Competitive Equilibrium Analysis for Repeated Procurement Auctions

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In this study, we conduct a competitive equilibrium analysis for the repeated (sequential) procurement auctions. We consider capacitated suppliers (bidders), each with a U-shaped cost function that captures the economies (and dis-economies) of scale in bidding quantity. Cases with both homogenous and non-homogenous bidders are considered in a symmetric incomplete information setting. First we analyze a pair of bidders and derive their expected winning probabilities based on the way their cost functions interact. We then try to generalize the two-bidder settings to N-bidder settings. We derive key mechanism design results for this repeated auction using Myerson's framework.

Key words: Repeated (Sequential) Procurement Auctions; Competitive Bidding; U-shaped Cost Functions; Mechanism Design.

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1. Introduction. Since the 1990s, increasing numbers of online auctions have led to a significant increase in the number of theoretical and experimental studies about auctions in the recent years. Analyzing different aspects of various auction types, the majority of these studies focus on forward auctions, where there are many buyers competing to obtain an item provided by a single seller. However, as procurement has evolved from a secondary function to a strategic tool in today's competitive industrial markets, the need for analytical studies of these settings has increased. Suppliers' competitive bidding strategies depend on how the mechanism is defined and affect the equilibrium allocation that determines the buyer's total procurement cost. Therefore, an analytical study can provide valuable insights to all decision makers in a procurement setting.

In this paper, we conduct an equilibrium analysis for repeated procurement auctions. We consider capacitated suppliers, each with a U-shaped cost function that captures the economies (and diseconomies) of scale. On one hand, they realize economies of scale when the quantity demanded is less than their capacity, due to setups and other fixed costs, or due to the learning effect and increased productivity in proportion to the increasing quantity. On the other hand, they face diseconomies of scale as the quantity demanded exceeds their capacity, either due to the overtime/outsourcing costs, or due to a required increase in the existing capacity to meet the increased demand. Hence, the quantity demanded might play a crucial role in determining the equilibrium bidding strategies that might have major impact on the possible allocation outcomes and total procurement cost for the buyer.

Another important property of the industrial context is that many procurement auctions are recurring events. Suppliers participating in a sequence of reverse auctions might gain useful information about their competitors and might also reveal crucial information about themselves. The limit on information availability is an important design factor affecting the equilibrium bids in sequential (repeated) auctions, since the suppliers will submit their bids for the next period based on the beliefs that are updated according to the information feedback. Besides this informational aspect, there is also an economic aspect in cases in which capacity allocated in the prior stage is not released in the second stage. This feature allows us to consider capacity constraint in a more realistic way than just putting a hard constraint that restricts winning to one period at most.

As a benchmark comparison to the repeated setting, we also include the case of a multi-period single-shot auction. We consider the participating suppliers acting as if they have a constant unit production cost, which might be approximated from the U-shaped cost function based on the available demand information. This benchmark will help us to distinguish the differences between our study and the classic single-period static analysis.

Motivated by these aspects of industrial procurement, we have two main goals in this paper. First we will explore the single period procurement game fully for the cases with homogenous and non-homogenous suppliers in a symmetric incomplete information setting, so that we can discuss the analytical properties of the suppliers' competitive bidding strategies under incentive-compatible and individual-rational mechanisms. Second, we will expand the single-period game to a two-stage sequential (repeated) game setting, in which we explore the analytical changes in the equilibrium outcomes, caused by the informational and economical considerations. In order to reach these two goals, we begin by analyzing a pair of bidders and deriving the respective results, which we then will generalize to N-supplier settings.

We discuss related literature in section 2 and introduce the single-period procurement model in section 3. In section 4, we analyze repeated procurement auction settings. After providing numerical study in section 5, we conclude our study in section 6 by stating the main observations and future research directions.

2. Literature Review. In our review of the broad auction theory literature, we will focus on the fundamental points of the studies that are directly related to our current paper. In the economic theory literature, which concentrates mainly on forward auctions, the analysis of auctions as games of incomplete information originates in the William Vickrey's [12] seminal work. Vickrey proposes the second-price sealed-bid auction and proves analytically the incentive compatibility of the mechanism. Krishna [9] discusses the auction theory in this tradition and gives an account of developments since Vickrey's pioneering paper. Klemperer [8] also provides an extensive survey of the economics of auction theory. Another keystone study in auction theory is Myerson's study [11], in which he examines the optimal auction design for forward auctions as a mechanism design problem and develops key concepts, such as the revelation principle and general revenue equivalence. Although these studies mainly analyze forward auctions, they are also important for our paper because they provide essentials that can be used to derive the corresponding results for procurement auctions.

Besides the literature of auction theory in economics, there has been recent research about auctions in the operations research (OR) and operations management (OM) literature. Kalagnanam and Parkes [7] provide an overview of the various auction mechanisms commonly encountered both in practice and in the literature, and also state possible alternative classification schemas for auctions. Although there are several studies that consider procurement auctions in the OR literature, they focus mostly on the winner-determination problem, rather than on the analytical derivation of competitive bidding strategies. The studies that deals with bid derivation focuses on the iterative bidding strategies that use primal-dual information.

All of these studies provide a static analysis of single-period procurement settings, mainly from the buyer's view. Among these studies, Gallien and Wein [4] study the problem of designing multi-item procurement auctions in capacity-constrained environments. They propose an iterative mechanism in which the suppliers bid their unit costs strategically and their capacity constraints truthfully. Their key assumptions are that each supplier has her own fixed unit-production cost for each item and that each supplier is constrained exogenously by her own capacity level. Even in the single-period procurement auction model, our current study differs from their study in three major ways. First, we consider production cost as a function of the quantity demanded. Second, capacity is not a hard constraint in our study; suppliers can provide quantities more than their capacity level at an increased cost. Finally, we focus on the analytical derivation of the equilibrium bidding functions, instead of considering iterative mechanisms.

There has been a limited number of studies on the dynamic aspects of auctions. The dynamics of forward auctions are totally different from the dynamics of procurement auctions recurring over time. As a pioneering paper in this field, Luton and McAfee [10] considers two auctions held in sequence with the possibility of learning between the auctions. They analyze the case of two independent indivisible projects for which firms have independent cost draws. They model the possibility of learning by allowing the second project cost to be the minimum of the two draws. Hence, the link between the costs of two projects is defined independent of the structure.

There have been recent empirical studies that consider repeated procurement auctions in the highway construction environment and assume that capacity is the binding constraint throughout the periods, i.e. winning in one period might prevent the winning supplier from participating in future auctions.

Jofre-Bonet and Pesendorfer [6] propose an estimation method for a repeated highway construction procurement auction, and also take into account the capacity constraints. In the first stage, they assume that the bid distribution in each period is affected by state-defining variables, such as backlog, cost and contract characteristics, and they implement a Kernel type estimator for each period's bid distribution. In the second stage, costs are inferred from the first order condition of the optimal bids. Voicu [13] analyzes the repeated auction in highway construction by focusing on the properties of Markov perfect equilibrium. He implements functional forms for the bid functions that depend on the period and analyzes their properties.

In a generalized setting, Elmaghraby [2] studies the importance of ordering in sequential procurement auctions when the suppliers have capacity constraints and therefore a single supplier cannot win both auctions. Their study highlights the influence of ordering on the efficiency and optimality of an auction.

In another recent study, Elmaghraby and Oh [3] focus on the design of the optimal erosion-rate policy and compare its performance against a second-price sealed-bid auction under the learning effect. They study mainly the design of the optimal erosion policy and characterize the optimal discount price schedule as a function of the market structure. They also consider the case when suppliers take into account the impact of current actions on future periods.

Caillaud and Mezzetti [1] study two sequential ascending-price auctions in which bidders' valuations of the items in the auctions are perfectly correlated. Due to this perfect correlation of valuations, their concern is more on the informational aspect. They show that strategic non-disclosure of information takes the form of non-participation in the first auction by bidders who have valuations below a threshold.

Grimm [5] compares sequential and bundle auctions in the procurement setting of two complementary goods, where success in the first period affects future period opportunity positively. She divides the suppliers into two groups: those who can provide both items and those who can only provide the second item. She also distinguishes the winner of the first period, by assuming that the incumbent will have a comparative cost advantage in the second period.

Although each of aforementioned studies provides valuable insights for analyzing repeated auctions, our study aims to explain the repeated procurement setting in an industrial context more efficiently by providing theoretical analysis of competitive bidding strategies. First, we consider the production cost as a function of the quantity in order to capture the (dis)economies of scale that each supplier might face. Second, we consider capacity in a more realistic way; we allow suppliers to participate in the auctions even if the quantity demanded exceeds their capacity level. Further, we do not restrict our analysis by assuming that each supplier can win only one of the two auctions due to hard capacity constraint. Our sole restriction on capacity is that the winner of the first period cannot utilize the first-period allocated capacity for the second-period demand. Therefore, depending on the quantities demanded in both periods and on the cost structure of the winning supplier, the incumbent may or may not have a comparative cost advantage in the second period. Hence, we believe that our study will provide valuable insights for evaluating the competitive bidding strategies of capacitated suppliers participating in repeated procurement auctions. We also discuss possibility of strategic behavior on the suppliers' side and illustrate the effects of demand uncertainty on the mechanism, specifically on the payment rule.

3. Single-Period Procurement Model After stating our main assumptions to define the scope of our study in this section, we will provide an in-depth analysis of several single-period procurement models. We will follow Myerson's framework in discussing the mechanism design properties of the defined games. We will state the results in the main body and will have the derivations and the proofs for these results in the appendix.

We consider a risk-neutral buyer in need of Q_t units of an item in each period. In the main body of our study, we assume that the demand is deterministic for each period, since it is a realistic assumption to model real life single-period procurement setting. As an extension of our main study, we will briefly discuss the stochastic demand scenario and its impact on the equilibrium bids. In a single-period procurement setting, stochastic demand might be reasonable only if the buyer requests for quotes from the suppliers without being able to give them an exact demand amount. Another option, in which stochastic demand might be applicable, is to incorporate demand uncertainty for future periods in cases the suppliers behave strategically, taking into account the future period demands while bidding in the current period.

Assuming sole sourcing, the buyer chooses the awarded supplier for each period via a well-defined procurement mechanism, trying to minimize the total procurement cost. Although in many situations there might be non-price attributes affecting the procurement decisions, it might still be reasonable to assume that price is the sole factor used to choose the winning supplier. This can be stated with the loss of generality if the item to be procured is somehow standardized, or the non-price attributes are fixed beforehand for all eligible participating suppliers.

Having defined the buyer side of our study, we now state the supplier-related assumptions to complete the scope definition. We assume that all suppliers are risk-neutral and that each bids to maximize her own expected profits. We assume that all suppliers are aware of the number of suppliers participating in the procurement auction and that it is fixed exogenously, i.e. the supplier pool is closed to new entries, and no supplier goes out of business during the sequence of procurement auctions.

Our study depends on the private value assumption, which means that each supplier's cost is totally independent of the other suppliers' costs. Although it might seem more reasonable to consider interdependent (affiliated) value models due to similarities among the technologies used to produce a certain product in real life, we assume that there are sufficient differences among the technologies used by each supplier to support the use of private value models.

