Managing High-Tech Capacity Expansion
Via Reservation Contracts

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We study capacity reservation contracts in high-tech manufacturing, where the manufacturer (the supplier) shares the risk of capacity expansion with her OEM customer (the buyer). We focus on short-life-cycle, make-to-order products under stochastic demand. The supplier and the buyer are partners who enter a "design-win" agreement to develop the product, and who share demand information. The supplier would expand her capacity in any case, but reservation may encourage her to expand more aggressively. To reserve capacity, the buyer pays a fee upfront while (a pre-specified portion of) the fee is deductible from the order payment. As capacity expansion demonstrates diseconomy of scale in this context, we assume convex capacity costs. We first analyze the players' incentives as a means to evaluating the value of reservation. We show that as the buyer's revenue margin decreases, the supplier faces a sequence of four profit scenarios with decreasing desirability. We examine the effects of market size and demand variability to the contract conditions, and show that it is the demand variability that affects the reservation fee, and that the convex cost assumption leads to somewhat different insights than the linear cost cases in the literature. We propose two channel coordination contracts, and discuss how such contracts can be tailored for situations where the supplier has the option of not complying with the contract, and when the buyer's demand information is only partially updated during the supplier's capacity lead time. We conclude the paper by summarizing insights useful for high-tech capacity management.

1. Introduction

Manufacturing capacity plays a significant role in high-tech industries such as semiconductors, electronics, and telecommunications equipment. This paper is motivated by our involvement with designing capacity reservation contracts for a US telecommunications integrated circuit (IC) device manufacturer. During the upside market in the late 1990's, the manufacturer has constantly suffered from capacity shortages, resulting in lost revenue, eroding their long-term market position. Despite the need for improved service levels and higher revenue, aggressive capacity expansion would expose the firm to significant financial risk due to high capacity cost, long (capacity) lead times, and high demand volatility. For instance, a state-of-the-art semiconductor manufacturing Fab costs \$500 million to \$2 billion to build, but the demand volatility might be as high as 80% deviation from the forecast during a particular quarter. Moreover, the overall market size might drop sharply in reaction to economic contraction, as evident from the decline of telecommunications demand that took place in the early 2000's. In this environment, the device manufacturers (the supplier) are forced to adopt a conservative capacity expansion policy, limiting their downside risk at the expense of upside potentials. Consequently, their downstream buyers (e.g., OEM manufacturers) may not have adequate supplies to fill the market orders. However, if the buyer is willing to mitigate the supplier's downside risk by assuming a certain level of liability, the supplier might be willing to expand capacity more aggressively. For instance, the customer may offer early commitment on a certain portion of future capacity, in exchange, the supplier commits to increase her capacity to meet the customer's demand. This provides a win-win situation for the supplier and the buyer by creating additional surplus in the channel.

In this paper, we propose *capacity reservation contracts* designed for short life-cycle, make-to-order high-tech products under stochastic demand. We consider the case where the supplier's wholesale price and the buyer's selling price are exogenous. The wholesale price is typically determined exogenously by the market when the negotiating firms are of similar size, or when the firms settle wholesale price negotiation early-on thus decouples the decision from capacity reservation. The latter captures a reality in the consumer electronics and telecommunications equipment industries: oftentimes, an OEM manufacturers and their component suppliers agree on pricing when a "design-win" agreement is reached (this is when the OEM manufacturer gives a supplier the right to develop a product, such as a custom IC chip). This is followed by full-scale product design, development, testing, and production ramp-up. The issue of capacity reservation arises much later during, or right before, regular production. At this point, the wholesale price is considered a given. In this context, we examine capacity reservation contracts with *deductible reservation fees:* the buyer pays a fee upfront for each unit of capacity that she would like to reserve for a certain time in the future. When the buyer actually utilizes the reserved capacity (i.e., placing a firm order), the reservation fee is deducted from the order payment. However, if the reserved capacity is not fully utilized within the specified time, the reservation fee associated with unused capacity is not refundable. Since the reservation fee is typically lower than the wholesale price, the reservation contract provides the buyer flexibility in that it does not ordain a firm commitment prior to demand realization. Moreover, capacity reservation reduces the supplier's risk associated with capacity expansion since the buyer is liable for the capacity to a certain extent. This form of risk sharing is particularly attractive in high-tech manufacturing environment where the demand is volatile and the capacity is capital intensive.

It has been well established in the literature that capacity reservation using *options contracts* achieves channel coordination so long as the wholesale price is *endogenous* (see Cachon 2003 and references therein). In its simplest form, an *options contract* specifies the price to purchase the option before uncertainties are resolved, and the price to exercise the option after the uncertainties are resolved. Motivated by the high-tech product development setting discussed earlier, we consider a version of options contracts where the wholesale price is determined *exogenously*. We will show that the standard options contract do not lead to channel coordination when the wholesale price is *exogenous*, or when the uncertainty is not completely resolved at the time the option is exercised. More importantly, in this case the options contract may not be *individually rational* for the players involved. This is true because when the wholesale price is exogenous, the supplier may have incentive to expand capacity based on her knowledge of market demand, consequently, the buyer could count on the supplier to make extra capacity available without reservation. Therefore, there is an incentive issue that must be examined since we can no longer assume that entering a reservation contract is always for the best interests of both players.

We will start our exposition by investigating the conditions under which capacity reservation is beneficial for both the supplier and the buyer. It will be shown that both parties are willing to participate in the negotiations for capacity reservation if the supplier's capacity cost and the buyer's expected margin are both F, and the end-item demand is volatile. This helps to establish the relevance of capacity reservation in high-tech manufacturing since the above conditions are commonplace. At the same time, the analysis helps to identify situations when capacity reservation is not supported by both players' incentives, therefore undesirable. We then examine the implementation of capacity reservation contracts as a means to achieving channel coordination. Two generalized versions of capacity reservation mechanisms are proposed: *partial payment deduction* and *reservation with cost sharing*. It will be shown that although both mechanisms could achieve coordination with the same expected payoffs for the players, they have different cash flow implications when the capacity cost is nonlinear.

To consider further implications of capacity reservation contracts in the high-tech environment, we generalize our results to two important cases: 1) voluntary contract compliance, and 2) partial information updating. The notion of *voluntary compliance* in the context of supply contracting has been investigated by Cachon and Lariviere (2001). Tomlin (2003) extends their analysis by introducing the concept of "partial compliance." In both cases, the authors show that the equilibrium decisions deviate from the *forced compliance* case. In our analysis, we first determine the penalty scheme for noncompliance under which the coordination can be still achieved. It is shown that in a supplier-lead supply chain (with exogenous wholesale price), even with the absence of forced compliance, the supplier has incentive to signal the buyer that she will fully comply. Next we consider partial information updating; in our base model, a main assumption is that the demand uncertainty is *completely* resolved after the capacity is built, but before utilized by the buyer. This assumption is common in the options contract literature (see Cachon 2003). However, due to long production lead-time in high-tech manufacturing such as semiconductors, it is quite possible that the buyer has to finalize her order (utilize the reserved capacity) *before* knowing the actual demand, but *after* a partial information (forecast) update. In other words, the buyer must exercise her option with partially resolved demand uncertainty. We examine channel coordination in this situation, and find that coordination can be achieved if the contracts developed for the base case are coupled with buy-back agreements. Ultimately, we make the case that for the high-tech manufacturing environments, practical supply contracts should incorporate terms related to capacity reservation, compliance regime, partial information update, and buy-back agreements, while leaving wholesale price exogenously defined

The paper is organized as follows: in the next section, we summarize related work in the literature, which is followed by an introduction of the base model in Section 3. In Section 4, we analyze the capacity reservation contract with deductible fees, and we examine the effects of market size and demand variability to contract conditions. In Section 5 we introduce two mechanisms for channel coordination. In Section 6 we discuss the issue of voluntary compliance. Section 7 generalizes our contract framework to the case where the demand is only partially updated before the buyer decides on the final order amounts. We then summarize the managerial insights and discuss our results in Section 8. Proofs of the main Theorems are in the Appendix.

2. Related Work in the Literature

Earlier work on capacity reservation focuses on strategies from the buyer's perspective while the terms specified in the reservation are exogenous. Silver and Jan (1994) and later Jan and Silver (1995) consider the case where the buyer pays a nonrefundable premium and is guaranteed a certain level of supplies in an environment where the availability of capacity is highly uncertain. They propose methods for the buyer to determine the level of dedicated capacity to reserve and the size of each periodic replenishment. Brown and Lee (1998) also study capacity reservations in the context of semiconductor manufacturing. In particular, they discuss "pay-to-delay" capacity reservation contracts. The authors focus their analysis from the buyer's perspective and derive optimal policies for the buyer. A comprehensive review of recent literature on capacity procurement games is provided by Cachon (2003).

The research on capacity reservation contracts can be broadly categorized into two groups based on the buyer's incentive to reserve. Specifically, buyers are motivated to reserve capacity so as to 1) achieve potential cost reduction through early commitments, or 2) avoid future supply disruption through risk sharing with the supplier. Papers in the first category consider multiple ordering opportunities where the buyer has the option of committing to an order quantity in advance with the option of purchasing additional quantities at a higher cost, or through the spot market, after demand information is updated. Gurnani and Tang (1999) and Huang *et. al.* (2003) study the general problem from the buyer's perspective under different settings with demand information update. Donohue (2000) and Serel *et. al.* (2001) investigate the equilibrium strategies based on *full* (payment for the) *commitment.* The former considers forecast update in a two-stage setting where the costs of both the early and delayed commitments are decided by the supplier. The author proposes a supplier-initiated contract that utilizes buy-backs to coordinate the channel. The latter extends the capacity reservation problem to a multi-period setting with stationary demand. In their model, the buyer contracts a certain amount of products from the supplier for each period by paying a reduced rate. As such, the supplier guarantees the delivery of the buyer's order up to the contracted amount. Bonser and Wu (2001) proposes a similar multi-period setting, where future supplies are secured by long-term contract and spot market purchases. To minimize procurement costs, the buyer must fulfill long-term contract commitments to avoid "underlift" penalty while at the same time taking advantage of spot price fluctuations. Wu *et. al.* (2002), Glovachkina and Bradley (2002), Spinler *et. al.* (2002), and Araman *et. al.* (2003) consider *partial commitments* that employ options contracts under various assumptions regarding demands and spot market prices.

In this paper, we also use options and buy-back as part of the contract mechanism, however the paper makes very different assumptions about the availability of supply. While earlier work assumes that the buyer could buy from a selection of suppliers (or the spot market) and they always have the ability to fill the buyer's orders, we assume that the buyer is restricted to buy from a particular supplier who has limited capacity. While the former assumption is appropriate for commodity products, our assumption is in general true for high-margin products in the high-tech industry. For instance, each telecommunication IC device is custom designed for a specific family of products with low substitution opportunities. In this respect, our work is more in line with papers in the second category that are summarized in the following.

