Modeling Capacity Reservation in High-Tech Manufacturing

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Abstract: The high-tech manufacturing industry is characterized by rapid innovation and volatile demands. Capacity reservation provides a risk sharing mechanism that encourages the manufacturer to expand capacity more readily, while improving the revenue potential for the OEM customers. We propose a *deductible reservation (DR)* contract where the customer reserves future capacity with a fee that is deductible from the purchasing price. We show that the *DR* contract provides *channel coordination* and is *individually rational* for all players involved. This has practical importance since reservation has intuitive benefit for the manufacturer, but less so for the customers. We start the analysis with a one-manufacturer, one-customer system with stochastic demand, then generalize the analysis to the case of n customers. A unique feature of the *DR* contract is that the manufacturer announces *ex ante* the "excess" capacity she will expand in addition to (and regardless of) the customer reservation amount. We show that the reservation fee should be increasing in the excess capacity amount, and coordination could be achieved with different combinations of the two. To establish practical insights we compare the *DR contract* with a contract known in the industry as *take-or-pay*. We show that while the manufacturer is no worse off under *take-or-pay* , there may not exist a contract setting that guarantees to benefit the customers. We discuss the similarities and differences between the capacity reservation contracts and other well known supply chain contracts.

(*Capacity Reservation, Contracting, Supply Chain Coordination, Game Theory)*

1 Introduction

Capacity management is a significant issue in the high-tech industries such as semiconductor, telecommunications devices, and optoelectronics. In this environment, manufacturers are confronted with capital intensive facilities and highly skilled labor, operating under long manufacturing lead-time, short product life-cycle, and near-continuous technological innovation. Physical expansion of manufacturing capacity involves enormous risk. This involves building new facilities, purchasing new equipment, and/or automating existing production processes, all of which translate into significant capital investment. For instance, building a state-of-the-art semiconductor fab requires capital investment exceeding $2 Billion. Before the investment turns into profit, the manufacturer faces technological uncertainties during ramp up, followed by market uncertainties after full production. In the case where demands are not sufficient to cover revenue projections, or to recover the investment, significant consequences follow, e.g., several niche high-tech manufacturers declared bankruptcy during the economic down turn in 2001 due to the sharp demand shortfall.

Knowing what is at stake, high-tech manufacturers constantly seek opportunities for hedging and risk sharing when expanding their capacity. A growing trend in the industry is to structure capacity not only by physical expansions but also by strategic outsourcing. Since the mid 1990's, all major manufacturers have adopted aggressive outsourcing policy when technologically possible (Ngwenyama and Bryson 1999). In this paper, we address the issue of capacity management in this environment. We consider *capacity reservation* contracts between the manufacturer and her main customers: the customer reserves future capacity from the manufacturer, the manufacturer expands capacity by configuring in-house capabilities, or by making outsourcing arrangements. As outsourcing may involve lengthy processes such as technology certification and contract negotiation, early reservation provides the needed lead-time. Note that we do *not* consider capital expansion decisions in this context. This is because capacity reservation is a means of order management, while physical expansion (e.g., building new facilities, procuring capital equipment) requires long-term strategic planning taking into account multi-period market scenarios (Karabuk and Wu, 2001). These decisions are typically made at different levels in the organization, and clearly have different modeling implications, which we will discuss later.

Our research originates from a project completed at a major telecommunications component manufacturer in the U.S. Part of the capacity reservation contract described in the paper has been implemented at the firm. When we started the project in the fall of 2000, the telecommunications infrastructure market is growing at an enormous rate. With a rather conservative expansion policy in the past, the manufacturer's capacity is significantly below that of the market demand. In an upside market, the manufacturer has obvious incentive to expand their capacity for higher revenue, but the focus is on "soft" expansions mentioned above. This is due to the high volatility in market demands. For the family of devices we have examined, demand volatility (percentage change from the lowest to the highest) can be as high as 80% during a particular quarter, while only a few main customers dominate the demand for a particular family. Soft expansion provides the flexibility needed to react to market conditions. The relationship between the manufacturer and their main customers are critical: the manufacturer provides proprietary technology which the customer relies on, while the main customers provide a more stable stream of demands that help to dampen uncertainty. There are also "small" customers in the market, to whom the manufacturer typically releases the orders after satisfying the main customers' demands. Somewhat unique to this high-tech manufacturing environment is that the availability of the right capacity at the right time is more critical then the (wholesale) price. The price of a particular device is negotiated during the "design-win" phase early on, which does not fluctuate in any given quarter, or used as a bargaining tool. In some cases, the devices are produced and charged on a cost-plus basis. As the manufacturing lead-time is long while the capacity can be scarce, the customer often desire to make reservations for future capacity a few months before placing the firm order. Capacity reservation provides the manufacturer the assurance and lead time needed to pursue more aggressive outsourcing, while making plans for line reconfiguration, labor shift adjustments, etc.

2 Related Literature

The main issue involved in capacity expansion can be explained by the notion of *double marginalization*, first introduced in the economics literature by Spengler (1950). Consider a manufacturer-customer collective system. Without a deliberate coordination scheme, the manufacturer faces a local decision problem representing only part of the marginal revenue of the system, thus she does not have the incentive to expand capacity beyond what is locally optimal. This leads to insufficient capacity, which results in lower expected revenue for the customer, and reduced profit potential for the manufacturer. Contracts are deliberate coordination schemes that could help to alleviate this inefficiency. Most relevant to this paper is the line of research in supply chain contracting, which deals with the channel inefficiency created by the conflicting interests between suppliers and buyers. An excellent survey on supply chain contracting can be found in Cachon (2001). The capacity reservation contracts discussed in this paper are considered using the general setting of supply chain contract with stochastic demand. Reservation contracts are conceptually similar to return policies and buy-back contracts. Pasternack(1985) proposes return policy for perishable commodities and derives the condition between the wholesale and the buy back prices which guarantee channel coordination. He shows that there exists an infinite set of coordinating contracts, characterized by the wholesale and buy back prices, each represents a different profit sharing split between the suppliers and the retailer. Marvel and Peck (1995) and Lau et al. (2000) study the case where the buyer decides the retail price but demand is price sensitive. Tsay and Lovejoy (1999) and Tsay (1999) model the incentives of the supplier and the retailers under Quantity Flexibility (QF) contract. The supplier promises to supply the product at a quantity up to $q(1+u)$ and the retailer promises to order at a quantity no less than $q(1-d)$. Thus, the contract is featured by (u, d, w) where w is the wholesale price and u, d are flexibility percentages. They show that various contract settings could lead to channel coordination when w is adjusted according to (d, u), which also results in different profit splitting. Lariviere (1999) makes comparison between buy back, QF, and a number of alternative contracts considering stochastic demands. Eppen and Iryer (1997) consider backup agreements in the context of the fashion industry.

A majority of the supply chain contracting literature focuses on the business setting in a retail environment. While many of the insights are directly relevant to the manufacturing context, there are distinct features that are unique in the high-tech capacity reservation environment: (1) as mentioned above, the purchasing (wholesale) price is negotiated separately as part of the long-term agreement between the manufacturer and the customer, not a contract parameter for capacity reservation, (2) due to the interchangeable nature of manufacturing capacity, the manufacturer may choose to expand at least part of her capacity regardless of the reservation status (this information turns out to be critical to the customer's reservation decision), (3) on the same token, reserved capacity unused by one customer could be utilized by another customer, this creates a dependency among competing customers that is not the case in typical supply contracts.