Our crucial assumption about suppliers is that they will face both economies and diseconomies of scale as they are weakly constrained by capacity. We will consider U-shaped cost functions to capture these issues. We assume all suppliers are symmetric in the parametric form of their cost functions and that they are aware of this commonality. However, each supplier has her own cost function depending on the value of the parameter θ, which defines the supplier type, and supplier types are drawn from the same distribution $F(\theta)$, with a normalized finite support of $[\theta_{min}, \theta_{max}]$. Each supplier knows her own type exactly and knows that others' types are drawn independently from $F(\theta)$. Hence, our study will be based on the symmetric incomplete information setting.

Among various alternatives to represent U-shaped costs, we choose to work with the following form:

$$C(\theta, Q) = \alpha(\theta) + \gamma(\theta)Q + \delta(\theta)Q^2 \tag{1}$$

It will be possible to capture a wide variety of situations by the cost function defined in equation 1. For each supplier, the above equation will represent economies of scale for quantities less than $-\gamma(\theta)/2\delta(\theta)$, that is the capacity of a supplier with type θ, and diseconomies of scale for greater quantities. Figure 1 shows an example view of a θ-typed supplier's cost function, satisfying the following conditions: $\alpha(\theta) > 0$, $\gamma(\theta) < 0$ and $\delta(\theta) > 0$.

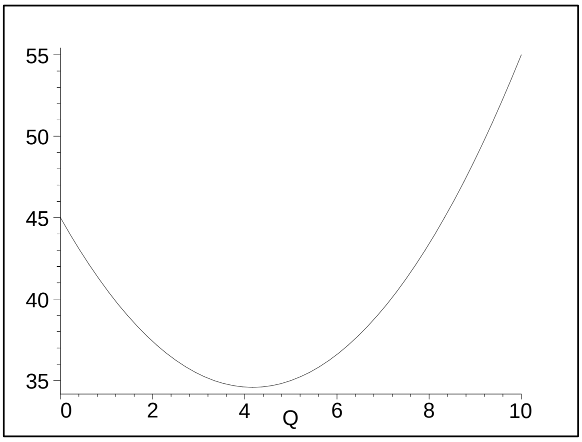

Figure 1: A Typical U-Shaped Cost Function

Based on the structural form of the cost function, we distinguish the suppliers as homogenous or non-homogenous. The above representation allows us to regenerate several different options that might occur in real life. These options might be listed as follows:

- Homogenous-Scale Suppliers: those having the same U-shaped function shifted by the supplier-specific capacity.

- Homogenous-Capacity Suppliers: those having the same capacity level differentiated by supplier-specific productivity(scale elasticity).
- Non-Homogenous Suppliers: those having both supplier-specific capacity and productivity.

Considering each of these options, we build our study by starting with two suppliers and then generalizing to N suppliers. Another important issue about the cost characteristics will be the number of times that suppliers' cost functions intersect. This property provides additional information about the suppliers' cost structures. The following alternatives are captured in our study:

- Pairwise Single Crossing
  - Single Crossing in N-Supplier Generalization in the case of non-homogenous suppliers.
  - Arbitrary Number of Crossing in N-Supplier Generalization in cases of homogenous-scale or non-homogenous suppliers.
- Pairwise Double Crossing
  - Double Crossing in N-Supplier Generalization in the case of non-homogenous suppliers.
  - Arbitrary Number of Crossing in N-Supplier Generalization in cases of homogenous-capacity or non-homogenous suppliers.

Before studying the above options in detail, we briefly discuss the mechanism design and implementation issues.

3.1 Mechanism Design and Implementation The mechanism design problem has been well studied by Myerson. In the mechanism design approach to auction problems, if the set of possible bids is unrestricted, it will not be possible to analyze the design problem due to the complication caused by the unlimited number of possible bids for each supplier. Therefore, the common approach is to implement the *revelation principle*, which states that for any given mechanism and an equilibrium for that mechanism, there exists a *direct mechanism* in which it is an equilibrium for each bidder to report his or her true type, and the outcomes are the same as in the given equilibrium of the original mechanism. Hence, restricting the analysis only to direct mechanisms does not cause any loss of generality.

Two important issues that should be considered in the design of auctions are *incentive compatibility* (IC) and *individual rationality* (IR). Under the procurement context, the IC property implies that the suppliers cannot gain any additional profit by pretending to be someone other than their true types. In the case of a direct mechanism, IC implies that revealing her true type is the equilibrium strategy for each supplier. However, this may not be in line with a supplier's best interest. Therefore, a proper incentive scheme should be constructed while designing the mechanism. This may be accomplished by defining the appropriate allocation and payment rules. Besides considering incentives, the payment rule should also make the mechanism more attractive than any possible outside option. IR implies that each supplier, by participating in the auction, has a higher expected payoff than she would have from the outside option.

We are currently more interested in the implementation of well-structured auction mechanisms, such as first-price or second-price auctions. During the implementation of such mechanisms, suppliers actually consider Bayesian incentive compatibility and individual rationality due to the private information assumption, and the revenue equivalence holds as long as the participating suppliers are symmetric.

If the second-price auction is chosen as the implementation procedure, the weakly dominant strategy of each supplier will be to reveal her true type, expecting to receive a payment that will be equal to the cost of the most competitive opponent, given that she is the least-cost-type supplier. However, if the first-price auction is chosen as the mechanism, each supplier will inflate her own cost by some amount that will provide sufficient incentive for her to act according to her true type. We analyze the first price auction in the following sections.

3.2 Pairwise Single-Crossing Models We start our detailed analysis of the pairwise single-crossing models with the single crossing in N-supplier generalization and then continue with the arbitrary number of crossing generalization.

3.2.1 Single Crossing in N-Supplier Generalization We assume that the cost functions of these suppliers will intersect at a single point, which we will denote by Q_c. For any $Q < Q_c$ the cost function will be an increasing function of θ, and for any $Q > Q_c$ it will be a decreasing function of θ. This model might be realized under the non-homogeneous suppliers option, and the crossing point can be considered as an item specific threshold value. After stating the required properties for parameters, we represent such a setting in Figure 2.

- $\alpha^{'}(\theta) > 0$ since $C^{'}(\theta, Q) > 0$ at $Q = 0$
- $C^{'}(\theta, Q) = 0$ at $Q = Q_c$
  - if $\gamma^{'}(\theta) \neq 0$, then $\delta^{'}(\theta) = \frac{\gamma^{'}(\theta)^2}{4\alpha^{'}(\theta)} > 0$, and $\gamma^{'}(\theta) < 0$ since $\frac{-\gamma^{'}(\theta)}{2\delta^{'}(\theta)} > 0$
  - if $\gamma^{'}(\theta) = 0$, then $\delta^{'}(\theta) < 0$ since $Q_c = \sqrt{\frac{-\alpha^{'}(\theta)}{\delta^{'}(\theta)}} > 0$
  - $\alpha^{''}(\theta) = \gamma^{''}(\theta) = \delta^{''}(\theta) = 0$

These conditions are required for the existence of a unique critical quantity Q_c that is independent of supplier type θ.

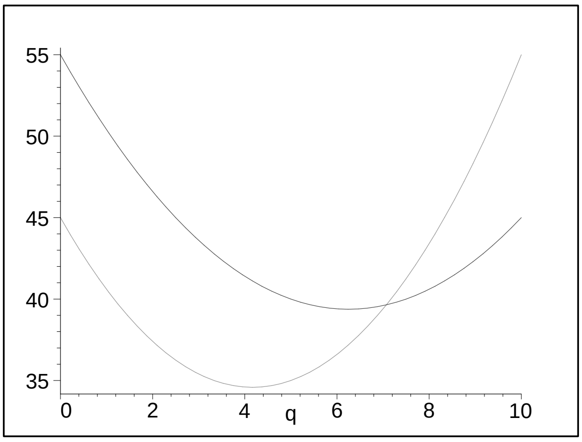

Figure 2: Single Crossing Point Setting

Actually, this environment can be considered as a variant of the N-suppliers case under the bounded rationality assumption. It might not be possible to know the exact number of participating suppliers or detailed individual information about each of them. Hence, each supplier will choose to bid against her most competitive rival, assuming that her competitor will have the same cost structure. After building our analysis on the two-suppliers game, as a mathematical extension in the appendix we discuss the generalization to N-suppliers game, which has the restrictive assumption that all suppliers' cost functions intersect at a single point.

The structure of the equilibrium bidding function depends on whether the amount of quantity demanded Q is less than or more than Q_c as the probability of winning changes depending on the relative location of quantity demanded.

Case I : $Q < Q_c$

We define the expected profit function of a supplier as follows:

$$\pi(\theta_j) = (1 - F(\phi(b_j)))\,(b_j - C(\theta_j, Q))Q \tag{2}$$

The first term in equation 2 denotes the probability of winning the auction while bidding b_j. A supplier j will win the auction if and only if she submits the lowest bid. Since β is an increasing function of θ, submitting the lowest bid is equivalent to stating that the supplier j will win whenever she has a lower type than her opponent. We denote the inverse bidding function by $\phi(b_j)$ to represent the relation between the submitted bids and the supplier's revealed type. The second term shows the profit margin of supplier j. Therefore, the supplier is actually facing a tradeoff while determining the optimal bid. An increase in the bid will increase the profit margin, but will also reduce the probability of winning at the same time. The optimal bid is determined at the point where these effects balance off.

Due to the symmetric Bayesian-Nash equilibrium result, we can argue that $\phi(b_j) = \theta_j$. Making this substitution in the necessary first-order condition equation for maximization and rearranging the terms to solve the first-order differential equation, the following equilibrium bidding function is obtained:

$$\beta(\theta_j) = C(\theta_j, Q) + \int_{\theta_j}^{\theta_{max}} C^{'}(x, Q) \left(\frac{1 - F(x)}{1 - F(\theta_j)} \right) dx \tag{3}$$

In equation 3, $C^{'}(x, Q)$ represents the partial derivative of $C(x, Q)$ with respect to x. It can be thought of as the marginal cost of type at the given quantity level.

Although $\beta(\theta_j)$ is derived from the necessary condition, the following proposition verifies that it is indeed the optimal strategy:

PROPOSITION 3.1 *If $Q < Q_c$, the symmetric equilibrium strategies of suppliers for a single-period procurement auction are given by equation 3.*

PROOF. See Appendix. □

Case II : $Q > Q_c$

We can now consider the case in which the quantity demanded exceeds the critical quantity. In this case, the winning probability definition changes because the most efficient supplier is now the one with the highest type. As β is now decreasing function of θ, submitting the lowest bid is equivalent to stating that supplier j will win whenever she has a higher type than her opponent. The expected profit function of a supplier will be defined as follows:

$$\pi(\theta_j) = (F(\phi(b_j)))\,(b_j - C(\theta_j, Q))Q \tag{4}$$

The first order condition yields the following equilibrium bidding function defined by:

$$\beta(\theta_j) = C(\theta_j, Q) - \int_{\theta_{min}}^{\theta_j} C^{'}(x, Q) \left(\frac{F(x)}{F(\theta_j)} \right) dx \tag{5}$$

As $C^{'}(x, Q)$ will be negative for all x in this case, the equilibrium bid submitted by any supplier will still be greater than the the corresponding supplier's average cost. Proposition 3.2 verifies the equilibrium bidding strategy as the quantity demanded is more than the critical quantity.

PROPOSITION 3.2 *If $Q > Q_c$, the symmetric equilibrium strategies of suppliers for a single period procurement auction are given by equation* **??**.