Papers in this group investigate capacity reservation motivated by ensuring future supply availability. Cachon and Lariviere (2001) and Tomlin (2003) focus on buyer-lead models and investigate forced and voluntary compliance regimes. The former examines capacity contracting in the context of supplier-buyer forecast coordination. Buyer provides an initial forecast and a contract consisting of firm commitments and capacity options. After supplier builds capacity, the buyer places an order based on the up-to-date forecast. They show that although supply chain coordination can be achieved through options contracts in the full information case, it is only possible under *forced compliance*. They conclude that in the absence of *forced compliance*, higher supplier capacity can not be induced (thus fails to coordinate the channel). Tomlin (2003) enhances this approach by introducing *partial* compliance. He shows that under nonlinear, price-only contracts, options do induce higher supplier capacity, but not necessarily sufficient to achieve coordination. In this paper, we show that in a supplier-lead channel the supplier has incentive to signal the buyer her willingness to comply by offering a noncompliance penalty that would be agreeable to the buyer. Thus, channel coordination can be achieved as in the forced compliance case. Also in the second group of papers is Eppen and Iyer (1997), who model capacity reservation through *backup* agreements: the buyer makes firm commitments at the beginning, but has the option to cancel the order with penalty. Barnes-Shuster *et. al.* (2002) study a generalized case where the buyer places firm orders while purchasing options at the same time, under a two-period setting. Li and Atkins (2002) study capacity coordination problems between the marketing and production divisions of a firm where the demand for the finished product is stochastic yet price sensitive. In their setting, *production* chooses the inventory (capacity) level while *marketing* decides on the product pricing. The transfer price between the divisions is set by the central management. To coordinate the system, the central management sets a transfer price and a penalty for unused inventory paid by the marketing to the production.

Most papers above focus on the procurement problem from the buyer's perspective, assumes either endogenous wholesale prices, or that the supplier builds nothing without a buyer commitment. This paper focuses on the context for high-tech products that differs from earlier work in all three aspects. We investigate capacity reservation contracts where the wholesale price is determined *exogenously* by the market or by earlier negotiations, while the supplier acts as a strategic player who makes decisions on the capacity reservation (options) fee as well as capacity expansions. As discussed in Section 1, this wholesale price assumption better captures the high-tech manufacturing setting where pricing and capacity reservation are fundamentally separate. This subtle difference in assumption has important implications; under the *endogenous* wholesale price assumption, options contracts can always produce win-win solutions for the players, but this is not necessarily the case under *exogenous* pricing. Thus, the players' incentive to participate becomes an issue. We investigate the viability of capacity reservation contracts under various cost/margin scenarios and market conditions. Moreover, with knowledge of market demand, the supplier has incentive to build capacity even without buyer commitment. We examine the impact of capacity cost (i.e., linear vs.. convex) on the supplier's capacity expansion decisions.

Jin and Wu (2001) also study capacity coordination under exogenous wholesale price by utilizing *take-or-pay* contracts. In their setting, a per unit penalty for unused capacity is charged to the buyer only if the utilized portion of the reserved capacity falls below a certain threshold. The contract specifies both the penalty and the threshold. Although the proposed contracts lead to channel coordination, oftentimes, they are not Pareto improving and as such the supplier may not have incentive to participate. The coordination mechanisms proposed in this paper provide higher efficiency for the supplier in that they are individually rational and could generate higher payoffs for the supplier. Ferguson *et. al.* (2003) examines two commitment strategies for the buyer: 1) place all orders in advance before any demand update, or 2) delay orders until a portion of the demand uncertainty is resolved. In both cases the buyer's orders are *firm*. Their analysis explains the trade-offs between early and delayed commitments from the buyer and the supplier's perspectives. However, the authors do not consider channel coordination. In this paper, we consider the effect of *partial* commitments to supply chain coordination in the context of capacity reservation.

By incorporating exogenous wholesale prices, partial commitments, partial information update, and the compliance regime into the capacity reservation framework, we offer an options contract structure that captures the business environment unique to the high-tech industries.

3. Model Setting

As a basic building block for the analysis of capacity reservation, we consider a one-supplier, one-buyer system in a single-period setting where the supplier faces stochastic demand. The supplier and the buyer are manufacturing partners (e.g., OEM manufacturer and their key component supplier) in the supply chain who operate in a make-to-order fashion. Before placing a firm order to the supplier, the buyer has the option to reserve future capacity in advance so as to assure availability. In the case of insufficient capacity, there will be no backorder and the unmet demand will be lost. This setting is realistic in the high-tech manufacturing environment where the product life-cycle is short while the manufacturing lead-time is long. The buyer pays a "wholesale price" w to the supplier, and incurs a revenue of p for each unit of product. Unique to this environment is that the buyer pre-negotiates the wholesale pricing (w) early on during the design-win phase, and does not consider pricing a subject of further negotiation. However, it is not uncommon for the suppliers to impose other forms of price adjustments in the forms of one-time charge ("engineering fees"), or variable service charges. We assume that the supplier produces with marginal cost c, and $p > w > c$.

At the beginning of the period, the supplier has the option to expand her capacity. We use a convex function $V(k)$ to characterize the capacity cost, where $V(k)$ is increasing in capacity k and $V'(0) = 0$. We assume that the residual value of capacity is concave in the capacity amount, while the cost of building capacity increases in a convex fashion. Thus, it is possible to incorporate the residual value of capacity as part of the capacity cost. For example consider $V(k) = \lambda k - \beta k^z$ where $\lambda > \beta$ and $z < 1$. The total cost of capacity is the procurement cost minus the residual value of the capacity. The residual value is generally represented by a concave function as each additional unit capacity returns a future value with a diminished rate. Without loss of generality, we assume that the initial capacity is zero. Product demand x follows a continuous distribution $F(x)$ when $x \geq 0$ with density $f(x)$, both are differentiable for $x \geq 0$. We also assume that $F(x)$ is invertible and $\overline{F}(x) = 1 - F(x)$. Furthermore, we assume complete information in that the supplier has full information regarding the buyer's demand distribution and revenue function, and vice versa.

First consider a centralized supply chain where the capacity decisions are made to optimize channel efficiency. In this case, the optimal capacity is defined by the classical newsvendor solution. Let $S(k)$ be the expected sale of the channel given capacity k. Given the demand distribution $S(k)$ can be written as

$$S(k) = k - \int_0^k F(x)dx.$$

For any given capacity k the integrated channel profit can be written as follows:

$$\Pi_I(k) = (p - c)S(k) - V(k) \tag{1}$$

It is straightforward to verify that $\Pi_I(k)$ is concave in capacity k and the optimal capacity solves the following equality:

$$k^o = F^{-1}\left(\frac{p - c - v^o}{p - c}\right),$$

where v^o is the derivative of the capacity cost evaluated at k^o. We know that the channel efficiency is maximized when the capacity is built up to k^o. However, the supplier may not have the incentive to expand her capacity to k^o. Specifically, for any given k the supplier's profit function is as follows:

$$\Pi_{\mathrm{S}}^0(k) = (w - c)S(k) - V(k) \tag{2}$$

Thus, the optimal capacity k^* for the supplier is her own newsvendor solution:

$$k^* = F^{-1}(\frac{w-c-v^*}{w-c}),$$

where v^* is the derivative of the capacity cost evaluated at k^*. Since $p > w$, we know that $k^* < k^o$, i.e., the supplier will not expand her capacity to the channel optimum. *If* the buyer does *not* take part in the capacity expansion decision, her profit is a function of the supplier's capacity choice k^*, i.e.,

$$\Pi_B^0(k^*) = (p-w)S(k^*). \tag{3}$$

In summary, since the supplier's profit margin is less than that of the integrated channel and the buyer does not share liability for the capacity, the supplier will build less capacity than what is optimal for the channel due to *double marginalization*. This leads to revenue loss in the channel. In general, if the buyer's marginal revenue and the cost of capacity are both sufficiently high, or the wholesale price is sufficiently low, the buyer will choose to make a *full* commitment and complete her purchase by paying the full price prior to the realization of her demand. Since full commitment is but a special case of the reservation contracts discussed in the next section, we exclude this consideration from the base model. We use the base model as a conceptual benchmark, and to analyze the players' incentives (or disincentive) to participate in capacity reservations. In the following sections, we explore various extensions to the base model that enhance the coordination between the supplier and the buyer.

4. Capacity Reservation with Fully Deductible Payments

We consider capacity reservation with the following sequence of events: (1) the supplier announces a unit reservation fee $r \leq w$; for the guaranteed use of future capacity (2) given *r,* the buyer places reservation of quantity $q(r)$, paying $r \cdot q(r)$, (3) after observing the reservation amount, the supplier determines her capacity, k ($k \geq q(r)$), (4) the buyer's demand x is realized, and the buyer orders x units; (5) the supplier produces min(x, k) units with marginal cost c. The supplier deducts the amount $r \cdot$ min(x, q) from the buyer's purchasing cost, but keeps the amount $r \cdot$ max$(0, q-x)$.

As described above, if the buyer's realized demand x, is less than the quantity reserved q, the supplier keeps the difference $r(q-x)$, i.e., the reservation fee for unused capacity is not refundable. Here, we assume that the supplier always builds sufficient capacity to cover the reservation amount (known as *forced compliance*). We will discuss the *voluntary compliance* case in Section 6. Note that the buyer has the option of not placing a reservation, knowing that the supplier will expand capacity to her newsvendor optimum k^*. Thus, in order to convince the buyer to place a reservation, the supplier must offer a contract where the buyer's expected profit is no worse than $\Pi_B(k^*)$.

The buyer does not order more than the realized demand since we assume that the items have no value if not sold in the current period, and no inventory can be carried to the next period. This is in general true in the high-tech environment where product specifications change frequently, and there is a high risk of carrying inventory as the product will likely be obsolete by the following period. Under this setting we can write the supplier's profit function as follows:

$$\Pi_S(k,\, r) = (w-c)S(k) - V(k) + rE[max\,(0, q-x)] \, = (w-c)S(k) - V(k) + r\int_0^q F(x)dx \quad (4)$$

There are three basic decision stages in our model. In the first stage, the supplier chooses the reservation fee r, which is followed by the buyer's reservation decision q. Then, the supplier chooses her capacity level k. We will start from the thrid-stage decision problem by the supplier.

4.1. The Supplier's Capacity Decision

Given r and q, the supplier solves the following decision problem to determine her capacity level k :

$$\underset{k \geq q}{Max}\, \Pi_S(k, r)$$

The profit function consists of the expected revenue from sales, the capacity cost, and the expected revenue from the buyer's "over" reservation (the portion of reservation that exceeds the realized demand). It is straightforward to see that if the buyer's reservation quantity $q \leq k^*$, the supplier's capacity level should be set at her newsvendor optimum k^*. Otherwise, the optimal capacity level is q since $\Pi_S(k,\, r)$ is concave in k, and thus each additional unit of capacity beyond q will return a negative margin, i.e., the supplier has no incentive to expand her capacity beyond the reserved amount.

4.2. The Buyer's When to and How to Reserve Strategy

In the second decision stage, the buyer's decision problem is as follows:

$$Max\, \Pi_B(q) = (p-w)S(q) - r\int_0^q F(x)dx \quad (5)$$

The first term is the buyer's expected profit through sales, and the second term is the expected loss for unused reserved capacity. As $\Pi_B(q)$ is concave in q, the optimal reservation quantity is

$$q(r) = F^{-1}\left(\frac{p-w}{p-w+r}\right) \quad (6)$$

Note that both the reservation quantity and the buyer's expected profit decrease in reservation fee r. Given the buyer's best response in (6), a basic question is "under what condition is the buyer better off making a reservation?" Clearly, if her optimal reservation quantity *q(r)* is less than or equal to the supplier's newsvendor capacity k^*, the buyer will not reserve, knowing that the supplier will build a capacity of at least k^* units anyway. Hence, the following inequality is necessary to hold:

$$r \leq (p-w)\frac{v^*}{w-c-v^*} \quad (7)$$

If the supplier announces a reservation fee r greater than the right-hand-side in (7), the buyer's best response is not to reserve at all (comparing (5) and (3), it is clear that the buyer is worse off if she reserves an amount $q \leq k^*$). Hence, (7) establishes an upper bound for the *acceptable* reservation fee.