In the context of capacity reservation, the interaction among competing buyers could be important. Several researchers examine the retail supply chain setting with competing retailers, focusing on the characterization of equilibrium behavior (c.f., Bernstein and Federgruen (2000), Carr et al. (1999), and Van Mieghem and Dada (1999)). When the supplier's capacity level is not sufficient to satisfy all buyers' demands, the allocation rule used for available capacity could be critical.

Cachon and Lariviere (1999b) study different capacity allocation rules and their effect on the players' strategic behavior. In a related work, Cachon and Lariviere (1999a) examine three particular capacity allocation schemes: proportional, linear, and uniform. Their equilibrium analysis shows that the proportional and linear allocation schemes induce unpredictable behavior because the *Nash equilibrium* may not exist. However, under uniform allocation (dividing the capacity equally) there always exists a unique *Nash Equilibrium*, and the retailer would order the optimal quantity. Serel et al. (2001) consider a capacity reservation contract where the supplier guarantees to deliver any order amount desired by the buyer up to a reserved fixed capacity, in exchange, the buyer offers guaranteed payment. The wholesale price to be charged by the supplier is the primary contract parameter. Jain and Silver (1995) also consider capacity reservation decisions. They develop an algorithm to determine the optimal level of capacity reservation from the buyer's perspective. They do not consider the interaction between the manufacturer and the customer. Other forms of capacity coordination are also been proposed: for instance, (Lee, et al., 1998, Cachon and Fisher 1997) propose mechanisms for customer to share forecast data with manufacturer, which reduce the risk of capacity expansion. In high-tech manufacturing, it is common practice where the manufacturers share demand forecast throughout the supply chain.

3 Model Description and Analysis

We consider a game theoretic setting where the customer, who faces stochastic demand, desires to reserve future capacity before placing a firm order. The manufacturer must specify the customer's obligation (financial or otherwise) when making the reservation, and decide the level of capacity to make available to the customer(s). We consider the players' decisions in a single-period setting while the players hold symmetrical information about the demand distribution and the profit rates. This setting is consistent with the high-tech environment described earlier: (1)

a single-period decision model is appropriate since the manufacturer adjusts capacity levels using outsourcing and other means of soft expansion, (2) symmetrical demand information can be viewed as the result of joint forecasting and the fact that the customer (typically a downstream OEM) does not have significant advantage on market demand information, (3) symmetrical information on the profit rates is a somewhat weaker but reasonable assumption in that most players in this market are long-term partners with repeated dealings, the expected profit rates are public knowledge. However, the manufacturer and the customers do have distinctively different financial incentives, which potentially lead to double marginalization. We will start our analysis with a one-manufacturer, one-customer setting, then generalize the results to two and more customers. We assume that the manufacturer makes capacity expansion decisions at the beginning of the period. At the end of the period, the contract customers' demands will be met first; the remaining capacity, if any, will be available to the spot market customers at a lower rate. We summarize the notations to be used as follows:

r_0 : the manufacturer's profit rate when the capacity is sold to the contract customers

r_s : the manufacturer's profit rate when the capacity is sold to the spot market

r_c : the customers' profit rate; here we assume the same profit rate for all contract customers

$f_i(D_i)$: the probability distribution function characterizing contract customer i's demand, where $D_i \geqslant 0$ and $f_i(D_i) > 0$.

$F_i(D_i)$: the cumulative distribution function characterizing the contract customer i's demand

$f(D)$: p.d.f. for the combined customer demands D; which is the convolution of all contract customers' demand functions

$F(D)$: c.d.f. of the combined demands D from all contract customers

c_0 : manufacturer's initial capacity

c_e : the marginal cost for increasing each unit of manufacturing capacity

c : manufacturer's capacity after expansion

We assume that $r_0 > c_e > r_s$ since the reservation contract is implemented in the context of a partnership between the manufacturer and the contract customers. The profit rate of the manufacturer from the contract customers shall be no less than the capacity expansion cost, while the spot market sales are used as a means to absorb excess capacity, with a lower expectation on the profit rate. We assume that the contract customers' demands are independent and their combined demand function is a convolution of their individual demand functions. The independence assumption is realistic, for example, in the telecommunications infrastructure market we have studied. There, all main customers are dominant infrastructure builders from independent markets in North America, Europe, or Asia. We further assume that the manufacturer's initial capacity satisfies scarce capacity assumption, or $c_0 < F^{-1}(\frac{r_0 - c_e}{r_0 - r_s})$, otherwise there would be no need for capacity reservation. Moreover, we assume that unused capacity has zero salvage value for the customer who reserves the capacity.

To establish a performance benchmark, we first define the system's profit which includes the profits for the manufacturer and the contract customers, but not the spot market customers. The system's expected profit as a function of the available capacity is as follows:

$$\pi_s(c) = \int_0^c [(r_0 + r_c)D + r_s(c - D)]f(D)dD - (c - c_0)c_e + c\int_c^\infty (r_0 + r_c)f(D)dD \quad (1)$$

Note that this profit function has the form of a standard newsvendor model. The capacity level that maximizes $\pi_s(c)$ is thus

$$c_s^* = F^{-1}(\frac{r_0 + r_c - c_e}{r_0 + r_c - r_s}) \quad (2)$$

Without a coordination contract, the manufacturer's expected profit is as follows:

$$\pi_m(c) = \int_0^c [r_0 D + r_s(c - D)]f(D)dD - (c - c_0)c_e + r_0 c(1 - F(c)) \quad (3)$$

The capacity level that maximizes $\pi_m(c)$ is $c_m^* = F^{-1}(\frac{r_0 - c_e}{r_0 - r_s})$. Since $r_c > 0$, and $F^{-1}(.)$ is non-decreasing, we have the following Theorem.

Theorem 1 Without a coordination mechanism, the manufacturer will always expand her capacity less than the system optimal, i.e., $c_s^* > c_m^*$. The system will not reach channel coordination.

This result is straightforward and the inefficiency is due to *double marginalization*, since the manufacturer does not consider the profit margin of the customer when making capacity expansion decision. Given the capacity level c_m^* determined by the manufacturer, the expected profit for the customer can be expressed by the following:

$$\pi_c(c_m^*) = \int_0^{c_m^*} r_c D f(D) dD + c_m^* \int_{c_m^*}^{\infty} r_c f(D) dD \tag{4}$$

3.1 Deductible reservation contract in a one-manufacturer, one-customer system

We first propose a deductible reservation contract in a one-manufacturer, one-customer system. In this contract setting, the customer makes capacity reservation by paying a fee before the manufacturer expands her capacity. The reservation fee is deductible from the purchasing price when the customer later places a firm order. We will refer to this scheme as the *deductible reservation (DR) contract*. The event sequence for the *DR contract* is defined as follows:

1. The purchasing price w is agreed upon *ex ante* between the manufacturer and the customer,

2. The manufacturer announces (a) the per-unit reservation fee r, where $r < w$, and (b) excess capacity E, which is the amount of capacity the manufacturer prepares to expand in addition to (and regardless of) the reservation amount.