PROOF. A proof similar to the one used for proposition 3.1 can be provided. □

We characterize the symmetric equilibrium bidding strategies of a capacity-constrained supplier in the following proposition:

PROPOSITION 3.3 *For $Q < Q_c$, the symmetric equilibrium bid of supplier j is an increasing function of her type θ_j, whereas for $Q > Q_c$, it is a decreasing function of her type θ_j.*

PROOF. We can prove analytically this proposition by taking the first-order partial derivative of $\beta(\theta_j)$ with respect to θ_j for both cases.

Case I : $Q < Q_c$

$$\frac{\partial \beta(\theta_j)}{\partial \theta_j} = \int_{\theta_j}^{\theta_{max}} C^{'}(x, Q) \left(\frac{f(\theta_j)}{1 - F(\theta_j)} \right) \left(\frac{1 - F(x)}{1 - F(\theta_j)} \right) dx \tag{6}$$

$\frac{\partial \beta(\theta_j)}{\partial \theta_j} > 0$ as $C^{'}(x, Q) > 0 \; \forall x$ when $Q < Q_c$.

Case II : $Q > Q_c$

$$\frac{\partial \beta(\theta_j)}{\partial \theta_j} = \int_{\theta_{min}}^{\theta_j} C^{'}(x, Q) \left(\frac{f(\theta_j)}{F(\theta_j)} \right) \left(\frac{F(x)}{F(\theta_j)} \right) dx \tag{7}$$

$\frac{\partial\beta(\theta_j)}{\partial\theta_j} < 0$ as $C^{'}(x, Q) < 0\ \forall x$ when $Q > Q_c$.

These analytical results also validate the intuition. As the cost of providing the item increases, the supplier will submit a higher bid to cover her costs. The cost is an increasing function of θ for $Q < Q_c$; therefore, the higher the type, the higher the submitted bid, and oppositely the lower the type, the higher the submitted bid when $Q > Q_c$ due to the change of most efficient supplier from θ_{min} to θ_{max}. □

Having derived symmetric equilibrium bidding strategies, we can now comment on the expected procurement cost of the mechanisms for the buyer. The expected procurement cost is defined below:

$$\begin{aligned} E[PC] &= 2 \times E[m(\theta)] \\ &= 2 \times \int_{\theta_{min}}^{\theta_{max}} C(y, Q)F(y)f_1^1 dy \\ &= E[C_2^2(\theta, Q)] \end{aligned} \tag{8}$$

$C_2^2(\theta, Q)$ represents the higher cost for providing Q units between 2 suppliers. Hence, the total procurement cost to the buyer is just the expectation of this cost. We state the revenue-equivalence result that holds when the participating suppliers are symmetric in the following proposition:

PROPOSITION 3.4 *Whether the auction format chosen is first-price or second-price, the expected total procurement cost for the buyer is the expectation of the higher cost of providing the required amount between 2 suppliers.*

3.2.2 Arbitrary Number of Crossing in N-Supplier Generalization As an extension to the previous model, we drop the assumption that there exists a single crossing point and allow that the most efficient technology type will change as a function of quantity demanded.

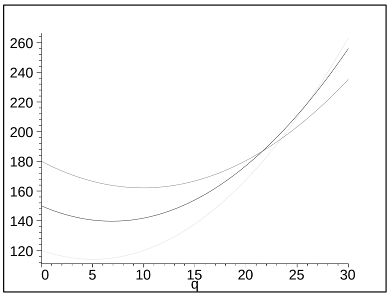

Figure 3: Single Crossing Generalization vs Quantity

Figure 3 represents an example of such generalized cost functions for three different supplier types, and figure 4 represents an example of such cost functions for three different quantity levels. As can be seen from these two figures, the cost functions are not only U-shaped in terms of quantity for a given supplier type, but also U-shaped in terms of supplier type for a given quantity level. Therefore, the most efficient supplier will be different for each quantity level.

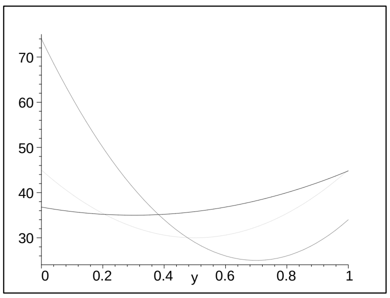

Figure 4: Single Crossing Generalization vs Supplier Type

Because all of the suppliers are aware of the common structure of the cost function, they are able to determine the most efficient technology type $\theta_e(Q)$ for the given deterministic demand Q. Based on the value of $\theta_e(Q)$, the suppliers are able to define their probability of winning and, hence, their equilibrium bidding function. Each supplier type θ_j has a cost equivalent type $\hat{\theta}_j$. Supplier type θ_j will be the winner if there is no supplier with a type $\theta \in [\theta_j, \hat{\theta}_j]$. The structure of the bidding function depends on the relative position of θ_j with respect to $\theta_e(Q)$.

Case I : $\theta_j < \theta_e(Q)$

$$\beta(\theta_j) = C(\theta_j, Q) + \int_{\hat{\theta}_j}^{\theta_{max}} C^{'}(x, Q) \left(\frac{1 - F(x)}{1 - F(\theta_j)} \right)^{N-1} dx - \int_{\theta_{min}}^{\theta_j} C^{'}(x, Q) \left(\frac{F(x)}{F(\theta_j)} \right)^{N-1} dx \tag{9}$$

Case II : $\theta_j > \theta_e(Q)$

$$\beta(\theta_j) = C(\theta_j, Q) + \int_{\theta_j}^{\theta_{max}} C^{'}(x, Q) \left(\frac{1 - F(x)}{1 - F(\theta_j)} \right)^{N-1} dx - \int_{\theta_{min}}^{\hat{\theta}_j} C^{'}(x, Q) \left(\frac{F(x)}{F(\theta_j)} \right)^{N-1} dx \tag{10}$$

Based on the equations 9 and 10, the equilibrium bidding function will be a decreasing function of θ for $\theta < \theta_e(Q)$, and an increasing function of θ thereafter.

3.3 Pairwise Double Crossing Having completed the detailed analysis of pairwise single crossing models, we continue our study by focusing on pairwise double crossing models.

3.3.1 Double Crossing in N-Supplier Generalization These models can be considered as a straightforward extension of the single crossing in N-Supplier generalization. In this case, there will be two critical crossing points, Q_c^1 and Q_c^2, which are item-specific threshold values. An example view of this model can be found in figure 5.

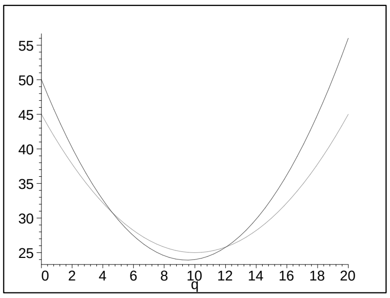

Figure 5: Pairwise Double Crossing

The structure of the equilibrium bidding function will depend on whether the quantity demanded Q is between Q_c^1 and Q_c^2 or not, as the probability of winning changes depending on the relative location of quantity demanded.

The equilibrium bidding functions are defined as follows:

If $Q \in [Q_c^1, Q_c^2]$

$$\beta(\theta_j) = C(\theta_j, Q) - \int_{\theta_{min}}^{\theta_j} C^{'}(x, Q) \left(\frac{F(x)}{F(\theta_j)} \right) dx \tag{11}$$

otherwise

$$\beta(\theta_j) = C(\theta_j, Q) + \int_{\theta_j}^{\theta_{max}} C^{'}(x, Q) \left(\frac{1 - F(x)}{1 - F(\theta_j)} \right) dx \tag{12}$$

4. Repeated Procurement Model Having completed the detailed analysis of the single-period procurement model in the previous section, we move on to the analysis of the repeated procurement model. We will first state our additional assumptions to define the scope of the repeated procurement setting as we move along the analysis. We will first study the short-term contracting setting, in which an individual auction is being held separately at each period. Next, we will focus on long-term contracting as a benchmark for the short-term contracting alternative.

4.1 Short-term Contracting We start our analysis by assuming that all suppliers are naive, such that they only consider the current period while setting their bids and do not take into account future periods. However, they do evaluate the information gathered from the previous period in updating their belief functions. We will briefly discuss strategic thinking at the end of our study.

We assume that the suppliers' types are drawn once and remain the same during both periods. At the end of the first period, each supplier will update her beliefs according to her private information, and this will cause informational asymmetry between the suppliers. Due to this assumption, information feedback provided to the suppliers at the end of the first period will play an important role in updating their beliefs for the other suppliers' types. Therefore, in terms of applicability information feedback is a crucial issue in the mechanism design.

4.1.1 Single Crossing in N-Supplier Generalization Initially, we will focus on the scenario in which the quantity demanded in each period follows Case I. The other scenarios will follow similar discussion.

If a supplier is the winner(supplier i) in the first period, she will update beliefs about the loser's type as:

$$F_i(\theta) = \frac{F(\theta) - F(\theta_i)}{1 - F(\theta_i)} \qquad \theta \in [\theta_i, \theta_{max}] \tag{13}$$

If she is a loser(supplier j) in the first period;

she will update her belief about the winner's type as:

$$F_{ji}(\theta) = \frac{F(\theta)}{F(\theta_j)} \qquad \theta \in [\theta_{min}, \theta_j] \tag{14}$$

Another crucial assumption is that winning in the first period reduces available capacity for the second period. Hence, whether winning in the first period brings cost advantage in the second period will depend on the quantity demanded in the second period and on the cost structure of the winning supplier. As a result, the winner of the first period will bid, realizing unit cost of $C(\theta_i, Q_1 + Q_2)$ while the loser is realizing unit cost of $C(\theta_j, Q_2)$.

Because we consider naive suppliers for our current study, the first-period bids will be defined by the bidding functions stated in the previous section. In this section, we will focus on the second-period bids, considering the assumptions that define the scope. Before deriving the equilibrium bidding strategies, we will point out an important problem in this repeated procurement setting. In the second period, the suppliers become ex-ante asymmetric not only in terms of costs, but also in terms of their beliefs about others. Due to this informational asymmetry, none of the suppliers will be able to define her own strategy based on Bayesian belief about others in a game-theoretic setting. An asymptotic behavioral analysis might be performed to derive the equilibrium strategies asymptotically. However, this is beyond the scope of our study.

As a design parameter of two-period game, we assume that a common information feedback θ_f is provided to both suppliers at the end of the first period in addition to their individual win/loss status. θ_f represents for the winner that her opponent has higher types than the announced type, whereas it represents for the loser that the winner has a lower type than the announced type. This common information feedback solves the informational asymmetry problem, leaving two groups of suppliers due to the type asymmetry, one winner (Incumbent) and one loser (Entrant).

The corresponding expected profit functions for the incumbent and the entrant are defined as:

$$\begin{aligned} \pi_I(b_I) &= (1 - F_i(\phi_E(b_I)))(b_I - C(\theta_i, Q_1 + Q_2))Q_2 \\ \pi_E(b_E) &= (1 - F_{ji}(\phi_I(b_E)))(b_E - C(\theta_j, Q_2))Q_2 \end{aligned} \quad (15)$$

If we substitute equation 13 and equation 14 into their respective places in the equation set 15, we will obtain the following set of equations:

$$\begin{aligned} \pi_I(b_I) &= \left(\frac{1 - F(\phi_E(b_I))}{1 - F(\theta_f)}\right)(b_I - C(\theta_i, Q_1 + Q_2))Q_2 \\ \pi_E(b_E) &= \left(\frac{F(\theta_f) - F(\phi_I(b_E))}{F(\theta_f)}\right)(b_E - C(\theta_j, Q_2))Q_2 \end{aligned} \quad (16)$$

The first relation defined by equation 16 shows the expected profit of the incumbent competing with the entrant for the second period. The second relation defines the expected profit of an entrant competing against the incumbent.