Moreover, the reservation fee must be less than the wholesale price w; clearly, if $r > w$ the buyer would have no incentive to reserve. Note that $r = w$ implies full commitment.

In general, the buyer's expected profit under early commitment must exceed her profit under no-commitment. To tighten the bound on r we can define a *threshold reservation quantity* q^t (where $q^t > k^*$) such that $\Pi_B(q^t) \geq \Pi_B^0(k^*)$. This *threshold quantity* is useful for the buyer in that she can negotiate a reservation fee that is lower than the *threshold reservation fee* r^t which corresponds to q^t. This is stated in the following Theorem.

Theorem 1. *It is individually rational for the buyer to reserve capacity if her optimal reservation quantity is no less than the threshold* q^t*, where* q^t *is the unique non-negative value satisfying*

$$S(k^*) = \frac{\int_0^{q^t} x f(x) dx}{F(q^t)} \tag{8}$$

The threshold reservation quantity q^t *has the following properties:*

(1) it provides a tight lower bound for the reservation quantity , i.e., $q^t > k^*$*,*

(2) it defines the threshold reservation fee r^t *(under which the buyer shall be willing to pay):*

$$r^t = (p - w)\frac{\overline{F}(q^t)}{F(q^t)} \tag{9}$$

(3) r^t *provides a tight upper bound for the reservation fee, i.e.,* $r^t < (p - w)\frac{v^*}{w-c-v^*}$.

A formal proof for the Theorem is given in the Appendix. The Theorem defines a tighter condition under which the buyer would have incentive to place capacity reservation. The Theorem stipulates that the expected increase in buyer's sales revenue must offset the expected liability from capacity reservation. Unfortunately, there is no closed form expression we can derive for the threshold value q^t. Nonetheless, since the right-hand-side of (8) is strictly increasing in q, for a given k^*, q^t can be found by a simple line search. Interestingly, q^t is independent of the buyer's selling price, p.

4.3. The Supplier's Individual Rationality Condition and the Optimal Reservation Fee

We now consider the *supplier's* perspective. We would like to know if the supplier is better off by choosing a reservation fee below the buyer's *threshold* r^t, or not offering reservation at all. As shown above, the buyer would only reserve if doing so improves her profit. However, unless the reservation induces additional surplus in the *channel*, the buyer's added profit would be the supplier's loss. This means that for the contract to be acceptable for both parties, it must generate more surplus in the channel compared to the no-commitment case. The following Theorem states this condition in terms of the channel profit Π_I.

Theorem 2. *Given the buyer's threshold reservation quantity* q^t*, the following are true:*

(1) the supplier will offer an acceptable contract (i.e. $r \leq r^t$*) iff*

$$\Pi_I(q^t) \geq \Pi_I(k^*) \tag{10}$$

(2) there exists a capacity level $\overline{k}$ such that $\overline{k} > k^o$ and $\Pi_I(k^*) = \Pi_I(\overline{k})$.

The first part of the Theorem is a straightforward statement from the previous observation, and the second part can be concluded from the fact that $\Pi_I(k)$ is strictly concave in k. Observe that Theorem 2 implies that the supplier will choose to offer a reservation contract only if $q^t \leq \overline{k}$. Using $q \equiv q(r)$, we can write the supplier's first stage decision problem as follows:

$$\underset{r \leq \min(w, r^t)}{Max} \; \Pi_S(r) = (w-c)S(q(r)) - V(q(r)) + r\int_0^{q(r)} F(x)dx. \tag{11}$$

For mathematical convenience, we rewrite the supplier's model by substituting the reservation fee, r with the reservation amount, q according to (6):

$$\underset{q \geq \max(q^t, q^w)}{Max} \; \Pi_S(q) = (w-c)S(q) - V(q) + (p-w)\frac{\overline{F}(q)}{F(q)}\int_0^{q} F(x)dx, \tag{12}$$

where $q^w \equiv q(w)$ is the quantity that the buyer is willing to reserve when the reservation fee is set at the wholesale price w (i.e., full commitment). In this representation of the supplier's expected profit function we are assuming that the supplier chooses the capacity level for which he will receive the highest reservation price that the buyer would be willing to pay. It is straightforward to see that the outcome will be the similar to that in (11). We may interpret the supplier's decision as follows: knowing the highest reservation fee that the buyer is willing to pay, the supplier is to choose a *capacity level* q^r that would maximize her profit. Thus, it is individually rational for the supplier to offer a reservation contract if the profit resulting from the optimal capacity level (reservation) q^r is no worse than her newsvendor profit (i.e., $\Pi_S(q^r) \geq \Pi_S^0(k^*)$).

Recall that in the environment we consider, the wholesale price w and the buyer's revenue margin p are both exogenous. Nonetheless, it is useful to know that given the pre-negotiated w, the (supplier's) individual rationality condition in (10) suggests a *threshold revenue margin p^t;* only when the buyer's margin is at or above this threshold would it make sense for the two parties to enter the reservation contract. This is stated in the following Theorem:

Theorem 3. *It is individually rational for the supplier and the buyer to enter the reservation contract provided that the buyer revenue margin p is no less than the threshold p^t which is defined as follows:*

$$p^t = c + \frac{V(q^t) - V(k^*)}{S(q^t) - S(k^*)} \tag{13}$$

From Theorem 1 we know that the buyer will reserve if and only if the reservation fee is less than r^t and her minimum reservation amount will be q^t and $q^t > k^*$. Theorem 2 implies that the supplier will announce a reservation fee that is less than or equal to r^t if and only if $\Pi_I(q^t) \geq \Pi_I(k^*)$. Observe from (13) that p^t is the value at which $\Pi_I(q^t) - \Pi_I(k^*) = 0$ and from (8), its value is independent of p. It is straightforward to see that $\Pi_I(q^t) - \Pi_I(k^*)$ is strictly increasing in p. Therefore, when the revenue margin $p \geq p^t$ would there be sufficient surplus generated in the channel to benefit both players. In essence, p^t is the *effective marginal cost* of a sold product: marginal production cost plus the cost of capacity expansion for each product sold beyond k^*. To answer the earlier question whether "the supplier is better off accepting the buyer's *threshold reservation fee* r^t, or not offering reservation at all", we conclude that the supplier's decision should be based on the potential surplus that could be generated from a coordinated channel, i.e., $\Pi_I(q^t) - \Pi_I(k^*)$. Only when the buyer's profit margin is sufficiently high would she be able to (adequately) offset the supplier's risk of expanding beyond her newsvendor capacity. In reality, the buyer may not have a sufficiently lucrative margin to support such a win-win outcome, in which case the supplier should reject the reservation strategy and stick with her newsvendor model for capacity expansion. From the above results, we may conclude that entering capacity reservation is not necessarily desirable for both players at all times. As discussed earlier, this produces an outcome different from the *endogenous* wholesale price analysis in the literature, where capacity reservations can always improve on "no-commitment" cases. With *exogenous* wholesale price, capacity reservation is only justified under the conditions given in Theorems 1 to 3. With this understanding, we continue our analysis on the supplier's optimal strategy.

A closer inspection of (12) reveals that the supplier's profit function may have multiple local maximum, that is, if one exists at all. The shape of the profit function $\Pi_S(q)$ is primarily driven by the demand distribution. To derive some insights, we focus our analysis on increasing failure rate (IFR) distributions, in which case the first order condition for $\Pi_S(q)$ has a unique solution. For any IFR distribution, the ratio $f(x)/\overline{F}(x)$ (known as the *failure rate*) increases in x. IFR distributions represent a rather general class of which many widely used distributions (e.g., Uniform, Weibull, Gamma, and Normal) belong. The following theorem shows that the supplier's profit function $\Pi_S(q)$ is well behaved if the demand follows any IFR distributions. For notation convenience, we denote $q^m \equiv Max\ (q^t, q^w)$.

Theorem 4. *The supplier's decision problem specified in (12) has the following properties:*

(1) The profit function $\Pi_S(q)$ is strictly decreasing in $[k^o, \infty)$,

*(2) If the demand distribution is IFR, then $\Pi_S(q)$ is either **decreasing** or **unimodal** on $[q^m, k^o)$, i.e., the decision problem has unique optimal solution.*

(3) To find the unique optimal reservation quantity, q^r, we have

*(i) If $\Pi_S(q)$ is **decreasing** in $[q^m, k^o)$, then $q^r = q^m$, and*

*(ii) if $\Pi_S(q)$ is **unimodal** in $[q^m, k^o)$, then q^r is the unique point in $[q^m, k^o)$ that satisfies the first order optimality condition for $\Pi_S(q)$.*

As pointed out in the theorem, should both the supplier and the buyer choose to initiate a reservation (i.e., $q^t < \overline{k}$) the supplier's best action is to set the reservation price at $r(q^r)$, and the buyer will respond by reserving q^r. Recall that for technical convenience, we rewrote the supplier's profit as a function of q (by substituting the reservation fee *r)*. To provide some practical insights from the Theorem, we convert the supplier's decision back as a function of the reservation fee *r*. Figures 1a-1d depict a sequence of four scenarios faced by the supplier as the buyer's revenue margin p decreases: (1) the supplier's optimal reservation fee is equal to the wholesale price, implying full commitment, (2) the optimal reservation fee is strictly less than the wholesale price and the threshold reservation fee; (3) the supplier's optimal reservation fee r^* coincide with r^t, indicating that the buyer is indifferent between reserving or not reserving; (4) the supplier's optimal strategy is not to offer capacity reservation.

<< Insert Figure 1 here >>

4.4. The Effects of Market Size and Demand Variability

The above results only require that the demand distribution is IFR. If additional information is known about the market that drives the demand, we may be able to derive additional insights. In this section we examine the impact of market size and demand variability on the player incentives. To facilitate our analysis, we consider a demand distribution that is in the *shifted family*. A distribution is said to be from shifted family if $F(x|\theta) = F(x-\theta|0)$ holds. In this representation, θ is the scale parameter which encapsulates the market size information. An example to shifted family distribution is the Normal distribution which is widely used to capture uncertainty in real life practice.

Theorem 5. *If the demand distribution $F(x|\theta)$ is IFR and is in the shifted family, then*
(1) The supplier's optimal reservation price is $r^* = min(r^t, w)$.
(2) If the capacity cost v is linear then neither the optimal reservation fee, r^* nor the supplier's surplus ($\Pi_S(q^r) - \Pi_S^0(k^*)$) are dependent on the market size θ, however, the threshold reservation quantity q^t does increase in θ.
(3) If the capacity cost $V(k)$ is strictly convex, then the threshold reservation fee r^t is increasing in market size θ. Furthermore, there exists a unique market size threshold, θ^t, above which the capacity reservation is no longer favorable for the channel.

The first part of the Theorem states that if the demand distribution is in the *shifted family,* the supplier's optimal reservation price should be set as $min(r^t, w)$. This follows the same intuition as the first part of Theorem 4. In the second part of the Theorem, we see that if the cost for capacity expansion is linear, the supplier has incentive to increase her capacity to match the growing market size, resulting in a higher threshold reservation quantity. Interesting, in this case the increase in market size neither affects the optimal reservation fee, nor the supplier's surplus (for accepting reservation). This is because for any demand distribution of *shifted family* the coefficient of variation decreases as the market size (the mean) increases, which in turn reduces the supplier's risk exposure. In essence, the increase in the capacity cost due to market size is balanced with the reduction in risk. However, this does not suggest that the profits are independent of the market size. In fact, the supplier and the buyer's profits increase with a factor of $(w-c-v)$ and $(p-w)(w-c-v)/(w-c)$ with the market size, respectively. As a result while the profits increase with the market size, the value of reservation remain the same. One may thus conclude that if the capacity cost is linear, as the market size increases, the merit of reservations does not change in an absolute sense, but its relative benefit diminishes. Contrary to this observation, last part of the Theorem shows that if the capacity cost is *convex*, an increased market size indeed affects the reservation fee and the supplier's surplus. This is rather intuitive since the supplier needs more assurance from the buyer when the capacity cost demonstrates diseconomy of scale. In this case, the decrease in the coefficient of variation due to market size is not sufficient to offset the risk of expansion.