3. Based on the announced (r, E) pair, the customer decides the reservation amount R, and pays rR to the manufacturer.

4. Given the reservation amount R, the manufacturer expands the total capacity c to $E + R$. In the case where the initial capacity level is sufficient $c_0 \geq E + R$, no expansion will take place, i.e., $c = c_0$.

5. After the customer demand D is realized, the customer places an order D. When $D \leqslant c$, the customer receives the full ordered amount, otherwise the customer receives c. The payment from the customer to the manufacturer is specified as follows:

$$P = \begin{cases} (w-r)D & D \leqslant R \\ (w-r)R + w(D-R) & R < D \leqslant c \\ (w-r)R + w(c-R) & D > c \end{cases} \tag{5}$$

It should be clear that the capacity reservation only make sense when $r \leqslant w$. The customer's profit function under the contract can be expressed as follows:

$$\begin{aligned} \pi_c(R) \;\; = \;\; & \int_0^R [r_c D - r(R-D)] f(D) dD + \int_R^{R+E} r_c D f(D) dD \\ & + \int_{R+E}^{\infty} r_c (R+E) f(D) dD \end{aligned} \tag{6}$$

Lemma 2 There is a unique optimal reservation amount R for the customer under any reasonable policy (i.e., $r < w$ and $E < c_s^*$).

Proof: The first order derivative for the customer's profit function $\pi_c(R)$ is

$$\frac{d\pi_c(R)}{dR} = r_c - r_c F(R+E) - rF(R) \tag{7}$$

For any given $E < c_s^*$, $\frac{d\pi_c(R)}{dR} > 0$ when $R = 0$; $\frac{d\pi_c(R)}{dR} < 0$ when $R = \infty$; moreover, $\frac{d\pi_c(R)}{dR}$ is decreasing in R,and $\frac{d^2\pi_c(R)}{dR^2} < 0$. Thus, equation (7) has one and only one solution R^*, which maximizes the customer's profit. □

Under the *DR contract,* the manufacturer is the *Stackelberg* leader who sets the pair (r, E). In the following Lemma, we state the condition under which the reservation policy achieves channel coordination, which in turn specifies the relationship between r and E.

Lemma 3 To achieve channel coordination, the reservation policy offered by the manufacturer must satisfy the following condition:

$$r = \frac{r_c(1 - F(c_s^*))}{F(c_s^* - E)} \tag{8}$$

The Lemma would be straightforward to proof. First, we know that the manufacturer will announce an excess capacity amount E less than the newsvendor capacity, i.e., $E < c_s^* = F^{-1}(\frac{r_0+r_c-c_e}{r_0+r_c-r_s})$. Second, under channel coordination, the total capacity expansion would be $R + E = c_s^*$. Also, we know from (7) that for any given $E < c_s^*$, there is one and only one reservation fee r in $(0, \infty)$ that would result in channel coordination. The relationship (8)thus follows. The Lemma implies that if channel coordination is of interest, the more excess capacity E is to be built, the higher the reservation fee would be, i.e., r is increasing in E. The Lemma also implies that multiple (r, E) pairs (ones which satisfy (8)) could lead to channel coordination under the *DR contract*. One possible strategy is to use the reservation fee r to guarantee channel coordination while using E to influence the split of system profit generated by coordination. In practice, different $E's$ lead to different expansion strategies for the manufacturer. $E = 0$ represents *exact* capacity expansion, expand to the reservation amount with no excess; $E > 0$ represents *aggressive* expansion policy, allowing potentially higher gains in an upside market; $E < 0$ represents *overbooking*. However, credibility with the contract customer is crucial for the manufacturer in the high-tech industry, and *overbooking* is not considered an acceptable business practice. Thus, we will only consider the cases where $E \geqslant 0$.

Under the *DR contract*, the manufacturer's profit can be expressed as follows:

$$\begin{aligned}\pi_m(r, E) \;&=\; \int_0^{R+E} [r_0 D + r_s(R + E - D)] f(D) dD + r \int_0^R (R - D) f(D) dD \\ &\quad -(R + E - c_0)c_e + r_0(R + E)[1 - F(R + E)] \qquad (9)\end{aligned}$$

Under channel coordination the manufacturer may charge a reservation fee r defined by (8) in Lemma 3, we can rewrite the manufacturer profit function in terms of E as follows:

$$\begin{aligned}\pi_m(E) \;&=\; \int_0^{c_s^*} [r_0 D + r_s(c_s^* - D)] f(D) dD + \frac{r_c(1 - F(c_s^*))}{F(c_s^* - E)} \int_0^{c_s^* - E} F(D) dD \qquad (10) \\ &\quad -(c_s^* - c_0)c_e + r_0 c_s^* [\frac{c_e - r_s}{r_0 + r_c - r_s}]\end{aligned}$$

Observe that the decision on excess capacity E only influences the second term of the profit function. The optimal value of E can be thus determined by the first order condition expressed as follows:

$$\frac{d\pi_m(E)}{dE} = r_c(1 - F(c_s^*))(\frac{f(c_s^* - E)\int_0^{c_s^*-E} F(D)dD}{F^2(c_s^* - E)} - 1) \tag{11}$$

Thus, the optimal value of E depends on the demand distribution. While the value of E will not affect the channel coordination status, it does affect the manufacturer's profit.

We will now determine if the *DR Contract* is individually rational for the players involved, i.e., are the players better off with capacity reservation. We first consider the *exact capacity expansion case* ($E = 0$), then discuss the *aggressive expansion case* ($E > 0$). Under the exact expansion policy, to reach channel coordination the manufacturer should set the reservation price as follows (from (2) and Lemma 3):

$$r = \frac{r_c(c_e - r_s)}{r_0 + r_c - c_c} \tag{12}$$

The following Lemma considers incentive compatibility from the manufacturer's perspective.

Lemma 4 Under the exact capacity expansion policy ($E = 0$), the manufacturer gains more profit by entering a channel-coordinated reservation contract than not accepting reservation.

The formal proof is given in the Appendix. The Lemma makes clear the incentive for the manufacturer to enter the reservation contract and to achieve channel coordination. An interesting observation is that when $E = 0$ the manufacturer does not need to know the demand distribution function when she implements the *DR contract* . It should be clear from (10) and (11) that if the manufacturer is free to adjust the value of E (using the *aggressive expansion policy*), she stands to gain more under some demand distribution functions. We now consider the customer's incentive in the following Lemma.

Lemma 5 When $E \rightarrow c_s^*$, the customer gains more profit by entering a channel-coordinated reservation contract than not making reservation.

