 160, lines 15: the ) should not be there.

Page 162, line -12: $\mathbf{v}_q^1$ should be $\mathbf{v}_p^1$ and $\mathbf{v}_q^2$ should be $\mathbf{v}_p^2$.

Page 162, line -11 should be: $c^1\mathbf{v}_p^1 + c^2\mathbf{v}_p^2 = k_*(c^1\mathbf{w}_q^1 + c^2\mathbf{w}_q^2)$.

Page 163, line -6: restriction of $f$ should be restriction of $\varphi$.

Page 166, lines 4 and 5: Definition 1.2.17 should be Example 1.2.17 and Definition 1.2.21 should be Example 1.2.21.

Page 184, lines -3 and -2: *a linear transformation* should be *an isomorphism*.

Page 185, lines 8, 11, and 13: $[L(v)]$ should be $[L]$ (seven times).

Page 190, line 16: There should be a space after the word "If".

Page 192, line -14: Definition 4.6.17 should be Definition 4.5.25.

Page 193, line 10: we will call the standard basis of $T_0\mathbb{R}^{n-1}$ should be we will call the standard basis of $T_0\partial\mathbb{R}^n_-$.

Page 193, lines -10 and -9: $\mathbb{R}^n_-$ should be $\mathbb{R}^n_+$ (twice).

Page 195, lines -7 and -5: $(k, \varepsilon_k)$ should be $(k, \sigma_k)$.

Page 196, line -2: The correct definition of the function $f_3(x)$ is

$$f_3(x) = f_2(x)/(f_2(x) + f_2(1 + x)).$$

Page 197, line -13: The first occurrence of the word "the" should be eliminated.

Page 197, line -3: $\mathcal{R}^n$ should be $\mathbb{R}^n$.

Page 198, line 3: $x_i^0$ should be $x_1^0$.

Page 198, line 10: subscript $s - 1$ should be $j - 1$.

Page 198, line 16 $k^{-1}$ should be $k_j^{-1}$ (twice).

Page 201 lines 12 and 24: $\ell$-form(s) should be $\ell - 1$ form(s) (twice).

Page 201 lines -3, -2, -1 should read:

$$F(p,0) = f_0(p) \text{ and}$$
$$F(p,1) = f_1(p)$$

for every $p \in M$.

Page 209, line 2: $k_i(a,v)$ should be $\tilde{k}_i(a,v)$.

Page 209, line -3: $F : E \longrightarrow E'$ should be a smooth map $F : E \longrightarrow E'$.

Page 212, lines 16-17: replace *where* $\tilde{\ell}_{i_2 i_1} = (\ell_{i_2 i_1}, L_{i_2 i_1})$, *with* $\ell_{i_2 i_1}$ *a diffeomorphism and* $L_{i_2 i_1}$ *a vector space isomorphism* by *where* $\tilde{\ell}_{i_2 i_1}(q_1, v_1) = (\ell_{i_2 i_1}(q_1), L_{i_2 i_1}(q_1)(v_1))$, *with* $\ell_{i_2 i_1}$ *a diffeomorphism and for every* $q_1$, $L_{i_2 i_1}(q_1)$ *a vector space isomorphism.*

Page 212, line 20: replace $L_{i_3 i_1} = L_{i_3 i_2} \circ L_{i_2 i_1}$ by $L_{i_3 i_1}(q_1) = L_{i_3 i_2}(q_2) \circ L_{i_2 i_1}(q_1)$, *where* $q_2 = \ell_{i_2 i_1}(q_1)$.

Page 214, line -8: *Remark 5.1.8(ii)* should be *Remark 5.1.8(i).*

Page 217, line 3: the second occurrence of $(M^*)^{-1} \circ (L^*)^{-1}$ should be $(M^{-1})^* \circ (L^{-1})^*$.

Page 217, line -11: Theorem 4.4.9 should be Theorem 5.1.9.

Page 219, line 5: ${}^t g\, {}^t B v$ should be ent$({}^t g\, {}^t B v)$.

page 227, line 5: add We implicitly assume throughout this chapter, unless explicitly stated otherwise, that $\varphi$ has compact support (which is automatically true if $M$ is compact).

Page 231, line -12: $\varepsilon$ should be $\varepsilon_k$.

Page 232, lines 16 and 24: $f(u)$ should be $f(k(u))$.

Page 235, line 10: insert $\varepsilon_k$ after $=$.