The first order conditions of the equation set 16 can be rearranged to get:

$$\begin{aligned} \phi_E^{'}(b) &= \frac{1 - F(\phi_E(b))}{f(\phi_E(b))} \cdot \frac{1}{b - C(\phi_I(b), Q_1 + Q_2)} \\ \phi_I^{'}(b) &= \frac{F(\theta_f) - F(\phi_I(b))}{f(\phi_I(b))} \frac{1}{b - C(\phi_E(b), Q_2)} \end{aligned} \quad (17)$$

Using the boundary conditions $\beta_I(\theta_{min}) = \beta_E(\theta_f)$ and $\beta_I(\theta_f) = \beta_E(\theta_{max})$, differential equations defined by equation set 17 should be solved to define the equilibrium bidding functions for the incumbent and the entrants. Even under the two-supplier game, it will be possible to get a nice closed form analytical solution only for some special cases. Therefore, we focus more on some properties of the equilibrium strategies indirectly.

Before going into further analysis of equilibrium strategies, we need to define some more concepts. Due to the capacity allocation assumption, the incumbent has cost equal to $C(\theta_i, Q_1 + Q_2)$ which can be equivalent to $C(\hat{\theta}_i, Q_2)$ and bids accordingly. Therefore, we will define $\hat{\theta}_i$ as the transformed type of the incumbent.

We further distinguish different scenarios to point out some properties of the equilibrium bidding strategies:

- Scenario I: $\hat{\theta}_{min} < \theta_f \,\&\, \hat{\theta}_f < \theta_{max}$
- Scenario II: $\hat{\theta}_{min} > \theta_f \,\&\, \hat{\theta}_f > \theta_{max}$
- Scenario III: $\hat{\theta}_{min} > \theta_f \,\&\, \hat{\theta}_f < \theta_{max}$
- Scenario IV: $\hat{\theta}_{min} < \theta_f \,\&\, \hat{\theta}_f > \theta_{max}$

Proposition 4.1 *Under scenario I the incumbent will bid more aggressively compared to the entrant, while under scenario II the entrant will bid more aggressively.*

In scenario I, the entrant's type is stochastically higher than the incumbent's type, i.e. $F_I(\theta) > F_E(\theta)$. Therefore, $\phi_I(b) \leq \phi_E(b) \; \forall \, b$. This can be proven by using contradiction. Intuitively, when the entrant knows that she does not have comparative advantage against the incumbent, she will choose to bid more conservatively. Similarly, in scenario II, just the opposite holds true as the incumbent's type becomes stochastically higher than the entrant's types. In this scenario, because she has lost cost advantage due to winning in the first period, the incumbent intuitively chooses to submit a more conservative bid.

Proposition 4.2 *Under scenario III and scenario IV, there exists a single switching point θ_s. In scenario III, up to θ_s the*

entrant is more aggressive, after that the incumbent. In scenario IV, the opposite holds.

In these two scenarios, it is not possible to talk about the stochastic dominance of one group over the other in the complete possible range of types. There exists a single switching point, where the stochastic dominance passes from one group to another. Depending on this switch, the aggressiveness also changes from one group to another. In scenario III, $F_E(\theta) \geq F_I(\theta)\ \forall\, \theta \leq \theta_s$ and $F_E(\theta) < F_I(\theta)\ \forall\, \theta > \theta_s$. The opposite holds true for the last scenario.

An alternative scenario combination will be the setting at which both period demands follow Case II. The first-order conditions for the equilibrium bids will be defined as:

$$\begin{aligned} \phi_E^{'}(b) &= -\frac{F(\phi_E(b))}{f(\phi_E(b))} \cdot \frac{1}{b - C(\phi_I(b), Q_1 + Q_2)} \\ \phi_I^{'}(b) &= -\frac{F(\phi_I(b)) - F(\theta_f)}{f(\phi_I(b))} \frac{1}{b - C(\phi_E(b), Q_2)} \end{aligned} \tag{18}$$

the differential equations defined by equation set 18 should be solved to define the equilibrium bidding functions for the incumbent and the entrants, by using the boundary conditions $\beta_E(\theta_{min}) = \beta_I(\theta_f)$ and $\beta_E(\theta_f) = \beta_I(\theta_{max})$. Since these equations can only be solved for only special cases, we comment on the characteristics of the equilibrium strategies based on the incumbent's transformed type.

PROPOSITION 4.3 *Under the scenario $\hat{\theta}_f < \theta_{min}\, \&\, \hat{\theta}_{max} < \theta_f$, the entrant will bid more aggressively compared to the incumbent, while under the scenario $\hat{\theta}_f > \theta_{min}\, \&\, \hat{\theta}_{max} > \theta_f$, the incumbent will bid more aggressively.*

PROPOSITION 4.4 *Under the scenarios $\hat{\theta}_f < \theta_{min}\, \&\, \hat{\theta}_{max} > \theta_f$ and $\hat{\theta}_f > \theta_{min}\, \&\, \hat{\theta}_{max} < \theta_f$, there exists a single switching point θ_s. In the first of these scenarios, up to θ_s entrant is more aggressive, after that the incumbent, and the opposite holds for the last scenario.*

The intuitions beyond these propositions are exactly the same as those explained for the propositions given in the case-I demand combination. Proposition 4.3 results from the stochastic dominance of the entrant over the incumbent for the first scenario, and from just the opposite for the second scenario. However, under the scenarios defined in proposition 4.4 there is no strict stochastic dominance of one type over the entire type range, and therefore the switching type exists.

Defining the appropriate belief update and expected profit equations based on the observed demand pattern, the first-order condition equations required for the optimal bidding strategies can be obtained for any scenario case combination. The major problem in the equation sets defining the first-order conditions is that the variables are not separable for all distributions. If the underlying distribution for types allows the separability, then it is possible to get the closed form analytical solution for the competitive bidding function.

4.1.2 Arbitrary Number of Crossing in N-Supplier Generalization At the end of the first period, each supplier will update her beliefs according to her private information, and this will cause asymmetry between the suppliers.

If she is the winner(supplier i) in the first period, then she will update beliefs about the losers' types as follows:

$$F_i(\theta) = \frac{F(\theta) - F(\theta_i)}{1 - F(\theta_i)} \qquad \theta \in [\theta_i, \theta_{max}] \tag{19}$$

If she is a loser(supplier j) in the first period, then she will update for the winner's type as follows:

$$F_{ji}(\theta) = \frac{F(\theta)}{F(\theta_j)} \qquad \theta \in [\theta_{min}, \theta_j] \tag{20}$$

She will update as follows for the other losers' types:

$$\begin{aligned} F_{jk}(\theta \mid \theta_i) &= \frac{F(\theta) - F(\theta_i)}{1 - F(\theta_i)} \\ F_{jk}(\theta) &= \int_{\theta_{min}}^{min(\theta, \theta_j)} \left(\frac{F(\theta) - F(\theta_i)}{1 - F(\theta_i)} \right) \frac{f(\theta_i)}{F(\theta_j)}\, d\theta_i \end{aligned} \tag{21}$$

An important structural property of the setting is that the winner of the first period will bid realizing an average cost of $C(\theta_i, Q_1 + Q_2)$ while the losers are realizing an average cost of $C(\theta_j, Q_2)$. In order for the types of suppliers comparable at the second period quantity level Q_2, the winner's type θ_i will be transformed to $\hat{\theta}_i$ by solving the relation $C(\hat{\theta}_i, Q_2) = C(\theta_i, Q_1 + Q_2)$. Hence, the losers will also define their updated belief about the winner's type in terms of this transformed type as:

$$F_{ji}(\hat{\theta}) = \frac{F(\theta)}{F(\theta_j)} \qquad \theta \in [\theta_{min}, \theta_j] \Leftrightarrow \hat{\theta} \in [\hat{\theta}_{min}, \hat{\theta}_j] \tag{22}$$

Before going into deriving the equilibrium bidding strategies, we will point out an important problem in repeated procurement setting defined above. In the second period, not only the suppliers become ex-ante asymmetric in terms of costs, but their beliefs about others become also asymmetric. This additional degree of informational asymmetry caused by belief updates complicates the derivation of equilibrium bidding strategies. Indeed, it is not possible to derive closed form expressions even for simple cases.

2-Supplier Game

In the case of two suppliers, the model is somewhat simplified as there will be only one loser, although there will still be asymmetry between the bidding strategies of the winner and the loser in the second period. Denoting the winner by i, and the loser by j, the expected profit functions for them will be defined as:

$$\begin{aligned} \pi_i(b_i) &= (1 - F_i(\phi_j(b_i)))(b_i - C(\hat{\theta}_i, Q_2))Q_2 \\ \pi_j(b_j) &= (1 - F_{ji}(\phi_i(b_j)))(b_j - C(\theta_j, Q_2))Q_2 \end{aligned} \tag{23}$$

If we substitute equation 19 and equation 20 into their respective places in the equation set 23, we will obtain the following set of equations:

$$\begin{aligned} \pi_i(b_i) &= \left(\frac{1 - F(\phi_j(b_i))}{1 - F(\theta_i)}\right)(b_i - C(\hat{\theta}_i, Q_2))Q_2 \\ \pi_j(b_j) &= \left(\frac{F(\theta_j) - F(\phi_i(b_j))}{F(\theta_j)}\right)(b_j - C(\theta_j, Q_2))Q_2 \end{aligned} \tag{24}$$

The first order conditions of the equation set 24 can be rearranged to get:

$$\begin{aligned} \phi_j^{'}(b_i) &= \frac{1 - F(\phi_j(b_i))}{f(\phi_j(b_i))} \cdot \frac{1}{b_i - C(\phi_i(b_i), Q_1 + Q_2)} \\ \phi_i^{'}(b_j) &= \frac{F(\theta_j) - F(\phi_i(b_j))}{f(\phi_i(b_j))} \cdot \frac{1}{b_j - C(\phi_j(b_j), Q_2)} \end{aligned} \tag{25}$$

Using the boundary conditions $\beta_i(\theta_{min}) = \beta_j(\theta_i)$ and $\beta_i(\theta_j) = \beta_j(\theta_{max})$ differential equations defined by equation set 64 should be solved to define the equilibrium bidding functions for the winner and the loser. Although we cannot get a nice closed form solution without explicitly defining the distribution, we can still observe some basic properties of the equilibrium bidding functions. Based on the support range of the transformed type of the winner the following statements can be derived:

- If $\hat{\theta}_{min} > \theta_j$ & $\hat{\theta}_j < \theta_{max}$, two bidding functions will never intersect, that is $\phi_i(b) < \phi_j(b) \ \ \forall\, b$.
- If $\hat{\theta}_{min} > \theta_j$ & $\hat{\theta}_j < \theta_{max}$, then $\exists\, \theta_s$ such that the two bidding functions intersect at θ_s and $\forall\, \theta < \theta_s \Rightarrow \beta_j(\theta) \geq \beta_i(\theta)$ while $\forall\, \theta > \theta_s \Rightarrow \beta_j(\theta) \leq \beta_i(\theta)$.
- If $\hat{\theta}_{min} > \theta_j$ & $\hat{\theta}_j > \theta_{max}$, then $\phi_j(b) < \phi_i(b) \ \ \forall\, b$.