Proof: Denote the customer's expected profit with and without reservation as π_c^1 and π_c^0, respectively. Thus, we have the following profit functions:

$$\pi_c^0 = r_c c_m^* - r_c \int_0^{c_m^*} F(D)dD \tag{13}$$

$$\pi_c^1(E) = r_c c_s^* - r_c \int_0^{c_s^*} F(D)dD - r \int_0^{c_s^*-E} F(D)dD \tag{14}$$

Under channel coordination, the reservation fee $r = \frac{r_c(1-F(c_s^*))}{F(c_s^*-E)}$, thus the difference in customer's expected profit is as follows:

$$\begin{aligned}\Delta(E) &= \pi_c^1(E) - \pi_c^0 \quad (15)\\ &\geqslant r_c \int_{c_m^*}^{c_s^*} (1-F(D))dD - rF(c_s^*-E)(c_s^*-E) \\ &= r_c \int_{c_m^*}^{c_s^*} (1-F(D))dD - r_c(1-F(c_s^*))(c_s^*-E)\end{aligned}$$

since $c_s^* > c_m^*$, $\Delta(E) > r_c(1-F(c_s^*))(c_s^*-c_m^*) > 0$ when $E \to c_s^*$ $\square$

>From the above two Lemmas, we will state the following theorem concerning individual rationality.

Theorem 6 In the one-manufacturer, one-customer system, there is always a channel-coordinated DR contract that is also individually rational for both players.

Proof: From Lemma 3 we know that there are one or more (r, E) pairs exist that lead to channel coordination, but for a given value of $E < c_s^*$, there exist one and only one r that would lead to channel coordination. In the special case where the manufacturer chooses the *exact capacity expansion policy* ($E = 0$) while the customer's expected profit $\pi_c^1(0) > \pi_c^0$, the theorem follows directly from Lemmas 4 and 5. In general, the manufacturer may choose an excess capacity amount $E \in (0, c_s^*)$ for such that the customer's profit $\pi_c^1(E)$ defined in (14) satisfies

$$\pi_c^1(E) = \pi_c^0 + \min\{r_c(1-F(c_s^*))(c_s^*-c_m^*), \frac{\pi_s(c_s^*) - \pi_s(c_m^*)}{2}\} \tag{16}$$

From (15) in Lemma 5 and continuity of the customer profit function $\pi^1_c(E)$ we know that the manufacturer can always find an E that satisfies (16). This contract is individually rational for the manufacturer since she receives extra profits no less than the amount $\frac{\pi_s(c_s^*)-\pi_s(c_m^*)}{2}$, comparing to the case without reservation (i.e., the customer receives no more than $\frac{\pi_s(c_s^*)-\pi_s(c_m^*)}{2}$ according to (16)). On the other hand, since the second terms in (16) is non-negative, entering the contract is also individually rational for the customer. □

Essentially, in order for the *DR contract* to satisfy individual rationality for both parties, the manufacturer must adjust the excess capacity value E such that system profit generated from channel coordination (i.e., $\pi_s(c_s^*) - \pi_s(c_m^*)$) can be properly shared with the customer. The above analysis is based on the defined event sequence for the *DR contract.* We now consider the basic questions of whether the manufacturer has the incentive to commit to the true value of E *ex ante,* or to announce the value of E at all. It is not difficult to verify that announcing the true value of E is the best strategy for the manufacturer: firstly, the manufacturer has no incentive to announce a false E'. If the announced E' is more than the real E, the exaggerated amount $E' - E$ expected by the customer would cause her to reserve less, which would degrade the manufacturer's profit. On the other hand, if the manufacturer announces an E' that is less than the true E, the manufacturer would be forced to charge a lower reservation price r (recall that r is increasing in E as defined in (8)), which again degrades the manufacturer's profit. Besides, as the customer is not aware of the amount $E - E'$, the manufacturer is more likely to use this part of the capacity to satisfy the spot market customers, which further degrades her profit since we assume $c_e > r_s$. Next, we consider the option where the manufacturer does not announce E at all before the customer places her reservation. In other words, the manufacturer decides on E *after* the customer places the reservation with amount R. This would replace the sequence of events 2-3 in the *DR contract* to as follows:

1. The manufacturer announces the reservation price r,

2. The customer places reservation with quantity R, and
3. The manufacturer determines the excess capacity level E, then expands the capacity c to $R+E$.

The manufacturer's profit function in Stage 3 is the same as in (9) with fixed r and R, while E is the only decision variable. Since the second term is not a function of E, the optimization problem is the same as (3) with the additional constraint: $c \geqslant R$. The function (9) is concave in E, so the best strategy for the manufacturer in Stage 3 is as follows:

$$E = \begin{cases} c_m^* - R & \text{when } R \leqslant c_m^* \\ 0 & \text{when } R > c_m^* \end{cases} \tag{17}$$

In anticipation of the manufacturer's best strategy, the customer knows (in Stage 2) that if her reservation amount R is no more than c_m^*, there is no need to make reservation at all since the manufacturer would expand the capacity to $E = c_m^*$ anyway. In this case, the customer's profit is determined by (4). On the other hand, if her reservation amount R is more than c_m^*, the customer's profit maximizing problem is characterized by (6), but with $E = 0$. Thus, her optimal reservation amount can be computed as $R^*(E = 0) = F^{-1}(\frac{r_c}{r_c+r})$, and it must satisfies the condition $F^{-1}(\frac{r_c}{r_c+r}) > c_m^*$. However, the customer still does not have incentive to make reservation unless her optimal profit $\pi_c(R^*|E = 0)$ with reservation (6) is larger than her profit π_c^0 without reservation (4). In summary, the customer's best strategy in Stage 2 is as follows:

$$R = \begin{cases} F^{-1}(\frac{r_c}{r_c+r}) & \text{when } F^{-1}(\frac{r_c}{r_c+r}) > c_m^* \text{ and } \pi_c(R^*|E=0) > \pi_c^0 \\ 0 & \text{Otherwise} \end{cases}$$

Recall that $c_m^* = F^{-1}(\frac{r_0-c_e}{r_0-r_s})$, we know that if $\frac{r_c}{r_c+r} \leq \frac{r_0-c_e}{r_0-r_s}$ or $\pi_c(\frac{r_c}{r_c+r}) \leq \pi_c^0$, the customer will not make reservation unless the manufacturer announces E ex ante. On the other hand, if the conditions $\frac{r_c}{r_c+r} > \frac{r_0-c_e}{r_0-r_s}$ and $\pi_c(\frac{r_c}{r_c+r}) > \pi_c^0$ are satisfied, the customer will make reservation, but the case is equivalent to a special case of the *DR contract* where the manufacturer announces the (r, E) pair but always set $E = 0$. Thus, the manufacturer will be the same or better off under the general *DR contract* (with $E \geq 0$) since she has the freedom to choosing r for *any* $E \in [0, c_s^*)$.

Recall that the value of E alters profit sharing between the two players.

Intuitively, the *DR contract* reduces the manufacturer's capacity expansion risk, while preserving flexibility in the reservation fee and committed expansion amount. When the customer demand is lower than expected, the manufacturer receives compensation from undeducted reservation payment, which reduces the variance of the manufacturer's profit. The customer benefits from increased revenue because, with reservation, the manufacturer is willing to expand her capacity more aggressively. This is critical particularly in a competitive upside market.