There is an interesting implication of Theorem 5 that is noteworthy. One should observe that when the capacity cost is strictly convex the supplier may not have incentive to build sufficient capacity to satisfy all demands when θ is large. This is due to the fact that the marginal profit is linear in sales while the cost is convex in capacity. We note that this is not true for any convex cost function. An example is $V(k) = \lambda k - \beta k^z$ where $w - c > \lambda > \beta$ and $z < 1$.

We explore the effect of capacity cost further in the following Theorem.

Theorem 6. *If the demand is Normally distributed, then the threshold reservation fee r^t increases in standard deviation. If the capacity is costly, i.e., $V'(k^*) \geq \frac{1}{2}(w-c)$, then the value of capacity reservation (the channel's surplus) increases in standard deviation as well.*

The above two Theorems reveal that *demand variability* and the *cost structure of capacity* are two main factors determine the value of capacity reservation. Clearly, as the variability increases the supplier is exposed to more risk, and would be reluctant to build more capacity unless the buyer raises her threshold reservation fee. When the capacity cost is high relative to the supplier's profit margin, it is better for her (and for the channel) to take reservation. This is consistent with the intuition that the higher the demand variability, the more appealing it is to take capacity reservation. The result shows that when the availability of capacity is at stake (volatile market environments accompanied with high capacity costs) the supplier and the buyer are compelled to enter in a reservation contract. In summary, the foregoing observations point out that (1) it is the demand variability (standard deviation) that affects the reservation fee (Theorem 6), (2) the reservation *quantity* (thus the total profit) does increase with market size, and (3) when the capacity cost is *convex*, the reservation fee should indeed increase with market size.

5. The Coordination Mechanisms

Our analysis so far reveals that capacity reservation could provide real benefit for the buyer, the supplier, and the supply channel. This is especially the case when the capacity is costly and market is volatile; both are characteristics of high-tech manufacturing. The contract we have considered allows the buyer to *fully deduct* the reservation fee from the final payment. Under this setting, the channel profit generated from capacity reservation may not be optimal. In fact, in a capacity reservation contract with fully deductible reservation fee, the channel profit is suboptimal unless the threshold reservation quantity $q^t = k^o$. Thus, with *fully deductible* reservation fee, channel coordination is only achievable under a very restrictive case. In the following, we propose two variants of the base contract that will coordinate the channel.

5.1. Capacity Reservation with Partially Deductible Payments

We first propose a *partially deductible reservation (PARD)* contract where only a portion of the reservation fee r is deductible from the final payment, where the deductible portion r_2 is predefined as a contract parameter. The sequence of events is as follows:

(1) the supplier announces the reservation fee r for each unit of capacity reserved, and the refund rate r_2 for each unit of capacity actually utilized, such that

$$r_2 = \frac{p-c}{v^o}r - (p-w) \tag{15}$$

(2) given r and r_2, the buyer places reservation of quantity q, paying $r \cdot q$,

(3) the supplier expands her capacity to q,

(4) the buyer's demand x is realized, and the buyer orders x units;

(5) the supplier produces $\min(x, k)$ units with marginal cost c. The supplier deducts the amount $r_2 \cdot \min(x, q)$ from the buyer's purchasing cost.

With the added dimension (r_2) in PARD contract, the supplier has more flexibility in setting the reservation fee (r). We do not assume any restriction on r_2 except that $r_2 \leq w - c$, which implies that

$$0 < r \leq v^o \tag{16}$$

Theorem 7. *Under partially deductible (PARD) contract specified by (15) and (16), the buyer will reserve and the supplier will build the channel optimal capacity, i.e., $k = q(r) = k^o$. Moreover,*
(1) The buyer's profit is increasing in r and her expected profit can be no more than $\frac{r}{v^o}\Pi_I^o$
(2) The supplier's profit is decreasing in r and her expected profit will be no less than $\frac{v^o - r}{v^o}\Pi_I^o$.

While the PARD contract allows for a range of possible reservation fees, the actual fee corresponds to a specific split of the channel surplus between the supplier and buyer. The Theorem shows that as the reservation fee decreases the supplier's share rises. This may seem counter intuitive at first, but a quick reflection would reveal that this is exactly to be expected: each unit of increase in the reservation fee r corresponds to a higher rate of increase in the refund r_2 as $\partial r_2/\partial r = (p-c)/v^o > 1$ since $p > v^o + c$. Thus, the penalty charged for the buyer's unused reservation, $r - r_2$, actually decreases in r.

Note that the above discussion is only relevant when the capacity cost is strictly convex. A majority of the work in the literature assumes linear capacity cost. With linear cost, v, the supplier only stays profitable if w is greater than $v + c$. In such case it is straightforward to see that the proposed contract implies that buyer and supplier's split of channel surplus will be *exactly* $r/v \cdot \Pi_I^o$ and $(v-r)/v \cdot \Pi_I^o$, respectively. Clearly, the supplier will not make a positive profit if $r \geq v$. On the other hand, with the capacity cost being strictly convex, the supplier can still make positive profit when the reservation fee is equal to the marginal capacity cost at k^o. An advantage of the linear capacity cost is that it allows for the design of coordination contracts independent of the buyer's demand distribution. Thus the contract designed for one buyer can be used for any other buyer with a similar cost structure (but possibly with a different demand pattern). However, with convex capacity cost, computing v^o independent of the demand distribution would not be possible.

Conditions (15) and (16) define a continuum of contracts that correspond to varying degree of profit realizations for both parties. On one extreme the supplier collects all of the buyer profits. On the other extreme the buyer takes on all the capacity procurement cost however enjoys all the supplier profits from sales. In selecting the (r, r_2) pair the supplier could make the contract as favorable to the buyer (or herself) as their relative bargaining power dictates. While we do not model the bargaining outcome of the channel surplus (see Wu, 2004), one may think of the reservation fee a lever that the supplier uses to share channel surplus with the buyer, possibly based on the buyer's other capacity options, or the general market conditions. Note that by increasing the reservation fee, it is possible for the supplier to offer not only a full deduction of the fee upon use but also an additional discount (i.e., $r_2 > r$). It may be desirable for the supplier to offer a discount $r - r_2$ when there is a capacity surplus in the industry sector and the capacity (reserved for the buyer) is difficult to convert for other usage. Conversely, when there is a general capacity shortage in the industry, the supplier may wish to decrease r below $(p-w)v^o/(p-c)$, resulting in a no-deduction policy with an additional charge linear to the expected sales amount, *i.e.,* $r_2 < 0$. The surcharge, $r_2 - r$, can be interpreted as the "engineering fee" used in the semiconductor industry that are paid upfront by the buyer in return for a lower reservation fee. Thus, the partial deduction contracts can be viewed as a generalization, of which the no-reservation and reservation-with-full-deduction are both special cases.

If we exclude the consideration of exogenous factors such as market conditions, we may use the insights derived for the fully deductible contract (see Figure 1) on the PARD contract. Specifically, if the buyer's margin is sufficiently high, the supplier should choose r and r_2 in a way that $r > 0 > r_2$ (e.g., the case illustrated in Figure 1.a) or $r > r_2 \geq 0$ (Figures 1.b and c). Otherwise, she may have to select a deduction r_2 that is greater than the reservation fee (e.g., the case Figure 1.d).

5.2. Coordination via Cost-Sharing Contracts

We now consider a *cost-sharing (COSH) contract* where the buyer pays for a portion of the *capacity cost* associated with her reservation. Depending on her demand realization, the buyer either receives a refund or makes an additional payment for the utilized capacity. The *COSH contract* defined here is a generalization of *profit sharing contracts* of Cachon and Lariviere (2000) and Li and Atkins (2002). We propose an extension that the wholesale price cannot be treated as a contract variable (i.e., is exogenous).

We employ an additional modification where the supplier awards the buyer for utilizing the reserved capacity with deductions.

In the following we will describe how the contract coordinates the channel in the particular setting of high-tech capacity reservation. The contracts has the following sequence of events: (1) the supplier announces a reservation fee, $\alpha V(q)$, specified by parameter α ($0 \leq \alpha \leq 1$), and the refund rate r_2 for each unit of capacity actually utilized; (2) given α and r_2, the buyer places her reservation q, paying $\alpha V(q)$; (3) the supplier expands her capacity to q; (4) the buyer's demand x is realized, and the buyer orders x units; and (5) the supplier produces min(x, q) units with marginal cost c. The supplier deducts the amount $r_2 \cdot$ min(x, q) from the buyer's purchasing cost. Under this contract setting the buyer and supplier profits are respectively:

$$\Pi_B^C(\alpha, r_2) = (p - w + r_2)S(q) - \alpha V(q),$$
$$\Pi_S^C(q) = (w - c - r_2)S(q) - (1 - \alpha)V(q).$$

Theorem 8. *Suppose the supplier offers a cost-sharing contract (α,r_2) where*

$$r_2 = \alpha(p - c) - (p - w),\ 0 \leq \alpha \leq 1. \tag{17}$$

Then, the buyer will reserve and the supplier will build the channel optimal capacity, i.e., $q = k^o$. Furthermore, the buyer's profit is increasing and the supplier's profit is decreasing in α.

To prove the above theorem, first observe that under the *COSH contract* the critical fractile of the buyer is equal to that of the integrated channel. Hence, the buyer would indeed reserve the system-optimal capacity. Further, the derivative of the buyer's profit with respect to α is Π_I^o which is strictly positive, implying that buyer's profit is increasing in α. Thus, the supplier's profit is decreasing in α.

This theorem depicts a continuum of contracts where both the buyer's profit, $\alpha\Pi_I^o$, and the supplier's profit, $(1 - \alpha)\Pi_I^o$, are determined by α. Note that depending on α (i.e., for $\alpha < \frac{p-w}{p-c}$), r_2 may assume negative values, implying that the buyer has the option of taking less upfront responsibility in capacity investment (smaller α), but shares her revenue with the supplier (instead of receiving a refund). Observe that this coordination contract is independent of the demand distribution and can be employed for different buyers. As this is true for convex capacity cost in general, the COSH is more robust then the PARD. Comparing to the no-reservation case, we note that α must be greater than the ratio $\Pi_B^0(k^*)/\Pi_I(k^o)$ so that the buyer does not opt out by not reserving. If the buyer has an outside option, α should be increased to counter the competition.

Both PARD and COSH contracts specify a continuum of contracts that allow different allocations of the system surplus between the two players. In fact, a closer examination will reveal that the two contract types can be reduced to each other when the capacity cost is *linear*. Consider a linear capacity cost function with a margin of v. In this case, a COSH contract is identical to a PARD contract where $\alpha = r/v$. However, this is not the case with strictly convex capacity cost. First, as noted earlier, since a

COSH contract keys off the capacity cost function, it is independent of the demand distribution, while in a PARD contract, the supplier needs to compute the marginal cost at k^o each time she prepares a contract for the buyer. In this regard, COSH might be more straightforward to implement. On the other hand, the convex capacity cost brings about disparity in cash flows that would make PARD more appealing for the supplier. This is summarized as follows:

Remark. *For any given PARD and COSH contract pair that generate the same split of integrated channel profits between the players, the supplier receives a higher upfront payment from the buyer under a PARD contract if the capacity cost is strictly convex.*

Proof of this observation is given in the appendix. This result indicates that when both contracts provide the same profits in expectation, the supplier receives more cash under the PARD contract before uncertainty is resolved. Thus, when the capacity cost is increasing in a strictly convex fashion, the supplier may prefer to offer a PARD contract for a more favorable cash flow position.