3-Supplier Game

This form of the game will be more complicated due to the presence of an additional loser. The expected profit functions will be defined by:

$$\begin{aligned} \pi_i(b_i) &= (1 - F_i(\phi_j(b_i)))(1 - F_i(\phi_k(b_i)))(b_i - C(\hat{\theta}_i, Q_2))Q_2 \\ \pi_j(b_j) &= (1 - F_{ji}(\phi_i(b_j)))(1 - F_{jk}(\phi_k(b_j)))(b_j - C(\theta_j, Q_2))Q_2 \\ \pi_k(b_k) &= (1 - F_{ki}(\phi_i(b_k)))(1 - F_{kj}(\phi_j(b_k)))(b_k - C(\theta_k, Q_2))Q_2 \end{aligned} \tag{26}$$

Substituting the probability belief update equations into the equation set 26, we get the profit equations defined in terms of the original distribution:

$$\begin{aligned}
\pi_i(b_i) &= \left(\frac{1-F(\phi_j(b_i))}{1-F(\theta_i)}\right)\left(\frac{1-F(\phi_k(b_i))}{1-F(\theta_i)}\right)(b_i - C(\hat{\theta}_i, Q_2))Q_2 \\
\pi_j(b_j) &= \left(\frac{F(\theta_j)-F(\phi_i(b_j))}{F(\theta_j)}\right)\cdot \\
&\quad \left(1-\int_{\theta_{min}}^{min(\phi_k(b_j),\theta_j)}\left(\frac{F(\phi_k(b_j))-F(\theta_i)}{1-F(\theta_i)}\right)\frac{f(\theta_i)}{F(\theta_j)}\,d\theta_i\right)(b_j - C(\theta_j, Q_2))Q_2 \\
\pi_j(b_j) &= \left(\frac{F(\theta_k)-F(\phi_i(b_k))}{F(\theta_k)}\right)\cdot \qquad (27)\\
&\quad \left(1-\int_{\theta_{min}}^{min(\phi_j(b_k),\theta_j)}\left(\frac{F(\phi_j(b_k))-F(\theta_i)}{1-F(\theta_i)}\right)\frac{f(\theta_i)}{F(\theta_j)}\,d\theta_i\right)(b_k - C(\theta_k, Q_2))Q_2
\end{aligned}$$

We will conclude this short-term contracting section by mentioning briefly the impossibility of strategic behavior while setting the equilibrium bids. In a first-price auction setting, suppliers cannot behave strategically due to the intractability of the analysis, as a result of the asymmetry that occurs among suppliers in the second period. The suppliers are not able to define their second-period bidding strategies in a closed form even if we assume that they have symmetric bidding strategies in the first period. However, due to the asymmetries in the first period and in the cost advantage due to winning, it will be even more complex to define the second period strategies. Hence, being unable to derive them analytically, the suppliers choose to act naively in this short-term contracting setting.

4.2 Long-term Contracting Now, we will consider the long-term contracting environment, in which suppliers behave as if they have constant fixed unit cost throughout the entire project life. Since we consider this environment a benchmark for the short-term contracting setting, we will study two contracting alternatives that the buyer might offer. The first alternative will be the *declining-price contract*, whereas the second alternative will be the *fixed-price contract*. The intuition beyond the declining-price contract is that the buyer will be willing to realize productivity by having a price reduction in each period. We can link the contract environment to an auction setting by setting the reserve price of a given period to the winning bid of the previous period. We will show that this policy is a weakly dominant strategy for the buyer; the buyer's choice will depend on the suppliers being myopic or strategic. Myopic suppliers submit their bids by focusing only in the current period, while strategic suppliers consider the declining-price condition in setting their bids.

We start our analysis by considering myopic suppliers, and then we elaborate the discussion by considering strategic suppliers. In the declining-price contract, under an incentive-compatible and individual-rational mechanism, the minimum-cost myopic supplier is offered the price defined by the following equation:

$$\beta_t^M(c_j) = c_j + \int_{c_j}^{r_t}\left(\frac{1-F(x)}{1-F(c_j)}\right)dx \qquad (28)$$

The optimal price schedule for any project can be obtained by evaluating equation 28 at $r_t = \beta_{t-1}^M(c_j)$ for each period. The offered price schedule for a given myopic supplier during the project life will follow a pattern similar to the representative one shown in figure 6, in which it can be observed that the price converges exponentially to her true cost.

Next, we can analyze the case of strategic suppliers. We start with two periods in order to show the intuition beyond the equilibrium price offered to the minimum cost strategic supplier. The second-period expected profit, π_{j2}^S, is given by the following equation:

$$\pi_{j2}^S = (1 - F(\phi_2^S(b_{j2})))(b_{j2} - c_j)Q_2 \qquad (29)$$

Because prices are fixed in the initial period for both periods, and the same supplier is awarded in both periods, each supplier will set the reserve price to her own submitted first period bid. Therefore,

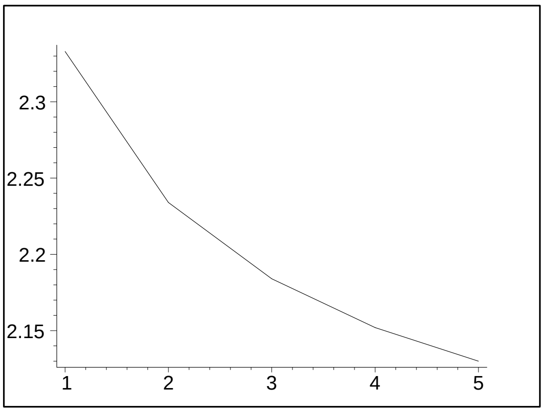

Figure 6: Submitted Bids of a Myopic Supplier During Project Life

there is no explicit term due to the reserve price affecting the expected profit. This will come into play while defining the optimal equilibrium price.

The optimal bidding strategy to maximize the expected profit defined by equation 29, as a result of solving the necessary first-order condition, is given by the following:

$$\beta_2^S(c_j) = c_j + \int_{c_j}^{r_2} \left(\frac{1 - F(x)}{1 - F(c_j)} \right) dx \tag{30}$$

The important point is that r_2 will be replaced by b_{j1} as a result of the above reasoning. Therefore, we can include the optimal strategy given by equation 30 after replacing r_2 by b_{j1} in the expected profit function of the project, denoted by $\tilde{\pi}_j^S$.

$$\tilde{\pi}_j^S = \left[(1 - F(\phi_1^S(b_{j1}))) \right] \cdot \left[Q_1(b_{j1} - c_j) + Q_2 \int_{c_j}^{b_{j1}} \left(\frac{1 - F(x)}{1 - F(c_j)} \right) dx \right] \tag{31}$$

The total expected profit from the project can be expressed as a function of the first period bid. As can be seen from equation 31, there is a tradeoff in choosing the optimal bid value. However, the main difference from the myopic suppliers case is that the positive effect of increasing the bid is weighted more due to the declining-price consideration. The optimal equilibrium price offered to the minimum cost supplier, is defined by the following:

$$\beta_1^S(c_j) = c_j + \int_{c_j}^{r_1} \left(\frac{1 - F(x)}{1 - F(c_j)} \right) dx + \frac{Q_2}{Q_1} \int_{\beta_1^S(c_j)}^{r_1} \left(\frac{1 - F(x)}{1 - F(c_j)} \right) dx \tag{32}$$

Proposition 4.5 *The optimal bidding strategy of a given strategic supplier j, $\beta_1^S(c_j)$, is defined by equation 32.*

Proof. The intuition behind the optimal bidding strategy given by equation 32 is that strategic suppliers ask for an additional subsidy in their initial-period price to recover expected opportunistic losses in the future due to declining price schedule, under an incentive-compatible and individual-rational mechanism. Under a fixed-price contract, the bidding strategies in each period will be given by the following:

$$c_j + \int_{c_j}^{r_1} \left(\frac{1 - F(x)}{1 - F(c_j)} \right) dx \tag{33}$$

and the total profit will be as follows:

$$(Q_1 + Q_2) \int_{c_j}^{r_1} \left(\frac{1 - F(x)}{1 - F(c_j)} \right) dx \tag{34}$$

However, due to the adjustment of the reserve price according to the winning bid, there is a difference of $\left(\int_{\beta_1^S(c_j)}^{r_1} \left(\frac{1-F(x)}{1-F(c_j)} \right) dx \right)$ that can be considered an opportunistic loss between equations 32 and 28.

Strategic suppliers will weigh this term with the quantity ratio and will add it to their first period bids in order to compensate the loss totally. It can be analytically shown that this bidding strategy satisfies the first-order condition. □

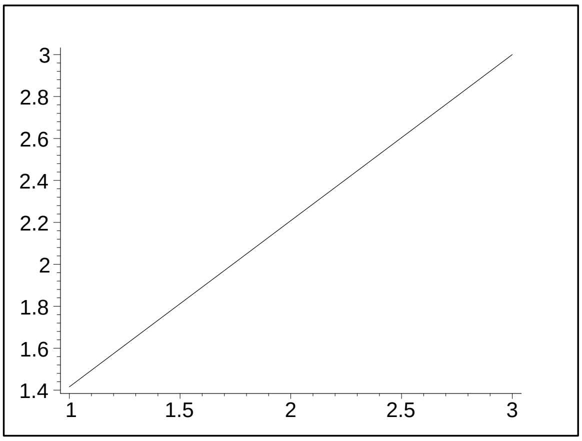

Figure 7: First-Period Equilibrium Price Offered to A Strategic Supplier

The graph in figure 7 represents a strategic supplier's first-period equilibrium price as a function of her true cost, while she is competing against five other strategic suppliers. We again assume that the costs are drawn from U(1,3).

PROPOSITION 4.6 *The submitted bids of the strategic players are more dense, i.e. the deviation of the equilibrium prices in the all strategic suppliers case is less than the deviation of equilibrium prices in the all myopic suppliers case.*

Analytically it is possible to show that the standard deviation of the submitted bids in the all strategic suppliers case is lower. The extra term, compensation amount, which strategic suppliers add to the myopic suppliers' bid, is a decreasing function of the true cost. Therefore, the submitted bids are closer to each other.

An observation about the optimal bidding strategy is that the quantity demanded has an impact on the magnitude of the markup added by the supplier to compensate the opportunistic loss. Specifically, there is an inversely proportional relation to the first-period quantity demanded and a directly proportional relation to the second-period quantity. As the second-period quantity increases, the opportunistic loss to be considered increases, and as the first-period quantity increases the opportunistic loss is compensated by adding smaller markup to unit item. Inflating the markup due to this intuition results in the following proposition.