3.2 Deductible Reservation Contract in a One-Manufacturer, Two-Customer System

We now consider the one-manufacturer, two-customer system. Without loss of generality, we will assume that the two customers have the same marginal profit, but each face a different demand distribution in two independent markets. A main issue in the two-customer system is that when the realized customer demands are more than the capacity available, an *ex ante* capacity allocation rule needs to be specified. We will use the *uniform capacity allocation* rule (i.e., dividing the available capacity equally) due to Cachon and Laraviere (1999a). Although "proportional" capacity allocation (i.e., dividing the available capacity proportionally to the announced demand, or the reservation amount) is more common in practice, the customers' behavior would be unpredictable as there is incentive to manipulate the demand information. In fact, in the latter case there is no *Nash equilibrium* exists for the customers' response (in the forms of announced demand, or reservation amount). In the following, we define the uniform capacity allocation rule more precisely. The allocated capacity B_1 for Customer 1 is as follows:

$$B_1 = \begin{cases} D_1 & D_1 \leqslant R_1 \\ D_1 & D_1 + D_2 \leqslant c \\ c - D_2 & D_1 + D_2 > c, D_2 \leqslant R_{2,} D_1 > R_1 \\ R_1 + \frac{c - R_1 - R_2}{2} & D_1 + D_2 > c, D_2 > R_{2,} D_1 > R_1 \end{cases} \tag{18}$$

The allocated capacity for Customer 2 is symmetrical to B_1. As shown by Cachon and Laraviere

(1999a), uniform capacity allocation induces each customer to reveal her true demand when it is realized. The manufacturer allocates capacity c to the customers based on (18) and the payment to the manufacturer is realized based on (5) defined earlier in the *DR contract*. In the two-customer setting, the *DR contract* defines two separate games: a *Nash Game* between the two customers, and a *Stackelberg Game* between the manufacturer and the customers. We first need to determine if there is a unique equilibrium point in the *Nash Game* between the two customers. If there is, the manufacturer will be able to compute the equilibrium point and makes decision on the reservation policy (r, E) as a *Stackelberg Leader* similar to the one-customer case.

Again, we assume the excess capacity $E \geqslant 0$. We may express the expected profit of Customer 1 as follows.

$$\begin{aligned}
\pi_{c1}(R_1, R_2) &= \int_0^{R_1} [r_c D_1 - r(R_1 - D_1)] f_1(D_1) dD_1 \\
&\quad + r_c \int_{R_1}^{R_1+R_2+E} r_c D_1 F_2(R_1 + R_2 + E - D_1) f_1(D_1) dD_1 \\
&\quad + \int_0^{R_2+E/2} r_c (R_1 + R_2 + E - D_2)[1 - F_1(R_1 + R_2 + E - D_2)] f_2(D_2) dD_2 \\
&\quad + \int_{R_1}^{R_1+E/2} r_c D_1 [1 - F_2(R_1 + R_2 + E - D_2)] f_1(D_1) dD_1 \\
&\quad + r_c [1 - F_1(R_1 + E/2)][1 - F_2(R_2 + E/2)](R_1 + E/2)
\end{aligned} \tag{19}$$

When Customer 1 makes decision on the reservation amount, she needs to consider the reservation amount of Customer 2, in addition to the reservation price r and excess capacity E. The first-order derivative of π_{c1} on R_1 is as follows:

$$\begin{aligned}
\frac{d\pi_{c1}}{dR_1} &= r_c - r_c F_1(R_1 + E/2) - r F_1(R_1) \\
&\quad - \int_0^{R_2+E/2} r_c F_2(D_1) f_1(R_1 + R_2 + E - D_1) dD_1
\end{aligned} \tag{20}$$

Lemma 7 For any given R_2, there exists one and only one optimal reservation amount R_1^* for Customer 1, given reservation policy (r, E) and demand distributions $f_1(D_1)$ and $f_2(D_2)$.

When we rewrite (20) into

$$\frac{d\pi_{c1}}{dR_1} = r_c + r_c F_1(R_1 + E/2)[F_2(R_2 + E/2) - 1] - rF_1(R_1) - \int_0^{R2+E/2} r_c F_1(R_1 + R_2 + E - D_2) f_2(D_2) dD_2 \quad (21)$$

Since $F_2(R_2 + E/2) - 1 < 0$ and the fact that $F_1(x)$ is strictly increasing, $\frac{d\pi_{c1}}{dR_1}$ is decreasing in R_1. In addition, when $R_1 = 0$, $\frac{d\pi_{c1}}{dR_1} \geqslant r_c - r_c F_1(R_1 + E/2) \geqslant 0$; when $R_1 = \infty$, $\frac{d\pi_{c1}}{dR_1} = -r$. Thus, there is one and only one best response R_1^* for Customer 1 given contract (r, E) and reservation amount R_2. By examining the relationship between R_1^* and R_2, we get the following Lemma:

Lemma 8 Customer 1's best response reservation amount R_1^* is decreasing in R_2 and satisfies the following relationship under any given reservation policy (r, E) and any value of R_2.

$$-1 < \frac{dR_1^*}{dR_2} < 0 \quad (22)$$

The proof is given in the Appendix. With Lemmas 7 and 8, we may now state the following Theorem.

Theorem 9 Under any DR contract (r, E), there is one unique Nash Equilibrium point (R_1^*, R_2^*) in the reservation game between the two customers.

Because of the symmetry, we also have $-1 < \frac{dR_2^*}{dR_1} < 0$. It is easy to see that $R_1^* = 0$ when $R_2^* = \infty$,and vice versa. Thus, the best response functions in the two player's reservation amount R_1 and R_2 are convex with a unique equilibrium point (R_1^*, R_2^*). Specifically, the *Nash equilibrium* is the solution to the following equations.

$$\begin{cases} r_c - r_c F_1(R_1 + E/2) - rF_1(R_1) - \int_0^{R_2+E/2} r_c F_2(D_1) f_1(R_1 + R_2 + E - D_1) dD_1 = 0 \\ r_c - r_c F_2(R_2 + E/2) - rF_2(R_2) - \int_0^{R_1+E/2} r_c F_1(D_2) f_2(R_1 + R_2 + E - D_2) dD_2 = 0 \end{cases} \quad (23)$$

Theorem 9 is critical for the analysis of the two-customer system, as the existence of unique Nash equilibrium allows the manufacturer to anticipate the result of the two-customer Nash game given a particular (r, E) pair. We can also determine the effect of reservation price r on the expanded capacity c, as stated in the following theorem.

Theorem 10 For a given E, the expanded capacity c is decreasing in the reservation price r, and there is a unique reservation price r which leads to channel coordination.

The Theorem follows intuitively from the convexity of the best response function. From (23), for a given E and R_2 Customer 1's best response R_1^* is decreasing in r (again, this is symmetrical to Customer 2's best response). Suppose the players' best response functions correspond to a particular value of r, as r increases, the sum $R_1 + R_2$ for any new equilibrium point would be no greater than the sum at the original point (since $-1 < \frac{dR_2^*}{dR_1} < 0$ and $-1 < \frac{dR_1^*}{dR_2} < 0$). Thus, we may conclude that the final capacity $c = R_1 + R_2 + E$ is decreasing in r. Further, for $r \to 0$, R_1, $R_2 \to \infty$, and for $r \to \infty$, $R_1 = R_2 = 0$. Since $E \leqslant c_s^*$, we may conclude that there is one and only one reservation price r that leads to channel coordination. As in the one-customer case, the manufacturer can adjust the reservation price r to reach channel coordination, while adjusting the excess capacity E to control profit sharing.