6. Voluntary Compliance

There are situations in practice where the supplier may have incentive not to comply to the reservation contract. For instance, the supplier may choose to "overbook" (i.e., under-expand) so as to ensure high utilization of her capacity. It is also possible that due to short-term market demand surge and/or manufacturing problems, the supplier is unable to provide the committed capacity at the right time. As is often the case in reality, the buyer only finds out that the supplier can not provide the reserved capacity when the realized demand (i.e., the buyer's firm order) exceeds the supplier's on-hand capacity. In the following, we consider "voluntary compliance" capacity reservation contracts, where the supplier may choose to expand her capacity below the committed level, but is subject to noncompliance penalties. We are interested to know if the PARD and COSH contracts above continue to coordinate the channel under voluntary compliance. Below, we show that in a supplier-lead channel, the supplier has incentives to tailor these contracts to assure compliance and thus coordination.

Suppose the supplier has to pay a noncompliance penalty u for each reserved capacity unit that she fails to provide when requested by the buyer. In this case, given the buyer's reservation quantity, the supplier's profit functions will be

$$\Pi_S^P = (p-c)\frac{v^o - r}{v^o}S(k) - V(k) + rq - u\left((q-k)\overline{F}(q) + \int_k^q (x-k)f(x)dx\right) \text{ and,}$$

$$\Pi_S^C = (1-\alpha)(p-c)S(k) - V(k) + \alpha V(q) - u\left((q-k)\overline{F}(q) + \int_k^q (x-k)f(x)dx\right)$$

under a PARD and a COSH contract, respectively. Since the supplier's noncompliance will not be revealed unless the buyer orders exceed the available capacity, the supplier may have incentive to "take a chance" and build her capacity below the reserved amount if u is sufficiently small. This phenomenon is addressed in the following Theorem:

Theorem 9. *For any given reservation amount, the supplier's capacity choice under a PARD contract will always be strictly below k^o if $u < \frac{p-c}{v^o}r$. Same is true for the COSH contracts when $u < \alpha(p-c)$.*

Theorem 9 underscores the fact that channel coordination cannot be attained under voluntary compliance if the noncompliance penalty is "small." Even if the buyer's reservation amount equals to the system optimal, the supplier's best response is not to deliver all the capacity reserved. However, under complete information the buyer would anticipate this behavior and alter her reservation policy, as stated in the following theorem.

Theorem 10. *The optimal buyer reservation amount in a PARD contract is strictly less than k^o if and only if $u < \frac{p-c}{v^o}r$. Same is true for the COSH contract when $u < \alpha(p-c)$.*

Consider the supplier's profit function and the results of Theorems 9 and 10 suggest that the supplier is actually worse-off by offering a small noncompliance penalty (i.e., the buyer will reserve less, leading to smaller system surplus and smaller supplier profit). In order to encourage the buyer to reserve at a sufficient level (as in the forced compliance case), the supplier must assure the buyer that she will comply by offering sufficiently large noncompliance penalties.

Corollary. *It is optimal for the supplier to offer a noncompliance penalty, u, such that $u = \frac{p-c}{v^o}r$ and $u = \alpha(p-c)$ in the PARD and the COSH contracts respectively. Then, both the reservation amount and the capacity level match the system optimal.*

The proof of the corollary follows directly from the proofs of Theorems 9 and 10.

7. Coordination under Partial Information Update

Up to this point, an implicit assumption of our model is that when the buyer place a firm order, the demand uncertainties are fully resolved. Although this is a common assumption, it may not be true for some high-tech operations such as the semiconductors; here, the production lead-time can be substantial, while the product life-cycle may be relatively short, creating tremendous pressure for the buyer (e.g., an OEM manufacturer) to place her order early. Thus, it is often the case that some level of uncertainty still remains at the time a firm order is placed. In this section, we investigate how the PARD and COSH contracts perform when the demand information is only partially updated. To model partial information update we employ the simple framework introduced by Ferguson *et. al.* (2003), where random variable X representing the final demand is composed of two components, *i.e.*, $X = Y + Z$. It is assumed that Y and Z are independent with continuous distributions F_Y and F_Z respectively. At the time when capacity reservation is committed both values are unknown; after the capacity expansion and right before the buyer places a firm order, the uncertainty represented by Y is resolved. The remaining component, Z, is resolved after the order is completed. We consider a two-stage setting as follows: in the first stage, the supplier chooses her contract parameters, the buyer makes reservation, and the supplier decides on the capacity level. In the second stage (after the capacity is built), the buyer decides how much to order subject to the capacity available. Note that our model differs from Ferguson *et. al.* (2003) in that they focus on the *timing* of (full) capacity commitments in an uncoordinated channel, whereas we are interested in *partial* commitments (reservations) and channel coordination.

We first analyze the problem from the perspective of the integrated channel. Let Q denote the order quantity to be determined in the second-stage for the integrated channel. Given the available capacity, the objective at this stage is

$$\underset{Q\le k}{Max}\ \Pi_I = pE[\min(Q, y+Z)] - cQ - V(k) = pE[\min(Q-y, Z)] + py - cQ - V(k)$$

where y is the observed value of Y. Using the first-order optimality conditions we get

$$Q^o = \min\left(k, y + F_Z^{-1}\left(\frac{p-c}{p}\right)\right) \tag{18}$$

For brevity, let $\Delta = F_Z^{-1}\left(\frac{p-c}{p}\right)$. Thus, we may write the first-stage decision problem as follows:

$$\underset{k}{Max}\ \Pi_I = pE[\min(k, Y+Z, Y+\Delta)] - cE[\min(k-\Delta, Y)] - c\Delta - V(k)$$

Let μ_Y be the expected value of Y. Then,

$$\begin{aligned}
&E[\min(k, Y+Z, Y+\Delta)] = \mu_Y + E[\min(k-Y, Z, \Delta)] \\
&= \mu_Y + \Delta F_Y(k-\Delta)\overline{F}_Z(\Delta) + \int_0^{\Delta} tF_Y(k-t)f_Z(t)dt + \overline{F}_Z(\Delta)\int_{k-\Delta}^{\infty}(k-t)f_Y(t)dt \\
&+ \int_0^{\Delta}\int_{k-s}^{\infty}(k-t)f_Y(t)dt f_Z(s)ds
\end{aligned} \tag{19}$$

Using (19) and taking the first derivative of Π_I with respect to k we get

$$\frac{\partial \Pi_I}{\partial k} = (p-c) - p\int_0^{\Delta} F_Y(k-t)f_Z(t)dt - V'(k) \tag{20}$$

From (20), it is clear that the profit function is concave and thus, there exists a unique optimal capacity, k^o.

Next we investigate how the contracts introduced in Section 5 can be tailored for coordination under this setting. First consider the PARD contracts. Given the reservation amount q, the buyer's objective when placing a firm order can be written as follows:

$$\underset{Q\le q}{Max}\ \Pi_B = pE[\min(Q, y+Z)] - wQ - rq + r_2Q \tag{21}$$

For the reserving stage we get

$$\underset{q}{Max}\ \Pi_B = pE[\min(q, Y+Z, Y+\Delta)] - (w-r_2)E[\min(q-\Delta, Y)] - (w-r_2)\Delta - rq \tag{22}$$

In a similar way we can write down the buyer objectives in the COSH contract simply by substituting rq in (21) and (22) with $V(q)$. A straightforward analysis of the first-order conditions of both objectives will show that neither contract forms could achieve coordination. The only exception is the trivial case when the supplier sells the product for free and the reservation fee is set at the marginal capacity cost at

k^o. Next, we show that channel coordination *can* be achieved if the proposed mechanisms are coupled with a buy-back agreement. Suppose under a PARD (COSH) contract, in addition to r (α) and r_2, the supplier introduces a new term specifying a buy-back price, b, to be paid by the supplier to the buyer for each ordered yet *unsold* products. Thus, the contract specifies that the buyer reserves capacity q by paying rq ($\alpha V(q)$ in COSH). Once the capacity is built and the demand information updated, the buyer places a firm order and pays $w - r_2$ for each unit. After the order is filled and delivered to the buyer, actual demand is realized. Finally, the supplier pays b to the buyer for each unsold unit. The timeline of these events are depicted in Figure 2. Under this arrangement, the buyer mitigates the supplier's downside capacity risk, while the supplier shares the buyer's overstocking risk. The following theorem shows that such synthesis between PARD or COSH contracts and buy-back agreements will coordinate the channel.

Theorem 11. *Suppose the supplier specifies a buy back price, b, to the capacity reservation contract . Then, the coordination mechanisms defined by (15) and (17) coordinate the channel if and only if*

$$b = p\frac{v^o - r}{v^o}. \tag{23}$$

In both types of contracts (PARD and COSH) the supplier (buyer) profits decrease (increase) in r.

<<Insert Figure 2 here>>

The Theorem concludes that the coordination mechanisms introduced in Section 5 can be indeed generalized to the partial information update case. Observe that for a fixed reservation fee, while the deductible fee is decreasing in the capacity cost, the buy-back price is increasing. However the rate of increase in b (*i.e.,* r/v^{o2}) is less than the rate of decrease in r_2 (*i.e.,* $(p-c)/v^{o2}$) implying that when the capacity becomes more expensive the buyer's risk share must increase. In summary, a complete PARD contract composed of (r, r_2, b) and/or COSH contract of (α, r_2, b) specified by (15), (17), and (23) achieve coordination under the generalized settings. We submit that this family of contracts uniquely satisfy the business environments of high-tech manufacturing, providing coordination mechanisms that are efficient, flexible, and versatile.

7. Conclusions

This paper examines capacity reservation contracts in the context of high-tech manufacturing. We propose reservation contracts with *deductible reservation fees* under exogenous wholesale price*:* the supplier announces a fee for capacity to be reserved for a certain time in the future, the fee is later deducted from the wholesale price for each unit of reserved capacity that is utilized by the buyer, the buyer places her reservation for capacity based on this reservation fee, and later, the supplier determines how much capacity to build. After some or all uncertainty around demand is resolved, the buyer utilizes the reserved capacity.

With the capacity reservation contracts, the buyer mitigates the supplier's capacity expansion risk by *partially* committing to utilize it. The commitment is partial in the sense that it only require a portion of the wholesale price as the reservation fee. The main motivation for the buyer to place reservation is to create incentives for the supplier to expand capacity more aggressively. We examine various implications of the contract concerning individual rationality and channel coordination, and we consider different compliance regimes and partial information update. Our analysis seeks to answer the following managerial questions:

1. Is it always profitable for the supply chain partners to enter a capacity reservation agreement? If not, what are the conditions that make capacity reservation appealing for both parties? What do the profit margins and the market conditions affect the players' incentives?

2. How can the capacity reservation contracts be tailored to coordinate the supply channel when the wholesale price is exogenously determined? How does the cost structure of capacity expansion influence the supplier's contract selection?

3. How does the compliance regime impact the supplier's and buyer's incentives? Can the channel coordination be achieved under voluntary compliance?

4. How can one adjust the contracts for coordination when the demand information is *partially* – instead of fully – updated at the time the buyer places a firm order? How do the players' incentives change under partial information update?