PROPOSITION 4.7 *In the case where all suppliers are strategic, asupplier's optimal bidding strategy will lead to a higher bid (or at least equal for the special case $c_j = r_1$) than the one resulting from a supplier's optimal bidding strategy in the case where all suppliers are myopic.*

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PROOF. The proposition can easily be proven by using the definitions of the optimal bidding strategies given by equations 28 and 32. The difference between the strategic bid and the myopic bid equals to $\frac{Q_2}{Q_1}\int_{\beta_1^S(c_j)}^{r_1}\left(\frac{(1-F(x))}{(1-F(c_j))}\right)dx$ and is positive since $\beta_1^S(c_j) < r_1$ as long as $c_j < r_1$. If $c_j = r_1$, then the submitted bids will be the same in both cases. As all integral terms equal to zero, the submitted bid equals to r_1 in both cases. □

We can easily generalize this intuition of compensating future losses to any finite T-period auction, such that the opportunistic loss in period t will be compensated partially in each of the prior periods. In other words, while determining the optimal bid in a given period t, the supplier will consider the opportunistic losses in all future periods, from $t+1$ to T, caused by bidding b_{jt} in the current period t. These considerations lead to an additional markup to compensate for each future period.

Therefore, the optimal bidding strategy for a given period t can be stated as follows:

$$\begin{aligned}\beta_t^S(c_j) &= c_j + \int_{c_j}^{\beta_{t-1}^S(c_j)} \left(\frac{1-F(x)}{1-F(c_j)}\right) dx \\ &+ \sum_{j=t+1}^{T} \left[\left(\frac{Q_j}{Q_t}\right) \cdot \int_{\beta_t^S(c_j)}^{\beta_{t-1}^S(c_j)} \left(\frac{1-F(x)}{1-F(c_j)}\right) dx \right] \end{aligned} \quad (35)$$

where the last period bidding strategy is given by the following:

$$\beta_T^S(c_j) = c_j + \int_{c_j}^{\beta_{T-1}^S(c_j)} \left(\frac{1-F(x)}{1-F(c_j)}\right) dx \quad (36)$$

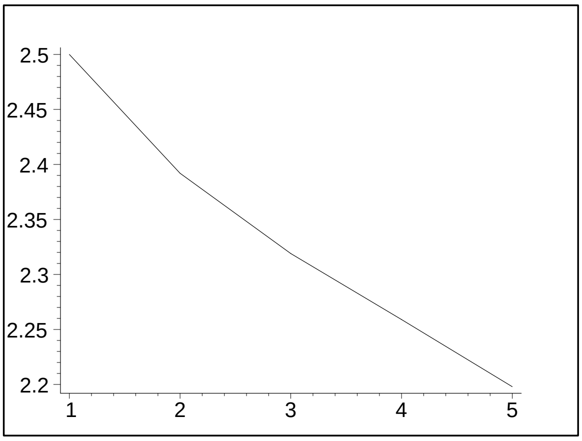

Figure 8: Equilibrium Price Schedule of A Strategic Supplier

Equations 35 and 36 are obtained using a dynamic program logic. First, we define the optimal bidding strategy for the last period in the project life. Next, we substitute this strategy in the expected joint-profit equation of the previous period in order to find the previous period optimal strategy. We continue to substitute each found strategy in the previous period's expected joint-profit equation until we reach the initial period. Finally, we can evaluate the bidding strategies of each period in a forward procedure. Figure 8 shows an example view of the equilibrium price pattern of a given strategic supplier.

Proposition 4.8 *The path followed by the submitted bids for a T-period auction will depend on the distribution of the quantity demanded throughout the periods.*

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Proof. As the optimal bidding strategy in any period t is defined by equation 35, and it is easy to see that the markup in any period is directly proportional to the ratio of the further period quantities to the current period quantity. The more fluctuation in the demand, the more fluctuation there will be in the mark-ups, leading to changes in the path that the optimal bid follows. □

An extension that we have considered for the case of strategic suppliers is the stochastic demand for the second period. As the quantity demanded does not have any role in the determination of the submitted bid for myopic suppliers, it is not important whether or not the demands are deterministic. However, the quantity demanded has a significant role in setting the equilibrium bids for strategic suppliers. The stochastic quantity demanded will be a straightforward extension of the studied model, because the costs are independent of the quantity demanded.

$$\beta_1^S(c_j) = c_j + \int_{c_j}^{r_1} \left(\frac{1-F(x)}{1-F(c_j)}\right) dx + \frac{E[Q_2]}{Q_1} \int_{\beta_1^S(c_j)}^{r_1} \left(\frac{1-F(x)}{1-F(c_j)}\right) dx \quad (37)$$

The only difference from the deterministic demand will be in the adjustment of compensation markup. The supplier will adjust the compensation markup based on the expected value of the quantity demanded in the second period, instead of the actual quantity demanded in the second period. The same type of extension will hold for all of the strategic supplier bidding strategies.

As we mentioned at the outset of this long-term setting analysis, the declining-price contract is a weakly dominant strategy for a buyer. Starting with the all strategic suppliers model, we would like to show that although the reserve price has an effect on the expected periodical payments, it does not change the expected total project procurement cost. Because the strategic suppliers consider the future periods while setting their current period bids, they compensate for future expected losses. Therefore, changing the reserve price will only change these adjustments, and at the end the total will remain the same as if the contract were a fixed price contract. In terms of mechanism design perspective, the total expected payment to a single supplier is given by the following:

$$\bar{M}_j(c_j) = \sum_{t=1}^{T} \left[Q_t \left((1 - F(c_j))c_j + \int_{c_j}^{r_1} (1 - F(x))\, dx \right) \right] \tag{38}$$

As can be seen from equation 38, the expected total procurement cost is independent of the reserve prices set in periods other then the first period.

However, in the case of all myopic suppliers, the reserve price policy significantly affects the expected total procurement cost since the myopic suppliers do not consider the future periods and do not compensate for future possible losses in their current period bids. Therefore, the buyer's best response is to set a declining-price policy to capture the surplus instead of leaving it totally to the suppliers.

5. Numerical Study. In this section, we try to visualize the stated propositions by providing basic examples. We assume that the suppliers' types are independently and identically distributed according to the uniform distribution defined on the closed interval [0,1], donated by U(0,1). We choose a uniform distribution because it is reasonable to assume supplier types are evenly distributed in a given range, and uniform distribution is commonly used in the auction literature.

We assume the any supplier's cost is defined by the following equation:

$$C(\theta, Q) = (45 + 10\theta) - 5Q + (0.6 - 0.2\theta)Q^2$$

.

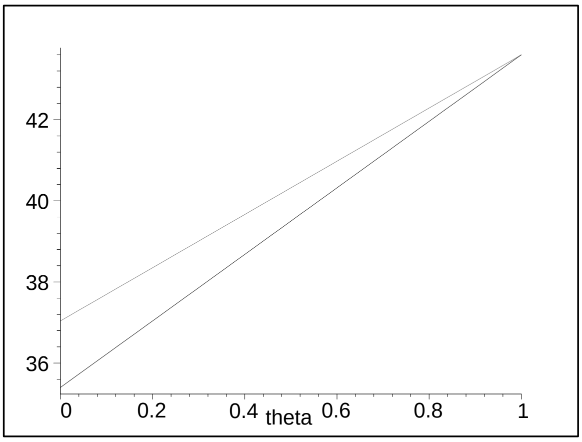

Figure 9: Bidding Function versus Type (Case-I)

The critical quantity Q_c for this example is seven lots. Assuming that the quantity demanded is three lots, i.e. Q=3, we will look at the case $Q < Q_c$. First we look at the relation between the submitted bid and the true type. In figure 9, the upper linear function represents the equilibrium bidding strategy of a supplier competing with four other suppliers, and the lower linear function represents the average cost of the same supplier, as functions of her true type.

As stated in the proposition, the submitted bid is an increasing function of the type. Another observation is that the profit margin, the difference between the submitted bid and the true cost, diminishes as the type increases. Intuitively, the supplier with a higher type needs to decrease the added markup due to the loss of competitive advantage as her cost comes closer to the upper limit of the possible cost space.

Next, in figure 10, we provide the supplier's equilibrium bidding strategy, with a true type of 0.5, as a function of the number of participating suppliers. And the lower straight line in this graph shows the same supplier's average cost. As seen from the graph, the submitted bid exponentially approaches true cost as the number of opponents increases.

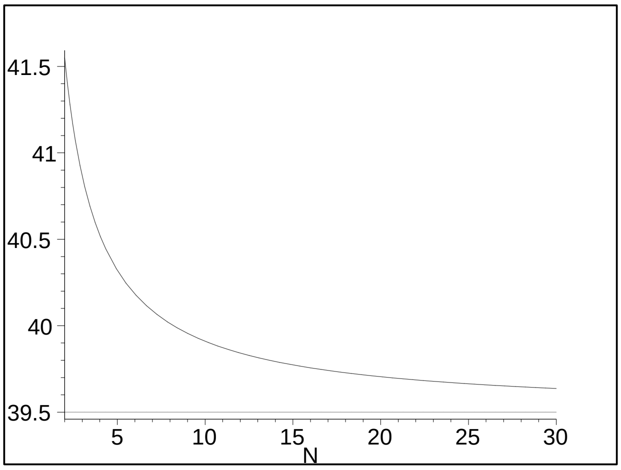

Figure 10: Bidding Function versus N (Case-I)

In the following example, we visualize the effect of stochastic demand. We assume that demand follows the uniform distribution in the finite range [2,6]. In figure 11, the line starting from a higher value shows the bidding function under demand uncertainty, while the other represents the bidding function in case of a deterministic demand, that is equal to $E[q]$, i.e. 4 lots in this example.

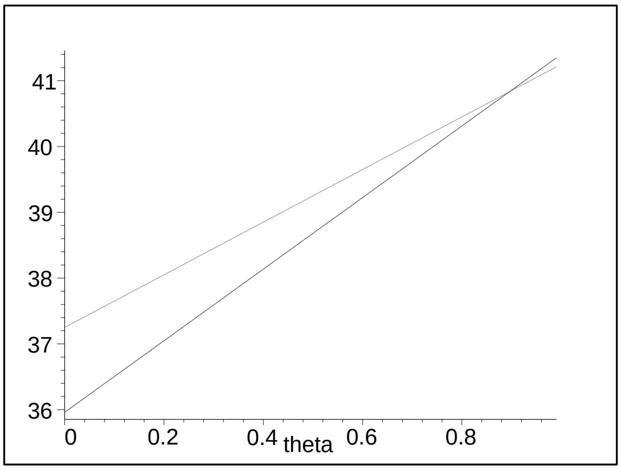

Figure 11: Effect of Demand Uncertainty (Case I)

In figure 12, the upper line represents the bidding function under stochastic demand, while the other line shows the deterministic-demand case bidding function. In this example, we assume that demand follows the uniform distribution defined on [8,10]. Thus, in the deterministic equivalent case, $Q = E[q] = 9$ lots.

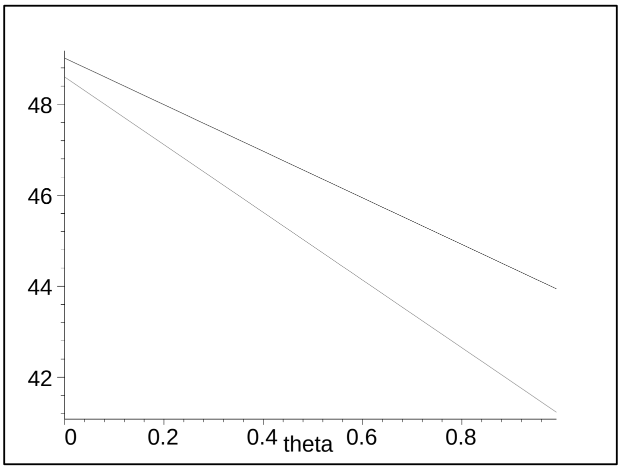

Figure 12: Effect of Demand Uncertainty (Case II)

6. Conclusion. In this paper, we consider the derivation of capacity-constrained suppliers' competitive bidding strategies in a repeated procurement auction setting. First, we discuss the detailed derivation of the equilibrium bids in a single period procurement setting. The single period setting provides the fundamental results for the repeated setting.