In the following, we will generalize the above results for systems with more than two customers. We first state the Lemma:

Lemma 11 For a given r, the total reservation amount $(R_1^* + R_2^*)$ and excess capacity E have the relationship $-1 < \frac{d(R_1^*+R_2^*)}{dE} < 0$

The proof is given in the Appendix. Now consider the three-customer case. Suppose the reservation amounts of the three customers are R_1, R_2 and R_3, respectively. Since the manufacturer could choose the excess capacity E, suppose she set $E = R_3$. Thus, for a given R_3, there is a unique equilibrium point (R_1^*, R_2^*) for the game between Customers 1 and 2, and $-1 < \frac{d(R_1^*+R_2^*)}{dR_3} < 0$

due to Lemma 11. Thus, for each given R_3, we may consider R_2^* a dependent variable on R_1^* as $R_2^* = g(R_1^*)$, where g is an increasing function. Using function g, we can find the pair R_1 an R_2 $(= g(R_1))$, a unique best response for Customer 3, and $-1 < \frac{dR_3^*}{d(R_1+g(R_1))} < 0$. Thus, there is a unique *Nash equilibrium* point for three-player game. In general, we may prove the $n-$customer case using the result from the $n-1$ customer case, and so on. The total reservation amount is still decreasing in r, so for any $E \in [0, c_s^*]$, there is a unique reservation price r that would lead to channel coordination.

4 The Take-or-Pay Contract

We have shown that the proposed *DR contracts* could lead to channel coordination, which increases the expected profits for the manufacturer and the customers. We now examine a capacity reservation contract commonly used in the high-tech industry known as the *take-or-pay contract*. In a one-manufacturer, one-customer setting, the contract uses the following sequence of events:

1. The manufacturer announces a triplet (p, E, a) where p is a non-zero penalty rate, E is the excess capacity, and a is a fractional number between $(0, 1]$.
2. The customer reserves capacity R but makes no payment to the manufacturer up front.
3. The demand D is realized, the customer places an order equal to this amount. She receives capacity equal to the amount $min(D, E+R)$, and makes the payment according to the following scheme:

$$P = \begin{cases} w \cdot min(D, E+R) & \text{if } D \geqslant aR \\ wD + p(aR - D) & \text{if } D < aR \end{cases} \tag{24}$$

Again, we assume $p < w$. In fact, the *take-or-pay contract* can be used to implement the *DR contract:* the *DR contract* with (r, E) where $E \geq 0$ can be implemented as a *take-or-pay contract* (p, E', a), where $a = 1$, $E' = E$ and $p = r$. However, there are main differences between to the two contracts. An obvious difference is that of cash flow: *take-or-pay contract* requires no up front payment from the customer, which makes enforcing the contract a potential difficulty. Another main difference is that of incentive compatibility, which we will discuss later in the section. In the following, we first explore main properties of the *take-or-pay* contract.

Let $r = p$, the customer's decision problem can be expressed as follows:

$$\begin{aligned}\pi_c'(R) &= r_c \int_0^{R+E} r_c D f(D) dD - p \int_0^{aR} (aR - D)] f(D) dD \\ &+ r_c \int_{R+E}^{\infty} r_c (R+E) f(D) dD \end{aligned} \quad (25)$$

Its first-order derivative of the profit function is $\frac{d\pi_c'(R)}{dR} = r_c - r_c F(R+E) - apF(aR)$.

Theorem 12 For any triplet (p, E, a) proposed by the manufacturer, there is a unique reservation amount R for the customer that achieves channel coordination.

The Theorem follows directly from the first order condition of $\pi_c'(R)$ as $F(.)$ is a nondecreasing function. Moreover, from the manufacturer's perspective, the channel coordinated solution satisfies $R + E = c_s^*$, thus, she must choose (p, E, a) that satisfies the equation:

$$r_c - r_c F(c_s^*) - apF(a(c_s^* - E)) = 0. \quad (26)$$

Observe that for any $E \in [0, c_s^*)$ and $a \in (0, 1]$, there is a unique p which leads to channel coordination, and $p = \frac{r_c - r_c F(c_s^*)}{aF(a(c_s^* - E))}$. Note that p increases when a decreases, or when E increases.

We now explore the relationship between the *take-or-pay* and the *DR Contract.* First, the manufacturer's profit function is as follows:

$$\begin{aligned}\pi_m(p, E, a) &= \int_0^{R+E} [r_0 D + r_s (R + E - D)] f(D) dD + p \int_0^{aR} (R - D) f(D) dD \\ &-(R + E - c_0) c_e + r_0 (R + E)[1 - F(R + E)] \end{aligned} \quad (27)$$

Lemma 13 To achieve channel coordination under the take-or-pay contract, the best strategy for the manufacturer is to set the excess capacity $E = 0$.

Proof: To achieve channel coordination, the manufacturer must choose (p, E, a) which satisfies the condition specified in (26) or $apF(a(c_s^* - E)) = r_c - r_c F(c_s^*)$ (note that the right-hand-side is a constant). For any given penalty rate p, equation (26) defines the relationship between a and

E. Specifically, a increases in $E \in [0, c_s^*)$. Further, in a channel coordinated solution $R = c_s^* - E$, thus the manufacturer's profit is only influenced by the second term of (27), or $p \int_0^{a(c_s^*-E)} (R - D) f(D) dD$. Since a increases in E (26) while $a(c_s^* - E)$ decreases in E, we know that the manufacturer's profit decreases in E. Since we do not allow negative E (overbooking), therefore the manufacturer's best strategy is to set $E = 0$. $\square$

It follows from the Lemma that under the *take-or-pay contract,* the manufacturer's decision degenerates from the triplet (p, E, a) to a pair (p, a). In the following, we state the effect of the above relationship from the manufacturer's perspective.

Theorem 14 Under channel coordination, the manufacturer is no worse off under the take-or-pay contract (p, a) than under the DR contract (r, E).

Proof: As noted before, the *DR contract* with (r, E) where $E = 0$ can be implemented as the *take-or-pay contract* (p, a) where $a = 1$ and $p = r$. However, the channel coordinated (p^*, a^*) yields the same or more profit for the manufacturer. *DR contract* with (r, E) where $E > 0$ can be implemented as the *take-or-pay contract* (p, a, E'), where $a = 1$, $E' = E$ and $p = r$. However, as shown in the proof of Lemma 13, we can find another channel coordinated policy $(p\prime, a\prime)$ where $p\prime = p$ and $a\prime < 1$ that generates more profit for the manufacturer. Thus, for the manufacturer we can find a (p^*, a^*) under *take-or-pay* that is the same or better than (r^*, E^*) under *DR*. $\square$

>From the above analysis, we know that the $(r, 0)$ *DR contract* could be implemented as $(p, 1)$ *take-or-pay contract,* while achieving channel coordination. Further, it is possible to improve the manufacturer's profit by adjusting (raising) the threshold level a. However, for the customer, given a channel coordinated pair (p, a) and the demand distribution, there is a unique reservation amount R (Theorem 12). The customer would have no other means to adjusting the contract parameters. Unlike the *DR contract (*Lemma 5 and Theorem 6), it is possible that no individually rational *and*

channel coordinated (p, a) pair exist for a *take-or-pay* contract, i.e., while the manufacturer has the incentive to take reservation, the customer may not have the incentive to make one.