In the following, we summarize key managerial insights under each of the above questions.

When is Capacity Reservation Beneficial? Without the presence of buyer commitment, the supplier would simply determine her capacity based on her knowledge of the demand, i.e., she would find the optimal capacity level based on a newsvendor-type decision model. We show that the supplier and the buyer could benefit from early commitment contracts under a specific set of conditions. First of all, the buyer's expected order size should be larger than the supplier's newsvendor capacity. Second, the buyer should be able to negotiate a threshold reservation fee (Theorem 1) below which her expected profit justifies her added liability. We also identify the conditions when it is beneficial for the supplier to accept the buyer's threshold requirement (Theorem 2). The basic insight here is that reservation is beneficial when the buyer's revenue margin is sufficiently high (beyond a certain threshold value, as specified in Theorem 3). We show that as the buyer's revenue margin decreases, the supplier will face a sequence of four scenarios with decreasing level of attractiveness (Figure 1 and Theorem 4). We observe that if the capacity cost is linear (e.g., outsourcing) neither the reservation fee nor the supplier's surplus are dependent on the market size. Intuitively, the increase in the capacity cost due to market size is balanced with the reduction in risk. However, if the capacity cost is convex (e.g., physical expansions), the reduction in risk is not sufficient to justify the increase in capacity cost (Theorem 5). Thus, the threshold reservation fee is increasing in market size. It should not be surprising that there exists a market-size threshold, above which capacity reservation (as defined in this context) is no longer favorable. Our results reveal that it is the demand variability that affects the reservation fee (Theorem 6).

Channel Coordination. The next research question is the design of contract mechanisms that would capture the potential benefits of capacity reservation. We first observe that, except for a special case, contracts with *fully deductible* or *non-deductible* fees both generate surplus that is suboptimal (for the channel). We propose two coordination contracts: first, the *partially deductible reservation* where a pre-specified portion of the reservation fee is deductible from the final payment. The deductible fee (refund) is a derived function of the reservation fee to achieve channel coordination (Theorem 7). We show that *partially deductible* (PARD) contract allows a range of possible reservation fees, each corresponding to a different split of the channel surplus between the supplier and buyer. An interesting finding here is that the supplier's share actually *decreases* in the reservation fee. This is due to fact that each unit of increase in the reservation fee corresponds to a higher rate of increase in the refund, thus, the penalty charged for the buyer's unused reservation decreases in the reservation fee. The second coordination mechanism is the *cost-sharing (COSH)* contract*,* where the buyer pays for a portion of the *capacity cost* associated with her reservation (Theorem 8). Depending on her demand realization, the buyer either receives a refund, or makes additional payment for the capacity utilized. We show that while the two contract schemes generate the same expected profits for the supplier, she is in a better cash flow position under the PARD contract if the capacity cost is strictly convex. However, the COSH contracts are less complex as the buyer's reservation fee is proportional to the capacity cost and the contract parameters are independent from the demand distribution.

Voluntary Compliance. Under voluntary compliance, the supplier can choose not to comply with the contract and pay a noncompliance penalty. We show that if the penalty is set too low, the buyer will not be able to trust the supplier to come through with the reserved capacity (Theorems 9 and 10) and will reserve less than optimal. It is shown that the supplier has incentives to set the noncompliance penalty sufficiently large so as to induce the buyer to behave optimally as in the forced compliance case. We derive the noncompliance penalty level that will coordinate the supply channel.

Partial Information Update on Demand. We consider a generalized case on demand information update, where the buyer has to place a firm order before the demand uncertainty is completely resolved. In this case, the buyer must take into account the partial information (forecast) update and decides on her order quantity subject to available capacity. It is shown that the contractual schemes proposed under the full information update case can no longer coordinate the channel. Nonetheless, by adding a buy-back agreements (Theorem 11), both the PARD and the COSH contracts will achieve channel coordination.

In conclusion, we submit that the various capacity reservation contracts along with their extensions concerning channel coordination, compliance regime, and information update are uniquely suited for the high-tech business environments. Our treatment of the wholesale price is particularly relevant in this environment where the wholesale price negotiation is independent from the capacity reservation decisions. An interesting extension would be to develop a three-stage decision model where the wholesale price negotiation, capacity reservation, and demand realization form the main decision stages.

APPENDIX

Proof of Theorem 1: It is individually rational for the buyer to make a reservation if $\Pi_B(q) \geq \Pi_B^0(k^*)$. The buyer's optimal reservation quantity *q* given reservation fee *r* is given in (6). Thus, r expressed in terms of the reservation quantity q is:

$$r = (p-w)\frac{\overline{F}(q)}{F(q)} \tag{A1}$$

From (5) and (A1), we may express the buyer's profit after reserving q as follows:

$$\Pi_B(q) = (p-w)S(q) - (p-w)\frac{\overline{F}(q)}{F(q)}\int_0^q F(x)dx = (p-w)\frac{\int_0^q xf(x)dx}{F(q)} \tag{A2}$$

First consider the definitions of $\Pi_B^0(k^*)$ and $\Pi_B(q)$ in (3) and (A2), respectively. We know that $\Pi_B(q) = \Pi_B^0(k^*)$ when the equality in (8) holds. Thus, if the buyer's optimal reservation quantity is at the threshold q^t, she is indifferent between reserving and not reserving. Observe from (A1) and (A2) both q and $\Pi_B(q)$ are strictly decreasing in r suggesting that the buyer indeed has incentive to reserve when $r < r^t$. Clearly for any $r < r^t$, $q > q^t$.

To prove property (1), observe that when $q^t = k^*$ the right-hand-side of equality (8), $\Pi_B(q^t)/(p-w)$, is less than the left-hand-side *S(*k^**)*. Again, since $\Pi_B(q)$ is strictly increasing (decreasing) in $q(r)$, q^t must be strictly greater than k^*. Property (2) follows directly from (A1).

Now consider Property (3). Recall that we derived the upper bound for the reservation fee (7) from the observation that the buyer would reserve no less than the supplier's newsvendor quantity k^*. Since property (1) states that $q^t > k^*$, and the fact that (A1) is decreasing in q, r^t must be a tighter upper bound, i.e., $r^t < (p-w)\frac{v^*}{w-c-v^*}$. □

Proof of Theorem 4. Let's first show that $\Pi_S(q)$ is decreasing in $[k^o, \infty)$. The first derivative of $\Pi_S(q)$ with respect to q can be written as follows:

$$\frac{\partial \Pi_S(q)}{\partial q} = \Pi_S'(q) = (p-c)\overline{F}(q) - V'(q) - (p-w)\frac{f(q)}{F^2(q)}\int_0^q F(x)dx$$

Note that the foregoing function is equivalent to:

$$\Pi_S'(q) = \Pi_I'(q) - (p-w)\frac{f(q)}{F^2(q)}\int_0^q F(x)dx \tag{A3}$$

Since $\Pi_I(q)$ is concave in q and k^o is the unique maximum, $\Pi_I'(q) \leq 0$ if $q \geq k^o$, and the second term is positive, thus for any q in $[k^o, \infty)$, the right hand side returns a negative value, i.e., $\Pi_S(q)$ is strictly decreasing in q in $[k^o, \infty)$.

From the supplier's decision problem (12), we know that $q^r \geq$ Max $(q^t, q^w) \equiv q^m$. If $q^m \geq k^o$ then $q^r = q^m$. If $q^m < k^o$, the maximizing point is in $[q^m, k^o)$. Since $\Pi_S(q)$ is compact and continuous within this interval, from Weierstrass' Theorem we know there exists a maximum solution. If there is no stationary point in $[q^m, k^o)$, then $q^r = q^m$. Otherwise, there is a stationary point, q^* that would be the unique maximizer for Π_S. Moreover, $q^m \leq q^* < k^o$. First note that, q^* must satisfy the first order optimality condition:

$$(p-c)\overline{F}(q^*) - V'(q^*) = (p-w)\frac{f(q^*)}{F^2(q^*)}\int_0^{q^*} F(x)dx \tag{A4}$$

The second derivative for $\Pi_S(q)$ is as follows:

$$\frac{\partial^2 \Pi_S(q)}{\partial q^2} = \Pi_I''(q) - (p-w)\frac{1}{F(q)}\left(f(q) + \frac{\int_0^q F(x)dx}{F(q)}\left(f'(q) - 2\frac{f^2(q)}{F(q)}\right)\right) \tag{A5}$$

Since $F(x)$ is an IFR distribution, we have

$$f'(x) \geq -\frac{f^2(x)}{\overline{F}(x)} \tag{A6}$$

Define a new function $H_1(q)$ by replacing $f'(q)$ in (A5) with $-f^2(q)/\overline{F}(q)$. After some manipulation $H_1(q)$ can be written as follows:

$$H_1(q) = \Pi_I''(q) - (p-w)\frac{f(q)}{F(q)}\left(1 - \left(\frac{1+\overline{F}(q)}{\overline{F}(q)}\right)\frac{f(q)}{F^2(q)}\int_0^q F(x)dx\right)$$

From (A6) we notice that $H_1(q) \geq \Pi_S''(q)$ and thus if $H_1(q) < 0$ at q, the second derivative of $\Pi_S(q)$ is negative. Using (A4) we can write $H_1(q^*)$ as

$$H_1(q^*) = \Pi_I''(q^*) - \frac{f(q^*)}{F(q^*)}\left((p-w) - \left(\frac{1+\overline{F}(q^*)}{\overline{F}(q^*)}\right)\Pi_I'(q^*)\right)$$

The foregoing function can be reduced to

$$H_1(q^*) = \tfrac{f(q^*)}{F(q^*)}\left(\Pi_I'(q^*) + (p-c)\overline{F}(q^*) - \tfrac{V'(q^*)}{\overline{F}(q^*)} - (p-w)\right) - V''(q^*) \tag{A7}$$

Let $H_2(q^*)$ denote the term inside the parenthesis in (A7). It is sufficient to show that q^* is a local maximum (i.e., $\Pi_S''(q^*) < 0$) if $H_2(q^*)$ is non-positive. Since $q^* \geq q^m > k^*$, from (2), $V'(q^*)/\overline{F}(q^*) > (w-c)$. Define $H_3(q^*)$, by replacing $V'(q^*)/\overline{F}(q^*)$ with $w-c$ in $H_2(q^*)$, thus

$$H_3(q^*) = (p-w)\overline{F}(q^*) - (p-c)F(q^*).$$

Note that $H_3(q^*) > H_2(q^*)$. Obviously if

$$(p-c) \geq (p-w)\frac{\overline{F}(q^*)}{F(q^*)}, \tag{A8}$$

then $H_3(q^*) \leq 0$ implying that $H_1(q^*)$ is negative, and q^* is a local maximum. Note that the right hand side of the foregoing inequality is $r(q^*)$. Since the capacity reservation price decreases in q, if (A8) holds, any $q^* \geq q^m$ has to be a local maximum, i.e., there exists at most one local maximum in (q^m, ∞). Suppose that inequality in (A8) does not hold, *i.e.*, $(p-c) < r(q^*)$, observe from (A4) that $(p-c)\overline{F}(q^*) > V'(q^*)$ implying that $r(q^*) > V'(q^*)$ as well. In other words at q^* the reservation price should be strictly greater than the marginal cost of capacity. Define $H_4(q^*)$ by replacing the last term in $H_2(q^*)$ with $V'(q^*)\frac{F(q^*)}{\overline{F}(q^*)}$ and then $V'(q^*)/\overline{F}(q^*)$ with $(w-c)$. Thus, $H_4(q^*) = 2\big((p-c)\overline{F}(q^*) - (w-c)\big)$. Since $r(q^*) > V'(q^*)$ and $w-c < V'(q^*)/\overline{F}(q^*)$,