Page 236, lines 2 and 3 should read:

$$\begin{aligned}\int_N \varphi &= \int_{sM} \big(k^*(\varphi)\big)(\mathbf{e}^1,\dots,\mathbf{e}^n)dV_{x_1,\dots,x_n} \\ &= \int_{sM} \varphi((k_*(\mathbf{e}^1),\dots,k_*(\mathbf{e}^n))dV_{x_1,\dots,x_n}.\end{aligned}$$

Page 242, line -7: 80 should be 100 and $-80$ should be $-100$.

Page 244, line -5: $\mathbf{e}^1$ should be $\mathbf{e}^1_t$ (twice).

Page 245, line 14: insert $dt$ after $(-2t)$.

Page 245, line -7: $-144$ should be $-656$ (twice).

Page 248, lines -4 and -3 should read:

$$\begin{aligned}k^*(\omega) &= \frac{\cos\theta}{\cos^2\theta+\sin^2\theta}(-\sin\theta d\theta) + \frac{\sin\theta}{\cos^2\theta+\sin^2\theta}(\cos\theta d\theta) \\ &= 0\end{aligned}$$

Page 249, line -10: $\varphi(k'(t))$ should be $\varphi(\mathbf{k}'(t))$.

Page 249, line -1: $\rho_1(\mathbf{v}_p) =$ should be $\rho_1(\mathbf{v}_p) = 1$.

Page 251, lines -4 to -1 should read:

$$\begin{array}{lll}\int_{C_1}\varphi_1 = 0 & \int_{C_1}\varphi_2 = 2 & \int_{C_1}\varphi = 2 \\ \int_{C_2}\varphi_1 = 2 & \int_{C_2}\varphi_2 = 0 & \int_{C_2}\varphi = 2 \\ \int_{C_3}\varphi_1 = 7/6 & \int_{C_3}\varphi_2 = 5/6 & \int_{C_3}\varphi = 2 \\ \int_{C_4}\varphi_1 = 8/5 & \int_{C_4}\varphi_2 = 2/5 & \int_{C_4}\varphi = 2\end{array}$$

Page 254, line 4: being should be beginning.

Page 256, line -4: $q_1$ should be $q$.

Page 256, line -3: line should read $r(t) = (x_0^1, \ldots, x_0^{i-1}, t, x_0^{i+1}, \ldots, x_0^n)$.

Page 257, line -5: $a'(x_0)$ should be $a'(x_0^i)$.

Page 260, lines 15 and 19: Corollary 6.4.14 should be Corollary 6.4.13.

Page 261, line 1: Corollary 6.4.14 should be Corollary 6.4.13.

Page 261, lines 15 and 16: $f^*$ should be $f_0^*$.

Page 264, line 7: line should end with a period instead of a comma.

Page 264, line 8: *where* $\varphi^1$ *is* should be *Let* $\varphi^1$ *be*.

Page 265, illustration following line 2: the region labelled $R_1$ should be labelled $R_t$.

Page 267, line 2: $a \geq 0$ should be $a \leq 0$.

Page 270, line 1: $\mathbf{F}(t)$ should be $\mathbf{F}(r(t))$.

Page 276 lines -8 and -7 should read:

$$\begin{aligned}\int_S \varphi &= \int_{s_R} k^*(\varphi)(\mathbf{e}^1, \mathbf{e}^2) dA_{uv} = \int_{s_R} \varphi(k_*(\mathbf{e}^1), k_*(\mathbf{e}^2)) dA_{uv} \\ &= \int_{s_R} \varphi(\mathbf{k}_u, \mathbf{k}_v) dA_{uv},\end{aligned}$$

Page 287, lines 6 and -5: delete second occurrence of $dy = dy$,.

Page 289, line 7: $H_N$ should be $H_S$.

Page 298, line -1: $|_0^2$ should be $|_0^3$.

Page 299, line -4: smooth map $k$ should be smooth map $c$.

Page 300, line 8: basic chain $k_i$ should be basic chain $c_i$.

Page 310, line 9: *compactly* should be *compactly supported*.

Page 319 line 12, should read:

$$\int_C \varphi = \int_{C' \cup C''} \varphi = \int_{C'} \varphi + \int_{C''} \varphi = 0$$

Page 320, line -8: $\mathbb{R}^2$ should be $\mathbb{R}^3$.

Page 341, line -9: $\varphi_i$ should be $\varphi^i$, $\psi_i$ should be $\psi^i$, and $d\varphi_i$ should be $d\varphi^i$.