A follow up question is whether the *take-or-pay* contract works for one-manufacturer, two-customer system (i.e., does the contract lead to channel coordination). Consider the response functions of Customers 1 and 2 as follows:

$$r_c - r_c F_1(R_1 + E/2) - apF_1(aR_1) - \int_0^{R_2} r_c F_2(D_1) f_1(R_1 + R_2 + E - D_1) dD_1 = 0$$

$$r_c - r_c F_2(R_2 + E/2) - apF_2(aR_2) - \int_0^{R_1} r_c F_1(D_2) f_2(R_1 + R_2 + E - D_2) dD_2 = 0 \qquad (28)$$

Observe that we still have the property $-1 < \frac{dR_2^*}{dR_1} < 0$ and $-1 < \frac{dR_1^*}{dR_2} < 0$ needed to prove the existence of a unique Nash equilibrium, and for any threshold value a, we can find the penalty rate p to reach channel coordination. We state this result in the following theorems.

Theorem 15 Under any take-or-pay contract (p, a), there is one unique Nash Equilibrium point (R_1^*, R_2^*) in the reservation game between the two customers.

Similar to the case under *DR contract*, the Theorem indicates that manufacturer can predict the result of the two-customer game based on its unique *Nash Equilibrium*.

Theorem 16 For a given a, the expanded capacity c is decreasing in the penalty rate p, and there is a unique penalty p which leads to channel coordination.

The proof for the above Theorems follow the same logic as in the *DR contract*.

5 Discussion and Conclusions

In this paper, we investigate capacity reservation in high-tech manufacturing where the manufacturer shares the risks of capacity expansion with her main customers. We first show that the system without reservation may result in lost revenue (when compared to the channel coordinated system) due to insufficient capacity. We propose the *deductible reservation contract* (r, E) which are individually rational while providing channel coordination. We start with a one-manufacturer, one-customer system where the contract follows the form of a *Stackelberg Game* with the manufacturer as the leader. We assume the manufacturer and the customer make their decisions based on a single-period newsvendor type model with known demand distribution, while the players hold complete information concerning each other's profit rates. A unique feature of the *DR* contract is that the manufacturer announces *ex ante* the "excess" capacity (E) she is going to expand in addition to (and regardless of) the customer reservation amount. We show that (Lemma 3) under channel coordination the reservation fee r is increasing in excess capacity E, and that multiple (r, E) pairs could achieve coordination. We suggest a strategy where the manufacturer sets r to guarantee channel coordination, while controlling E to share the benefit of coordination with the customer (i.e., to provide incentives for reservation). We show that it is always possible to design a channel-coordinated *DR contract* that is also individually rational (Theorem 6). This property is important since while reservation has clear economic benefit for the manufacturer, the benefit for the customer is less obvious. We show that in order for the manufacturer to attract the customer to place reservation, she must share a portion of the extra profit generated by coordination. Further, we examine the (common sense) alternative where the manufacturer does not announce the excess capacity in advance. We show that this is in fact equivalent to a $(r, 0)$ contract, thus it follows that the manufacturer could potentially improve her profits by announcing E *ex ante.* Moreover, it is to the manufacturer's best interest to commit to the announced quantity faithfully.

We extend the above analysis to the cases with two or more customers, assuming the customers are from separate and independent markets (e.g., telecommunications infrastructure builders). We model the (capacity) competition among customers as a *Nash Game,* while the interaction between the manufacturer and the customers remains a *Stackelberg Game*. Since there is a unique *Nash Equilibrium* for the competition among customers (Theorem 9), the manufacturer will be able to anticipate its outcome and choose a proper (r,E) pair to achieve channel coordination. Thus, the main results from the one-customer case can be generalized to *n* customers.

To establish practical insights we compare the *DR contract* with a contract known in the high-tech industry as the *take-or-pay* contract. In a *take-or-pay contract*, the customer agrees to a threshold $a \in (0, 1]$ in addition to the reservation amount R such that when she orders more than aR (*take*) there is no penalty, otherwise she *pays* penalty p for each unit short of aR. In fact, the *DR contract* can be implemented as a *take-or-pay contract* (p, a, E) with $a = 1$. We show that with the introduction of the threshold value a, the best strategy for the manufacturer (under channel coordination) is to expand her capacity exactly (set $E = 0$), and this is true regardless of the penalty rate. Thus E is dropped out of consideration from the *take-or-pay contract*. We show that the manufacturer is no worse off under the *take-or-pay contract* than under the *DR contract*. A critical difference between the two contracts is that under *take-or-pay* there may *not* exist a channel coordinated contract that is also individually rational. Specifically, there may not exist a *take-or-pay contract* that would guarantee to benefit the *customer*. Thus, in these cases the customer may have no incentive to place reservation in the first place. On the other hand, the *take-or-pay contract* does have its practical appeal due to the cash flow: the *DR contract* requires the customers to pay a fee upon reservation, but the *take-or-pay contract* does not require payment until the actual demand is realized. The cash flow implications could be significant when there is a long lead time between reservation and demand realization, which is not uncommon in high-tech industries. Nevertheless, the *take-or-pay* penalty can be difficult to enforce when the market demand sharply decrease as is the case throughout year 2001.

As mentioned earlier, both *DR* and *take-or-pay* contracts are conceptually similar to the *buy-back* contracts known in the supply contract literature (c.f., Donohue, 2000; Pasternack, 1985). For instance, when demand is *less* than the reservation amount R, the customer's cost would be the same in all three contracts. However, as pointed out in Section 2, since much of the supply contract literature are motivated by the retail (rather than manufacturing) settings, there are several main differences between these contracts. First, in the *buy back* contract literature, the wholesale price is a decision variable to be used in combination with the buy back price to achieve channel coordination. In our study, we assume the wholesale price is exogenous since it is seldom the subject of negotiation for capacity reservation. Second, the buy-back contract setting does not allow the customer to receive more than her ordered amount when the demand is *more* than the original order, while the customer can clearly get above and beyond her reserved amount under the reservation contract when $E > 0$, or when there are multiple customers. Third, for *buy-back* contract with multiple customers, a customer does not need to consider other customers' orders when the markets are independent, i.e., when one customer's order is not sufficient to cover demand, she does not have the option of utilizing the excess from others. Under the reservation contracts, the manufacturer has the flexibility to utilize unused reservation capacity from one customer to cover other customers' demands. Consequently, competing customers *are* dependent by the available capacity and they essentially play a Nash game before reservation.