$H_4(q^*) > H_1(q^*)$. The constraints in (12) enforce that $w \geq r(q^*)$ implying that $w/p \geq \overline{F}(q^*)$. Thus, obviously $(w-c)/(p-c) \geq \overline{F}(q^*)$ as well. Therefore if $(p-c) < r(q^*)$ then $H_4(q^*) \leq 0$. Since $H_4(q^*) > H_1(q^*)$, the second derivative at q^* must be negative. This shows that the second derivative of $\Pi_S(q)$ in $[q^m, k^o)$ at any stationary point is negative, i.e., there exists at most one stationary point in $[q^m, k^o)$, and if one exist it is the only local maximum. □

Proof of Theorem 5. To prove Part (1) we need to show that $\Pi_S'(\mathrm{q}^t) < 0$. To do this we will use the relationship between q^t and k^* defined in (8). Define $L(q^t, k^*)$ as follows:

$$L(q^t, k^*) = S(k^*) - \frac{\int_0^{q^t} xf(x)dx}{F(q^t)}$$

From (8) we know that $L(q^t, k^*) = 0$. Thus, the derivative of this function with respect to k^* must be 0 as well. Hence, from the chain rule

$$\frac{\partial L(q^t, k^*)}{\partial k^*} = \overline{F}(k^*) - \frac{f(q^t)}{F^2(q^t)} \int_o^{q^t} F(x)dx \cdot \frac{\partial q^t}{\partial k^*} = 0$$

Note also that derivative with respect to θ should be 0. That is,

$$\frac{\partial L(q^t, k^*)}{\partial \theta} = \overline{F}(k^*)\left(1 - \frac{\partial k^*}{\partial \theta}\right) - \frac{f(q^t)}{F^2(q^t)} \int_o^{q^t} F(x)dx \cdot \left(1 - \frac{\partial q^t}{\partial k^*}\frac{\partial k^*}{\partial \theta}\right) = 0 \tag{A9}$$

A straightforward analysis of (A9) will show that

$$\overline{F}(k^*) = \tfrac{f(q^t)}{F^2(q^t)} \int_o^{q^t} F(x)dx \tag{A10}$$

and $\partial q^t / \partial k^* = 1$ for any distribution that is from a shifted family. Thus,

$$\overline{F}(q^t) < \tfrac{f(q^t)}{F^2(q^t)} \int_o^{q^t} F(x)dx \tag{A11}$$

since $k^* < q^t$. We may write the first derivative of the supplier's profit function evaluated at q^t:

$$\Pi_S'(\mathrm{q}^t) = \big((w-c)\overline{F}(q^t) - V'(q^t)\big) + (p-w)\left(\overline{F}(q^t) - \frac{f(q^t)}{F^2(q^t)} \int_o^{q^t} F(x)dx\right)$$

Note that the term inside the first parenthesis is negative since $q^t > k^*$. From (A11), the second parenthesis is negative as well, thus $\Pi_S'(\mathrm{q}^t) < 0$. Since Π_S is unimodular in q (from Theorem 4), it is decreasing in $[q^m, k^o)$. Thus, the optimal quantity q^r should be at the boundaries of the feasible region, i.e., $q^r = Max\ (q^t, q^w)$, as shown in Theorem 4, *Part (3)-(i)*. From (9), we may conclude that $r^* = min(r^t, w)$.

To prove Part (2), first consider the threshold reservation fee with respect to θ, we have :

$$\frac{\partial r^t}{\partial \theta} = (p-w)\frac{f(q^t)}{F(q^t)}\left(1 - \frac{\partial q^t}{\partial k^*}\frac{\partial k^*}{\partial \theta}\right) \tag{A12}$$

When the capacity cost is *linear*, it can be verified using (2) that $\partial k^*/\partial \theta = 1$ for any distribution in the *shifted family*. From Part (1), we have $\partial q^t / \partial k^* = 1$. Thus, $\frac{\partial \mathrm{r}^t}{\partial \theta}$*=0.* Thus, we can conclude that r^* is independent of the market size. However, from (9) we can derive that $F(q^t - \theta|0) = (p-w)/p - w + r^t$. Since r^t and thus the foregoing ratio does not change in θ, clearly the quantity q^t must be increasing in market size θ.

To show that the supplier's surplus is not influenced by the market size, look at the first derivative of $(\Pi_S(q^{\mathrm{r}}) - \Pi_S^0(k^*))$ with respect to θ. Let $\Psi_\theta(q^t(k^*), k^*)$ denote this derivative for given q^t and k^*. Then, from the envelop theorem and the chain rule we can write the following;

$$\Psi_\theta = \frac{\partial\left(\Pi_S(q^r) - \Pi_S^0(k^*)\right)}{\partial\theta} = (w-c)\overline{F}(k^*) - (p-c)\overline{F}(q^t) + (p-w)\frac{f(q^t)}{F(q^t)}\int_o^{q^t} F(x)dx + \Pi_S'(q^t)\frac{\partial q^t}{\partial k^*}\frac{\partial k^*}{\partial\theta}.$$

Observe that since both $\partial q^t/\partial k^*$ and $\partial k^*/\partial\theta$ are equal to 1, thus $\Psi_\theta(q^t(k^*), k^*)$=0, i.e., the supplier's surplus does not change with market size θ.

Now consider Part (3). From (2), it is easy to see that if the capacity cost is strictly convex $\partial k^*/\partial\theta < 1$, but $\partial q^t/\partial k^* = 1$. From (A12), it follows that the reservation fee is increasing in θ. Further, from the envelop theorem, we can write the marginal change in surplus (as induced by the reservation) as $\Psi_\theta(q^w, k^*) = (w-c)\overline{F}(k^*) - V'(q^w)$. From the first order optimality condition we know that if $q^w > k^*$, then $\Psi_\theta < 0$, i.e., the value of reservations decreases in θ. Since the increment in k^* is less than the increment in q^w when θ increases, q^w will exceed k^* eventually, i.e., there is a market size threshold, above which the surplus generated by reservation decreases. □

Proof of Theorem 6. From (2) we know that $\overline{F}\,(k^*) - \dfrac{V'(k^*)}{w-c} = 0$

Differentiate the foregoing equality with respect to the standard deviation, σ, we have:

$$f(k^*)\left(\frac{k^*-\mu}{\sigma} - \frac{\partial k^*}{\partial\sigma}\right) = \frac{V''(k^*)}{w-c}\frac{\partial k^*}{\partial\sigma}$$

This implies that k^* increases (decreases) in σ when $k^* > \mu$ ($k^* < \mu$). Thus, $\left|\dfrac{\partial k^*}{\partial\sigma}\right| < \left|\dfrac{k^*-\mu}{\sigma}\right|$(A13)

Using the chain rule, we have $\dfrac{\partial r^t}{\partial\sigma} = (p-w)\dfrac{f(q^t)}{F^2(q^t)}\left(\dfrac{q^t-\mu}{\sigma} - \dfrac{\partial q^t}{\partial k^*}\dfrac{\partial k^*}{\partial\sigma}\right)$

As Normal distribution is from the *shifted family*, from Theorem 5, we have $\partial q^t/\partial k^*$=1. Thus, from (A13) and the fact that $q^t > k^*$, the term within the parenthesis in the above expression returns a positive value, i.e., the threshold reservation fee r^t increases in σ.

Now consider the channel surplus at q^t, which can be written as follows using (8)

$$\Pi_I(q^t) - \Pi_I(k^*) = (p-c)\frac{\overline{F}(q^t)}{F(q^t)}\int^{q^t} F(x)dx - V(q^t) + V(k^*)$$

Using (A10), the partial derivative of $\Pi_I(q^t) - \Pi_I(k^*)$ with respect to σ can be written as:

$$-(p-c)\int^{k^*}\frac{x-\mu}{\sigma}f(x)dx + \left((p-c)\left(\overline{F}(q^t) - \frac{f(q^t)}{F^2(q^t)}\int^{q^t}F(x)dx\right) - V'(q^t) + V'(k^*)\right)\frac{\partial k^*}{\partial\sigma}$$

The first term is positive. Since $V'(k^*) \geq \frac{1}{2}(w-c)$, $\partial k^*/\partial\sigma$ is negative. Thus, from (A11) and $q^t > k^*$ the second term is positive. In other words, the surplus created through reservations increases in the standard deviation. □

Proof of Theorem 7. Since the deductible reservation fee is $r_2 = \frac{p-c}{v^o}r - (p-w)$, for all r the critical fractile faced by the buyer is equivalent to that of the integrated channel. Thus, the buyer reservation equals the channel optimum capacity. From the results of section 4.1, the supplier's best response is to expand capacity to k^o. Now consider the buyer and the supplier's profit as a function of the reservation fee as follows:

$$\Pi_B(r,\, r_2) = (p - w + r_2)S(k^o) - rk^o = r\Big(\frac{p-c}{v^o}S(k^o) - k^o\Big) \le \frac{r}{v^o}\Pi_I^o.$$

$$\Pi_S(r,\, r_2) = (p-c)\frac{v^o - r}{v^o}S(k^o) - V(k^o) + rk^o \ge \frac{v^o - r}{v^o}\Pi_I^o.$$

The derivative with respect to r is positive for Π_B, and negative for Π_S, indicating that the buyer's profit increases in r whereas the supplier's profit decreases in r (Parts (1) and (2)). The inequalities are from the fact that $v^o k^o \ge V(k^o)$ for a convex increasing function. □

Proof of Theorem 9. First consider the PARD contracts. Given the reservation amount q, the supplier's last stage capacity decision problem is

$$\underset{k}{Max}\,(p-c)\frac{v^o - r}{v^o}S(k) - V(k) + rq - u\Big((q-k)\overline{F}(q) + \int_k^q (x-k)f(x)dx\Big)$$

Taking the first derivative of the profit function we get

$$\big((p-c)\tfrac{v^o - r}{v^o} + u\big)\overline{F}(k) - V'(k)$$

It is straightforward to see that the foregoing function returns a negative value at k^o when $u < \frac{p-c}{v^o}r$ implying (from concavity) that the supplier's optimal capacity choice is strictly less than system optimal. Observe from (17) that the supplier's profit function in the COSH contract is

$$\Pi_S = (1-\alpha)(p-c)S(k) - V(k) + \alpha V(q) - u\Big((q-k)\overline{F}(q) + \int_k^q (x-k)f(x)dx\Big),$$

and the first derivative with respect to k is

$$((1-\alpha)(p-c) + u)\overline{F}(k) - V'(k)$$

Observe that the first derivative is negative at k^o if $u < \alpha(p-c)$. □

Proof of Theorem 10. Under full information, the buyer will be able to predict the supplier's best response to her reservation decision. Under voluntary case she will expect that if the noncompliance penalty is sufficiently small, the supplier will not completely build her reservations. Considering this, her profit function in the PARD will be

$$\Pi_B = (p-c)\frac{r}{v^o}S(k) - rq + u\Big((q-k)\overline{F}(q) + \int_k^q (x-k)f(x)dx\Big),$$

and the derivative with respect to q at k^o will be $u\overline{F}(k^o) - r$, which is clearly negative when $u < \frac{p-c}{v^o}r$. Under the COSH,

$$\Pi_B = \alpha(p-c)S(k) - \alpha V(q) + u\big((q-k)\overline{F}(q) + \textstyle\int_k^q (x-k)f(x)dx\big),$$

and the first derivative is $u\overline{F}(k^o) - \alpha v^o$ which is also negative if $u < \alpha(p-c)$. Consequently the optimal reservation amount for the buyer is strictly less than k^o. □