Based on our best knowledge of the current literature, this study is the first to consider jointly capacity, cost functions capturing economies and diseconomies of scale, and repeated procurement setting. Our study captures various U-shaped functions that can be generalized to represent cost as a function of attributes other than quantity. Such attributes might be quality and delivery time related attributes. As a closely related extension, we plan to focus on such generalizations in future work. We model various U-shaped cost function possibilities to capture both homogenous and non-homogenous supplier structures while determining the competitive bids.

Since the cost is a function of quantity rather than being a static constant, the interaction of the suppliers' cost functions play an important role in deriving the winning probabilities, and hence the equilibrium bids. Therefore, we study pairwise single- and double-crossing scenarios. We start our analysis for both scenarios with the restricted setting and then generalize the results to the arbitrary number of crossing setting for N suppliers.

Although we cannot extensively characterize the equilibrium bidding functions for general distributions in the second period of a repeated setting, we observe how bid aggressiveness changes based on the quantities demanded in both periods and on the cost function structure. We show that there might still be scenarios in which the incumbent might have a winning chance in the second period even in the case of losing comparative advantage for the second period by winning in the first period.

Due to the cost asymmetries between the incumbent and the entrants in the second period, efficient allocation cannot be guaranteed. Because the equilibrium bids are asymmetric, there exists a positive probability that there might be a more efficient (i.e. less costly) supplier than the winner in the second period.

In the long-term contracting setting, we distinguish between myopic and strategic suppliers to emphasize the effect of the strategic thinking on determining the equilibrium prices, and hence the effect on the total procurement cost for the buyer. The strategic supplier of a given cost inflates the bid of a myopic supplier who has the same given cost. The inflation amount depends on different attributes for each period. This can be considered a compensation (hedging) amount against the possible loss in the future due to the current period bid. From the mechanism design perspective, if the suppliers are strategic, then the payment rule should provide an additional subsidy to the suppliers so that the corresponding direct mechanism could be incentive compatible and individually rational. The extent of compensating opportunistic losses depends on information availability and market structure.

From the buyer's point of view, offering a declining-price contract is a weakly dominant strategy in long-term contracting. This policy does not affect a strategic supplier's expected total profit. It only changes the periodical expected profits, but, on the overall, they end up with the same expected total profit. However, in the case of myopic players, the policy plays a significant role in determining the expected total profit. Therefore, the buyer could gain some of the surplus that would otherwise go totally to the suppliers by setting the reserve price to the winning bid in the case of myopic suppliers.

However, strategic behavior in the short-term contracting setting described by this study is intractable, and therefore it is reasonable to assume that suppliers will determine their bids naively. In the case of strategic thinking, due to the asymmetry among suppliers at the second period, the equilibrium bids submitted in the first period will also be asymmetric. Therefore it might not be possible to have full separation of types, and we need to deal with pooling which cannot be characterized under a first price auction setting.

Therefore, strategic thinking can only be considered by suppliers if the auction in the second period is a second-price auction instead of a first-price auction. Because bidding the true cost is a weakly dominant strategy even in the case of asymmetries, the second-period bids will be known while bidding for the first period, and these can strategically be taken into account in setting the first-period bid. This will be a challenging extension of the current study.

We also believe that this study provides a basic framework on which more advanced auction settings can be built. We will extend this study to consider non-price attributes that play crucial roles in selecting the awarded supplier in the industrial procurement setting. Though the price will always be a major decision factor, non-price attributes such as quality, delivery performance, or service level might be equally important in choosing the awarded supplier in practice. Another extension will be to study the suppliers' optimal strategies in case of bundling option for multi-item auctions in the presence of non-price attributes. As a final remark, an in-depth theoretical analysis of strategic thinking in all these settings will remain a challenging research question.

Appendix A. Detailed Mechanism Derivation. Given a direct mechanism $(\mathbf{P}, \mathbf{M})$ consisting of a pair of functions $\mathbf{P} : \Theta \to \Delta$ and $\mathbf{M} : \Theta \to \mathbb{R}^N$ where $P_j(\theta)$ is the probability that supplier j will be awarded and $M_j(\theta)$ is the expected payment received by supplier j, we represent the probability that supplier j will be awarded the contract when she reports her type to be ϑ_j instead of θ_j, while all other suppliers report their costs truthfully, by $p_j(\vartheta_j)$. Similarly, $m_j(\vartheta_j)$ represents the expected payment to supplier j when her report is ϑ_j, and all other suppliers tell the truth.

The probability of winning the contract and the expected payment received depend only on the reported type and not on the true type. Supplier j's expected payoff when she reports ϑ_j instead of her true type θ_j, again assuming that all other suppliers tell the truth, can then be defined as:

$$m_j(\vartheta_j) - p_j(\vartheta_j)C(\theta_j, Q) \tag{39}$$

Representing the expected profit of supplier j when she submits her true type θ_j, by $\pi_j(\theta_j)$, the direct revelation mechanism $(\mathbf{P}, \mathbf{M})$ is said to be IC if for all j, for all θ_j and ϑ_j:

$$\pi_j(\theta_j) \equiv m_j(\theta_j) - p_j(\theta_j)C(\theta_j, Q) \geq m_j(\vartheta_j) - p_j(\vartheta_j)C(\theta_j, Q) \tag{40}$$

Further derivation of the mechanism depends on the specific structure of the cost function and quantity demanded. Due to our single crossing point assumption, there will be two different mechanisms under deterministic demand: one for quantities less than Q_c and one for quantities more than Q_c. We call the first one as *LTP Mechanism*, because the lowest type supplier is the preferred supplier that provides the lowest cost for quantities less than Q_c. By following similar logic, we call the second one as *HTP Mechanism*. Due to the fact that both suppliers have the same cost when $Q = Q_c$, there will not be any competition between the suppliers. Hence, each supplier will bid exactly the same amount equivalent to the cost, resulting in neither gain nor loss.

Proposition A.1 *LTP mechanism is incentive compatible if and only if the associated p_j is non-increasing, whereas HTP is incentive compatible if and only if p_j is non-decreasing.*

Proof. IC implies that: $\pi_j(\theta) \geq \max_{\vartheta \in \Theta_j}\{m_j(\vartheta) - p_j(\vartheta)C(\theta_j, Q)\}$. In words, each supplier will maximize her expected payoff when she reveals her true type in a direct mechanism which has the appropriate payment and allocation rules that provide the required incentive scheme.

From equations 39 and 40, we can state that incentive compatibility is equivalent to the following requirement for all θ_j and ϑ_j:

$$\pi_j(\vartheta) \geq \pi_j(\theta) + p_j(\theta)(C(\theta_j, Q) - C(\vartheta_j, Q)) \tag{41}$$

The convexity of $\pi_j(\theta)$ depends on the structure of $C(\theta, Q)$. As stated in the U-shaped cost function conditions, cost being an affine function of type, the equilibrium expected profit $\pi_j(\theta)$ is a convex function.

The relation defined by equation 41 implies that for all θ, $-p_j(\theta)C^{'}(\theta, Q)$ is the slope of a line that supports the function π_j at the point θ. As π_j is absolutely continuous, it can be differentiable almost everywhere in the interior of its domain:

$$\pi_j^{'}(\theta) = -p_j(\theta)C^{'}(\theta, Q) \tag{42}$$

Under LTP mechanism, equation 42 implies that p_j is a nonincreasing function since $C^{'}(\theta, Q) > 0$ and π_j is convex.

In order to show that incentive compatibility is implied by the statement that p_j is a non-increasing function, we can rewrite equation 41 by using equation 44 as:

$$\int_{\vartheta_j}^{\theta_j} p_j(t_j)C^{'}(t_j, Q)\, dt_j \geq p_j(\theta_j)(C(\theta_j, Q) - C(\vartheta_j, Q)) \tag{43}$$

The relation defined by equation 43 certainly holds if p_j is nonincreasing.

However, under HTP mechanism p_j is nondecreasing based on equation 42. This is due to the fact that $C^{'}(\theta, Q) < 0$ while π_j is convex. In a similar way, it can be shown that $\int_{\vartheta_j}^{\theta_j} p_j(t_j)C^{'}(t_j, Q)\, dt_j \geq p_j(\theta_j)(C(\theta_j, Q) - C(\vartheta_j, Q))$ holds when p_j is nondecreasing. □

After discussing the incentive compatibility of the mechanisms, we continue our study by considering individual rationality. With the implicit assumption that the supplier has no other outside option where she can utilize her existing capacity to earn profit, we can set the outside option profit to zero. At the same, we also assume that the supplier does not incur costs if she chooses not to participate. Hence, a direct mechanism $(\mathbf{P}, \mathbf{M})$ is individually rational if, for all j and θ_j, the equilibrium expected profit $\pi_j(\theta_j) \geq 0$. The equivalence of this requirement under *LTP* and *HTP mechanisms* is stated by the following proposition.

Proposition A.2 *LTP mechanism is individually rational if* $m_j(\theta_{max}) \geq p_j(\theta_{max})C(\theta_{max}, Q)$, *while HTP mechanism is individually rational if* $m_j(\theta_{min}) \geq p_j(\theta_{min})C(\theta_{min}, Q)$.

Proof. In LTP mechanism, we can define $\pi_j(\theta_j)$ as the definite integral of its derivative:

$$\pi_j(\theta_j) = \pi_j(\theta_{max}) + \int_{\theta_j}^{\theta_{max}} p_j(t_j)C^{'}(t_j, Q)\, dt_j \tag{44}$$

Equation 44 implies that the shape of the expected profit function is completely determined by the allocation rule $\mathbf{P}$ alone. The payment rule $\mathbf{M}$ only determines the constant $\pi_j(\theta_{max})$.

The expected payment to j under an incentive compatible direct mechanism $(\mathbf{P}, \mathbf{M})$ is given by:

$$m_j(\theta_j) = m_j(\theta_{max}) - p_j(\theta_{max})C(\theta_{max}, Q) + p_j(\theta_j)C(\theta_j, Q) + \int_{\theta_j}^{\theta_{max}} p_j(t_j)C^{'}(t_j, Q)\, dt_j \tag{45}$$

Given LTP mechanism is incentive compatible, then IR is equivalent to the requirement that $\pi_j(\theta_{max}) \geq 0$, and hence $m_j(\theta_{max}) \geq p_j(\theta_{max})C(\theta_{max}, Q)$.