Another popular contract under stochastic demand is the *quantity flexibility* (QF) contract (c.f., Anupindi and Bassok, 1995; Tsay, 1999). A typical QF contract allows the customer to increase order quantity at some fixed percentage but forces the customer to commit to some percentage of initial order even if the demand is less than this value. Similar to the *buy back* contract, there is no interaction between the customers under the QF contract. When the demand of a customer is larger than her pre-specified QF percentage, she cannot get more under the contract even if other customers may have demand shortfall. This inflexibility could potentially degrade the system performance as in the buy-back contract. QF also allows the manufacturer to adjust the wholesale price for the interest of reaching channel coordination. Neither contract would apply directly to the capacity reservation setting.

This paper assumes a single-period decision model for the manufacturer. When the capacity expansion decision is considered explicitly through the physical expansion of manufacturing facilities, or procurement of capital equipment, it may be necessary to consider multiple-period capacity expansion models. This would lead to significantly more difficult contract analysis and some potentially interesting future research topics. Another useful extension is the case under asymmetric cost information. Without knowing the profit rates of the customers and certain parameters of the demand distribution, can the manufacturer still design a reservation contract to achieve channel coordination while remaining individually rational. In conclusion, we believe capacity reservation in the manufacturing setting suggests fruitful research topics which expand the scope of the supply contracting literature.

Appendix

To prove Lemma 4

Under the exact expansion policy ($E = 0$), the manufacturer's profit is as follows:

$$\begin{aligned} \pi_m^{'}(0) &= \int_0^{c_s^*} [r_0 D + r_s(c - D)] f(D) dD \\ &\quad + \int_0^{c_s^*} rF(D)dD - (c_s^* - c_0)c_e + r_0 c_s^*[1 - F(c^*)] \\ \text{and from (12) } r &= \frac{r_c(c_e - r_s)}{r_0 + r_c - c_e} \end{aligned}$$

Suppose

$$G(b) = (r_0 - c_e)b - (r_0 - r_s)\int_0^b F(D)dD + \frac{r_c(c_e - r_s)}{r_0 + r_c - c_e}\int_0^b F(D)dD$$

Then we can rewrite $\pi_m^{'}(0) = G(c_s^*) + c_0 c_e$

We can get

$$\begin{aligned} \frac{dG(b)}{db} &= r_0 - c_e - (r_0 - r_s)F(b) + \frac{r_c(c_e - r_s)}{r_0 + r_c - c_e}F(b) \\ &= r_0 - c_e - \frac{(r_0 - c_e)(r_0 + r_c - r_s)}{r_0 + r_c - c_e}F(b) \end{aligned}$$

When $b = c_s^*$, $\frac{dG(b)}{db} = 0$, and $\frac{dG(b)}{db}$ is decreasing in b. As $c_m^* < c_s^*$, we know that $G(c_s^*) > G(c_m^*)$.

Then we get

$$\begin{aligned}
\pi_m^{'}(0) &> G(c_m^*) + c_0 c_e \\
&= (r_0 - c_e)c_m^* - (r_0 - r_s)\int_0^{c_m^*} F(D)dD + \frac{r_c(c_e - r_s)}{r_0 + r_c - c_e}\int_0^{c_m^*} rF(D)dD \\
&> (r_0 - c_e)c_m^* - (r_0 - r_s)\int_0^{c_m^*} F(D)dD + c_0 c_e \\
&= \pi_m(c_m^*)
\end{aligned}$$

So, the manufacturer gains more profit by entering the reservation contract. □

To prove Lemma 8

>From (20), we observe that $\int_0^{R_2+E/2} r_c F_2(D_1) f_1(R_1+R_2+E-D_1)dD$ is strictly increasing in R_2, because $F_2(D_2)$ is a strictly increasing function and $f_1(D_1) > 0$. So $\frac{d\pi_{c1}}{dR_1}$ is strictly decreasing in R_2. Knowing the fact that $\frac{d\pi_{c1}}{dR_1}$ is strictly decreasing in R_1 and $\frac{d\pi_{c1}}{dR_1}(R_1^*) = 0$, we can get $\frac{dR_1^*}{dR_2} < 0$. For a specific R_2^0, we assume R_1^* is the optimal reservation amount for Customer 1. We assume R_1' is the optimal reservation amount for Customer 1 when Customer 2 orders $R_2^0 + \Delta$, here Δ can be any small positive real number. Suppose $g(R_1, R_2) = \frac{d\pi_{c1}}{dR_1}$, we have $g(R_1^*, R_2^0) = 0$ and

$$
\begin{aligned}
g(R_1^* - \Delta, R_2^0 + \Delta) &= r_c - r_c F_1(R_1^* - \Delta + E/2) - r f\imath_1(R_1^* - \Delta) \\
&\quad - \int_0^{R_2^0 + \Delta + E/2} r_c F_2(D_1) f_1(R_1^* + R_2^0 + E - D_1) dD_1 \\
&> r_c - r_c F_1(R_1^* - \Delta + E/2) - r F_1(R_1^*) \\
&\quad - \int_{R_1^* - \Delta + E/2}^{R_1^* + E/2} r_c F_2(R_1^* + R_2^0 + E - D_1) f_1(D_1) dD_1 \\
&\quad - \int_{R_1^* + E/2}^{R_1^* + R_2^0 + E} r_c F_2(R_1^* + R_2^0 + E - D_1) f_1(D_1) dD_1 \\
&= g(R_1^*, R_2^0) + \int_{R_1^* - \Delta + E/2}^{R_1^* + E/2} r_c f_1(D_1) dD_1 \\
&\quad - \int_{R_1^* - \Delta + E/2}^{R_1^* + E/2} r_c F_2(R_1^* + R_2^0 + E - D_1) f_1(D_1) dD_1 \\
&> 0
\end{aligned}
$$

Since $g(R_1', R_2^0 + \Delta) = 0$ and $g(R_1, R_2)$ is decreasing in R_1, $R_1' > R_1^* - \Delta$. This is true for any value of R_2^0 and any sufficiently small positive Δ. Then, we get $-1 < \frac{dR_1^*}{dR_2} < 0$ under any reservation contract and any R_2. $\square$

To prove Lemma 11

Assume the original equilibrium point is (R_1^*, R_2^*) for a particular value of E . Suppose we increase E with any positive ΔE to $E+\Delta E$. With uniform capacity allocation, this moves $R_1^*(R_2)$ and $R_2^*(R_1)$, thus the new equilibrium point to $(R_1^* - \frac{\Delta E}{2}, R_2^* - \frac{\Delta E}{2})$. For any (R_1', R_2') satisfies the first equation in (23), substituting the point $(R_1' - \frac{\Delta E}{2}, R_2' - \frac{\Delta E}{2})$ makes the left-hand-side of the equation > 0. Since $-1 < \frac{dR_1^*}{dR_2} < 0$ and $-1 < \frac{dR_2^*}{dR_1} < 0$, we may conclude that the new equilibrium point $(R_1^{*\prime}, R_2^{*\prime})$ (after increasing E by ΔE) has a sum $(R_1^{*\prime} + R_2^{*\prime})$ more than $R_1^* + R_2^* - \Delta E$ and less than $R_1^* + R_2^*$. This is true for any positive ΔE, thus we get $-1 < \frac{d(R_1^* + R_2^*)}{dE} < 0$. $\square$

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