Proof of Theorem 11. Consider the PARD. The buyer's last stage objective with (15) and (23) is

$$\underset{Q\le q}{Max}\ \Pi_B = pE[\min(Q, y+Z)] - rq - \left(p - (p-c)\frac{r}{v^o}\right)Q + p\frac{v^o - r}{v^o}(Q - y - Z)^+$$

It is easy to verify concavity. Observe that the first order optimality condition is identical to that of the integrated channel. Now, we write the buyer's objective for the reserving stage:

$$\Pi_B = pE[\min(q, Y+Z, Y+\Delta)] - rq - \left(p - (p-c)\frac{r}{v^o}\right)E[\min(q, Y+\Delta)]$$
$$+ p\frac{v^o - r}{v^o}(\min(q - Y, \Delta) - Z)^+. \tag{A14}$$

Then the first derivative with respect to q is

$$\Pi_B' = (p-c) - p\int_0^{\Delta} F_Y(q-t) f_Z(t)dt - v^o$$

Notice from (20) that the foregoing function returns zero at the channel optimal capacity level, k^o. From the derivative of (A14) with respective to r and the Envelop Theorem, optimal buyer profits increase in r implying the opposite for the supplier. The proof for the COSH can be completed by using the same analogy followed above. □

Cash Flow Comparison Between PARD and COSH Contracts

Consider any ($\widehat{r}$, $\widehat{\alpha}$) pair that generate the same expected profits for the buyer under both the partially-deductible and the cost sharing contracts. Then, we get

$$\frac{\widehat{r}}{v^o}((p-c)S(k^o) - v^o k^o) = \widehat{\alpha}((p-c)S(k^o) - V(k^o)).$$

Observe that $v^o k^o > V(k^o)$ for $k^o > 0$ since $V(k)$ is a strictly increasing convex function in $[0, \infty)$. Hence, the above equality implies that $\widehat{r} > \widehat{\alpha} v^o$. The total reservation fee to be paid to the supplier upfront is $\widehat{r}k^o$ and $\widehat{\alpha}V(k^o)$ under the PARD and the COSH contracts respectively. From the foregoing observation $\widehat{r}k^o > \widehat{\alpha}v^o k^o > \widehat{\alpha}V(k^o)$ implying that the reservation payment is higher under the PARD contract. □

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REFERENCES

Araman, V., J. Kleinknecht, and R. Akella. 2003. Coordination and risk sharing in e-business. Working paper, Stanford University, CA.

Barnes-Schuster, D., Y. Bassok, R. Anupindi. 2002. Coordination and flexibility in supply contracts with options. *M&SOM*. 4(3) 171-207.

Brown, A., and H. Lee. 1998. Optimal "pay-to-delay" capacity reservation with application to the semiconductor industry. Working paper, Vanderbilt University, Nashville, TN.

Bonser, J. and S. D. Wu. 2001. Procurement planning to maintain both short-term adaptiveness and long-term perspective, *Management Science,* 47 769-786.

Cachon, G. 2003. Supply chain coordination with contracts. S. Graves, T. de Kok (eds). *Handbooks in Operations Research and Management Science: Supply Chain Management*, Elsevier, The Netherlands.

Cachon, G., M. A. Lariviere. 2000. Supply chain coordination with revenue sharing: strengths and limitations. Working paper, University of Pennsylvania, Philadelphia, PA.

Cachon, G. and M. A. Lariviere. 2001. Contracting to assure supply: how to share demand forecast in a supply chain. *Management Science.* 47(5) 629-646.

Donohue, K. L. 2000. Efficient supply contracts for fashion goods with forecast updating and two production modes. *Management Science.* 46(11) 1397-1411.

Eppen, G. and A, Iyer. 1997. Backup agreements in fashion buying - the value of upstream flexibility. *Management Science*. 43(11) 1469-1484.

Ferguson, M. E., A. D. Gregory, and P. H. Zipkin. 2003. Commitment decisions with partial information updating. Working paper, Georgia Tech., GA.

Golovachkina, N. and J. Bradley. 2002. Supplier-manufacturer relationships under forced-compliance contracts. Working paper, Cornell University, NY..

Gurnani, H. and C. Tang. 1999. Optimal ordering decisions with uncertain cost and demand forecast updating. *Management Science*. 45 (10) 1456-1462.

Huang, H., S. P. Sethi, and H. Yan. 2002. Purchase contract management with demand forecast updated. Working paper, University of Texas at Dallas, TX.

Jain, K., E. A. Silver. 1995. The single period procurement problem where dedicated supplier capacity can be reserved. *Naval Research Logistics.* 42 915-934.

Jin, M. and S. D. Wu. 2001. Modeling capacity reservation in high-tech manufacturing. Working paper, Lehigh University, PA.

Li, Q. and D. Atkins. 2002. Coordinating replenishment and pricing in a firm. *MSOM*. 4(4) 241-257.

Serel, D. A., M. Dada, H. Moskowitz. 2001. Sourcing decisions with capacity reservation contracts. *EJOR.* 131 635-648.

Silver, E. A., K. Jain. 1994. Some ideas regarding reserving supplier capacity and selecting replenishment quantities in a project context. *Int. J. Production Economics.* 35 177-182.

Spinler, S., A. Huchzermeier, and P. R. Kleindorfer. 2002. The valuation of options on capacity. Working paper, Otto-Beisheim Graduate School of Management, WHU, Germany.

Tomlin, B. 2003. Capacity investments in supply chains: sharing the gain rather than sharing the pain. *MSOM.* 5(4), 317-333.

Wu, S.D. 2004. Supply chain intermediary: a bargaining theoretic framework, in *Handbook of Quantitative Supply Chain Analysis*, (D. Simchi-Levi, S. D. Wu, and M. Shen, eds.), Kluwer Academic Publishers, Norwell, MA.

Wu, D. J., P.R. Kleindorfer, J. E. Zhang. 2002. Optimal bidding and contracting strategies for capital-intensive goods. *EJOR*. 137 657-676.

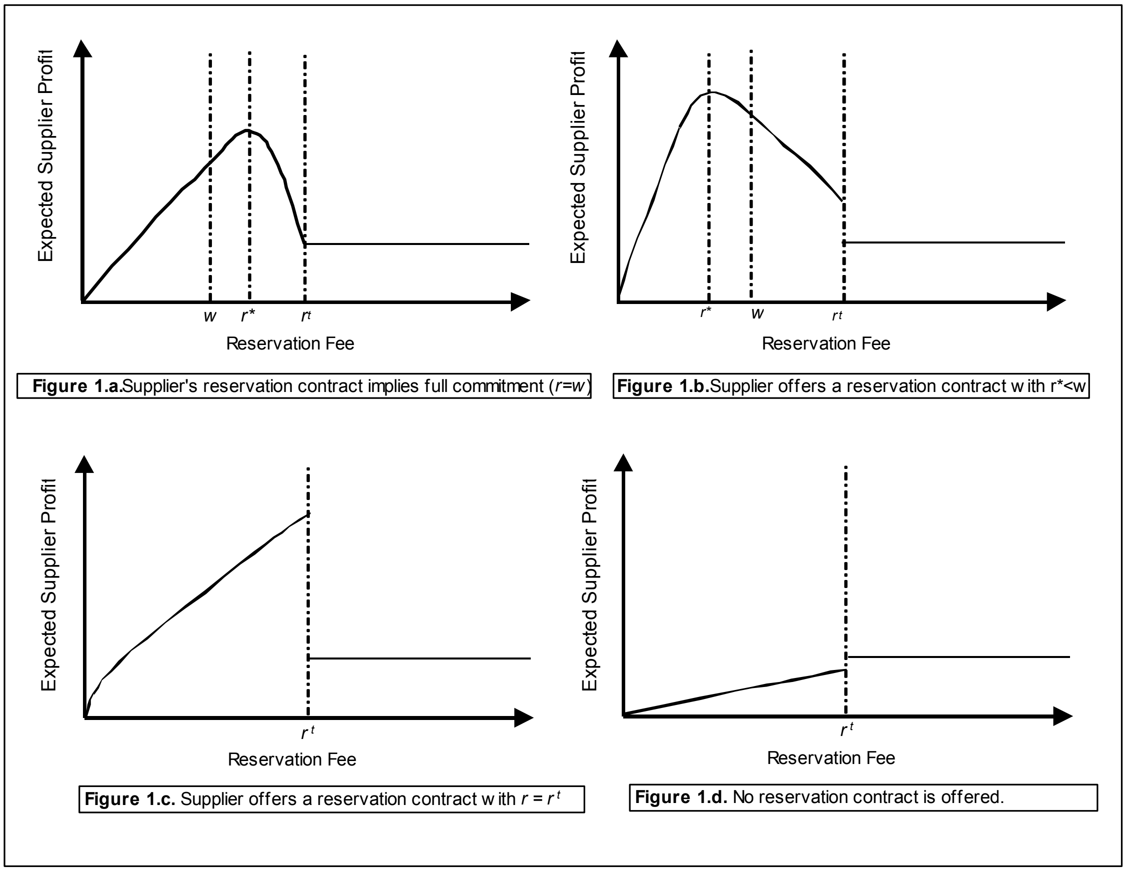

Figure 1.a. Supplier's reservation contract implies full commitment (*r=w*)

Figure 1.b. Supplier offers a reservation contract with r*<w

Figure 1.c. Supplier offers a reservation contract with $r = r^t$

Figure 1.d. No reservation contract is offered.

Figure 1. Possible outcomes of supplier's expected profit function

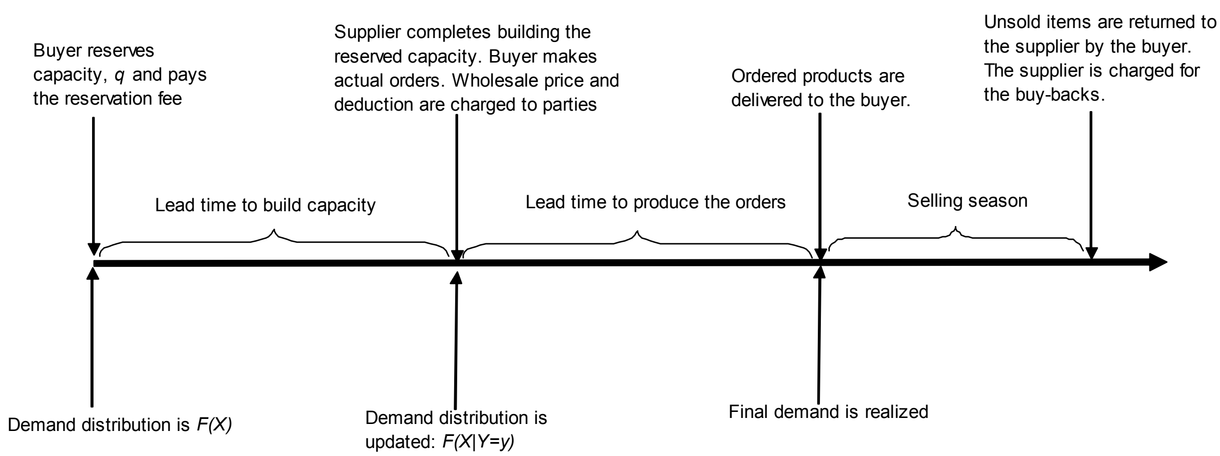

Figure 2. Timeline for the coordination contracts with buy-backs