Differential Forms: A Complement to Vector Calculus

Errata

Page 3 k or ℓ

Page 4 Ex. 2 $a)\ 3\varphi_3 - 4\varphi_4$ $b)\ x\varphi_3 + y\varphi_4$

Ex.5 $a)\ x\psi_3 + y\psi_4$ $b)\ 2y\psi_3 + \psi_4$

Ex. 6 $d')\ \psi_2\varphi_2$

Page 5 line 12 $(z + 1)e^z dz$

line -1 $d(x^4 + y^3 - z^2)dz$

Page 10 line 15 $\psi = x^5y^2z^3$

Page 17 line -15 Let $\varphi =$

line -8 $\frac{\partial}{\partial y}(x^2y^3 + x^4 + c(y))$

Page 18 line 1 Let $\varphi =$

line 6 Then $x^2z^3 +$

line 7 $= x^2z^3 + 2xy +$

line -14 $x^2yz^3 + xy^2 + 4xz + 2x + 2yz^3 - y - 2z^2 + c$

Page 19 line -12 $\varphi =$

Page 20 line 15 Let $\varphi =$

Page 21 line 14 Let $\varphi =$

line 17 Then $\psi = (x^2y^2z^3 + 2x^3y^3z - x^4yz^2)dydz$

Page 30 line 15

DEFINITION 3.12: $(d?_1)^* = \varepsilon d?^1$, $\varepsilon = \pm 1$, where $(d?_1)(\varepsilon d?^1) = dxdydz$.

Page 31 line 13 $dx_1, ..., dx_n$

Page 32 Ex. 4 b) $f = 2xy^3$

Page 33 Ex. 7e) $-4xy^2zdxdy$

Ex. 10

$$(d?_1)^* = \varepsilon d?^1 \text{ where } (d?_1)(\varepsilon d?^1) = dxdy.$$

Page 38 line 3 every

Page 54 line 11 then

Page 57 line -3 of footnote because that analogy lets us use

Page 65 line 12 $C(x,y,0)dz.$

Page 67 line 22

$$= x_1x_2\varphi(\mathbf{i},\mathbf{i}) + x_1y_2\varphi(\mathbf{i},\mathbf{j}) + x_2y_1\varphi(\mathbf{j},\mathbf{i}) + y_1y_2\varphi(\mathbf{j},\mathbf{j})$$

line 24

$$= x_1x_2(0) + x_1y_2\varphi(\mathbf{i},\mathbf{j}) + x_2y_1(-\varphi(\mathbf{i},\mathbf{j})) + y_1y_2(0)$$

line 25

$= (x_1y_2 - x_2y_1)\varphi(\mathbf{i},\mathbf{j})$

Page 68 last line of footnote in

Page 71 line -10 point (2,5,-3)

Page 71 line -2 lemma

Page 75 line 7 $r(0)$

Page 76 line 3 definition 3.3

Page 82 line 9 $C(k(t))h'(t)dt.$

Page 91 line 14 $\mathbf{w} = k_*(\mathbf{v}) = (kr)'(0)$

Page 94 lines 2,3,5,8 $\mathbf{w}$ should be $\mathbf{v}$

Page 97 line -9 $c_1\varphi_1$

Page 98 line 15 $g'(t)$

Page 101 line -4 $= \int\limits_I A(f(t))f'(t)dt$

Page 102 line 4 $= \int\limits_I A(f(t))f'(t)dt$

Page 102 line 13 $\varphi =$

line 14 $(6t + 2)^2$

line 18 $\varphi =$

line -1 $\varphi =$

Page 103 line 9 $\varphi =$

line -2 $\varphi^1 =$

Page 107 line -4 *on C*

Page 109 line 4 $(A(r(t)), B(r(t)), C(r(t)))$

line 6 $+ C(f(t), g(t), h(t))\big)dt$

Page 111 line -4 $\int_{C_3} \varphi_1 = 16/15 \qquad \int_{C_3} \varphi_2 = 14/15$

Page 112 line 10 $16/15 + 14/15$

Page 112 line -6 $\partial C = \{q\} \cup -\{p\}$

Page 144 line -10 $(1 \cdot 1 - 0 \cdot 0)$

Page 150 line -5

$$dy = \frac{-4uvdu + 2(-v^2 + u^2 + 1)dv}{(u^2 + v^2 + 1)^2}$$

Page 169: Replace lines -16 through -11 by:

Since $k_2 = k_1 \circ \ell$, $k_2^*(\varphi) = \ell^*(k_1^*(\varphi))$ by proposition III.3.61.

For simplicity we complete the proof when $n = 2$. As $k_1^*(\varphi)$ is a 2-form on a region in $I\!R^2$, we may write $k_1^*(\varphi) = f(x,y)dxdy$ for some function $f(x,y)$. Then

$$\ell^*(k_1^*(\varphi)) = \ell^*(f(x,y)dxdy) = f(\ell(u,v))J(\ell)(u,v)dudv \tag{5.17}$$

by proposition 3.34.

Then the conclusion of the theorem becomes, by definition 3.2,

$$\pm \int\limits_{T_1} f(x,y)dA_{xy} = \pm \int\limits_{T_2} f(\ell(u,v))J(\ell)(u,v)dA_{uv}. \tag{5.18}$$

We are assuming the orientations are all compatible. This implies that either T_1 and T_2 both have the same orientation as surfaces in $I\!R^2$, so both sides of (5.18) get the same sign, and that $J(\ell)(u,v)$ is always positive, or else T_1 and T_2 have opposite orientations as surfaces in $I\!R^2$, so the two sides of (5.18) have opposite signs, and that $J(\ell)(u,v)$ is always negative. In either case, then, (5.18) is just the standard change-of-variable formula for double integrals.

The case of general n is similar. (For $n = 3$, use 4.8 and 4.2 instead of 3.34 and 3.2.) $\square$

Page 190	Ex. 13	should be flush with left margin
Page 194	footnote line 1	invertible
Page 197	line 18	

$$a_2 \le x_2 \le b_2, \ldots, a_k \le x_k \le b_k$$

Page 216 line -3 involve dx_m.

Page 245 line -5 Jq should be Jg

Page 247 I.1 1 a) $(4x^2 - x)dx + 3xdy$

b) $3x^2dx + (-x^2 + 2xy + x + y)dy$

I.1 2 a) $(3x^3 - 4y^2z)dx + (3yz + 4xz)dy$

$-(3x^2 + 3y^2 + 3z^2 + 8x + 4)dz$

b) $(x^4 + y^3z)dx + (-x^3 - xy^2 - xz^2 + 2xy + y)dz$

Page 248 I.2 2 d) $xdydz + (y^2 - 2)dzdx + (-z - 2yz)dxdy$

Page 249 I.3 2 d) $x\mathbf{i} + (y^2 - 2)\mathbf{j} + (-z - 2yz)\mathbf{k}$

Page 251 III.1 2 e) 34 2 f) -17

III.2 2 d) 27

3 a) 85

Page 252 IV.2 7 a) $x^3 - 2xy + 2y^2$

Page 253 IV.4 2) 1/15

Page 254 V.3 13) $-79/5$

V.4 2) 1/15