In a similar logic, we can derive the equilibrium expected profit under HTP mechanism to be defined by:

$$\pi_j(\theta_j) = \pi_j(\theta_{min}) - \int_{\theta_{min}}^{\theta_j} p_j(t_j)C^{'}(t_j, Q)\, dt_j \tag{46}$$

And the payment function will be defined as:

$$m_j(\theta_j) = m_j(\theta_{min}) - p_j(\theta_{min})C(\theta_{min}, Q) + p_j(\theta_j)C(\theta_j, Q) - \int_{\theta_{min}}^{\theta_j} p_j(t_j)C^{'}(t_j, Q)\, dt_j \qquad (47)$$

Hence, IR is equivalent to $m_j(\theta_{min}) \geq p_j(\theta_{min})C(\theta_{min}, Q)$ under HTP mechanism. □

Considering the buyer as the designer of these mechanisms, we provide the properties of these incentive compatible and individually rational mechanism so that they are also optimal. By optimality of the mechanism, we mean that the buyer minimizes the total procurement cost defined by the following equation:

$$\sum_{j \in \mathcal{N}} E[m_j(\theta_j)] = \sum_{j \in \mathcal{N}} \int_{\theta_{min}}^{\theta_{max}} m_j(\theta_j) f_j(\theta_j)\, d\theta_j \qquad (48)$$

We claim the following proposition defines optimal allocation and payment rules for *LTP* and *HTP mechanisms.*

Proposition A.3 *The optimal LTP mechanism is given by:*

$$\begin{aligned} P_j(\theta) > 0 &\Leftrightarrow \psi_j(\theta_j) = \min_{i \in \mathcal{N}} \psi_i(\theta_i) \leq C(\theta_{max}, Q) \\ M_j(\theta) &= P_j(\theta)C(\theta_j, Q) + \textstyle\int_{\theta_j}^{\theta_{max}} P_j(\vartheta_j, \theta_{-j})C^{'}(\vartheta_j, Q)\, d\vartheta_j \end{aligned} \qquad (49)$$

where the optimal HTP mechanism is defined as:

$$\begin{aligned} P_j(\theta) > 0 &\Leftrightarrow \psi_j(\theta_j) = \min_{i \in \mathcal{N}} \psi_i(\theta_i) \leq C(\theta_{min}, Q) \\ M_j(\theta) &= P_j(\theta)C(\theta_j, Q) - \textstyle\int_{\theta_{min}}^{\theta_j} P_j(\vartheta_j, \theta_{-j})C^{'}(\vartheta_j, Q)\, d\vartheta_j j \end{aligned} \qquad (50)$$

Proof. We will go over the proof for *LTP mechanism* in detail and define the similarity for *HTP mechanism.* Substituting equation 45 to equation 48, we can rewrite the expected total procurement cost as:

$$\begin{aligned} \sum_{j \in \mathcal{N}} E[m_j(\Theta_j)] &= \sum_{j \in \mathcal{N}} (m_j(\theta_{max}) - p_j(\theta_{max})C(\theta_{max}, Q)) \\ &+ \sum_{j \in \mathcal{N}} \int_{\theta_{min}}^{\theta_{max}} \left(C(\theta_j, Q) + \frac{F_j(\theta_j)}{f_j(\theta_j)} C^{'}(\theta_j, Q) \right) p_j(c_j) f_j(c_j)\, dc_j \end{aligned} \qquad (51)$$

As the first term in equation 51 is constant, the buyer will focus on the second term, which is dependent on the allocation rule, to minimize the total expected procurement cost.

We can simplify equation 51 by defining $\psi_j(\theta_j) \equiv C(\theta_j, Q) + \frac{F_j(\theta_j)}{f_j(\theta_j)} C^{'}(\theta_j, Q)$. This term can be named as the virtual cost of a supplier with true type θ_j. When the virtual cost is an increasing function of the true type θ_j, the design problem is called regular. A sufficient condition for regularity is that $\frac{F_j(\theta_j)}{f_j(\theta_j)}$ is increasing.

Rewriting $p_j(\theta_j)$ explicitly as $\int_{\Theta_{-j}} P_j(\vartheta, \theta_{-j}) f_{-j}(\theta_{-j})\, d\theta_{-j}$, the expected total procurement cost can be reexpressed as:

$$\sum_{j \in \mathcal{N}} (m_j(\theta_{max}) - p_j(\theta_{max})C(\theta_{max}, Q)) + \int_{\Theta} \left(\sum_{j \in \mathcal{N}} \psi_j(\theta_j) P_j(\theta) \right) f(\theta)\, d\theta \qquad (52)$$

The function defined by equation 52 can be considered as a weighting function. Therefore, the optimal solution will be to assign positive weight to the smallest virtual costs since the objective is to minimize the function. If the buyer is not willing to pay more than the maximum possible cost of producing at the given quantity level, then the optimal allocation and payment rules can be defined as:

$$\begin{aligned} P_j(\theta) > 0 &\Leftrightarrow \psi_j(\theta_j) = \min_{i \in \mathcal{N}} \psi_i(\theta_i) \leq C(\theta_{max}, Q) \\ M_j(\theta) &= P_j(\theta)C(\theta_j, Q) + \textstyle\int_{\theta_j}^{\theta_{max}} P_j(\vartheta_j, \theta_{-j})C^{'}(\vartheta_j, Q)\, d\vartheta_j \end{aligned} \qquad (53)$$

The above defined mechanism is both incentive compatible and individually rational. In order to prove IC, based on proposition A.1 it is sufficient to show that p_j is non-increasing. Suppose $\vartheta_j < \theta_j$. Due to the regularity condition, $\psi_j(\vartheta_j) < \psi_i(\theta_j)$ and therefore for any θ_{-j}, it is true that $P_j(\vartheta, \theta_{-j}) \geq P_j(\theta, \theta_{-j})$. Hence, p_j is a non-increasing function. For IR, by definition it is clear that $m_j(\theta_{max}) = p_j(\theta_{max})C(\theta_{max}, Q)$.

The defined mechanism is optimal as it minimizes both parts of the equation 52. It minimizes the first part by setting the payment to the highest type at its possible minimum value, and particularly it minimizes the second part by giving positive weight only to the minimal terms that are lower than the maximum possible cost level.

Same methodology can be used to derive the optimality results for *HTP mechanism* by performing appropriate changes. The virtual cost will now be defined as

$$\psi_j(\theta_j) \equiv C(\theta_j, Q) - \frac{1 - F_j(\theta_j)}{f_j(\theta_j)} C^{'}(\theta_j, Q)$$

. For the regularity of the design problem, $\frac{1-F_j(\theta_j)}{f_j(\theta_j)}$ being a decreasing function is sufficient. The optimal allocation and payment rules are given by:

$$\begin{aligned} P_j(\theta) > 0 &\Leftrightarrow \psi_j(\theta_j) = \min_{i \in \mathcal{N}} \psi_i(\theta_i) \leq C(\theta_{min}, Q) \\ M_j(\theta) &= P_j(\theta)C(\theta_j, Q) - \textstyle\int_{\theta_{min}}^{\theta_j} P_j(\vartheta_j, \theta_{-j})C^{'}(\vartheta_j, Q)\, d\vartheta_j j \end{aligned} \tag{54}$$

This can be restated more specifically as:

$$P_j(\theta) = \begin{cases} 1 & if \psi_j(\theta_j) < \min_{i \neq j} \psi_i(\theta_i) \ and \ \psi_j(\theta_j) \leq C(\theta_{min}, Q) \\ 0 & otherwise \end{cases}$$

$$M_j(\theta) = \begin{cases} C(\kappa_j(\theta_{-j}), Q) & if P_j(\theta) = 1 \\ 0 & if P_j(\theta) = 0 \end{cases} \tag{55}$$

In equation 57, $\kappa_j(\theta_{-j})$ is defined by:

$$\kappa_j(\theta_{-j}) = sup\, \{\vartheta_j : \psi_j(\vartheta_j) \leq C(\theta_{min}, Q) \ and \ \forall\, i \neq j,\ \psi_j(\vartheta_j) \geq \psi_i(\theta_i)\} \tag{56}$$

□

Whenever the mechanism design problem is regular, the optimal *LTP mechanism* can be restated as:

$$P_j(\theta) = \begin{cases} 1 & if \psi_j(\theta_j) < \min_{i \neq j} \psi_i(\theta_i) \ and \ \psi_j(\theta_j) \leq C(\theta_{max}, Q) \\ 0 & otherwise \end{cases}$$

$$M_j(\theta) = \begin{cases} C(\kappa_j(\theta_{-j}), Q) & if P_j(\theta) = 1 \\ 0 & if P_j(\theta) = 0 \end{cases} \tag{57}$$

In equation 57, $\kappa_j(\theta_{-j})$ is defined by:

$$\kappa_j(\theta_{-j}) = inf\, \{\vartheta_j : \psi_j(\vartheta_j) \leq C(\theta_{max}, Q) \ and \ \forall\, i \neq j,\ \psi_j(\vartheta_j) \geq \psi_i(\theta_i)\} \tag{58}$$

In words, only the awarded(winning) supplier receives some payment, that is actually the highest value that would result in her winning. $\kappa_j(\theta_{-j})$ represents the most competitive opponent of supplier j among the remaining suppliers. Similar argumentation holds for the optimal *HTP mechanism*.

Appendix B. Proof of Proposition 3.1. PROOF. We can prove this proposition, by arguing that while all but supplier j follow the strategy $\beta(\theta)$ defined by equation **??**, it is optimal for a given supplier j to follow the same strategy $\beta(\theta_j)$.

In other words, we prove the proposition by showing that this bidding strategy is incentive compatible and individually rational for a given supplier.

The corresponding direct mechanism $(\mathbf{P}, \mathbf{M})$ for this mechanism can be defined:

$$\begin{aligned} P_j(\theta_j) &= (1 - F(\theta_j)) \\ M_j(\theta_j) &= P_j(\theta_j)C(\theta_j, Q) + \int_{\theta_j}^{\theta_{max}} C^{'}(x, Q)(1 - F(x))\, dx \end{aligned} \tag{59}$$

In order to show that the corresponding direct mechanism is incentive compatible, we need to show that $P_j(\theta_j)$ is a decreasing function of θ_j given by proposition A.1. Taking the partial derivative leads to the following equation: $\frac{\partial P_j(\theta_j)}{\partial \theta_j} = (N-1)(1-F(\theta_j))^{(N-2)}(-f(\theta_j)) \leq 0$. Hence, this proves that the corresponding direct mechanism is incentive compatible. As $M_j(\theta_{max}) = 0$, the corresponding direct mechanism is also individually rational due to proposition A.2.

Alternatively, we can prove that the bidding function defined in equation **??** is indeed the symmetric equilibrium strategy, by using contradiction. Suppose supplier j bids b instead of bidding $\beta(\theta_j)$. We can denote the type for which b is the equilibrium bid by η, i.e. $\beta(\eta) = b$. We can define supplier j's expected profit from bidding $\beta(\eta)$ although her true type is θ as:

$$\pi(\beta(\eta), \theta) = (1 - F(\eta))\left(\beta(\eta) - C(\theta_j, Q)\right)Q \tag{60}$$

We can call Δ to represent the difference between the actual profit $\pi(\theta)$, that the supplier gets when bids according to true type, and the deceived profit $\pi(\beta(\eta), \theta)$, that she gets when reveals her type other than her true type.

$$\begin{aligned} \Delta &= \pi(\theta) - \pi(\beta(\eta), \theta) \\ &= (1 - F(\eta))\left[C(\theta, Q) - C(\eta, Q)\right]Q + \int_{\theta}^{\eta} C^{'}(x, Q)\left(1 - F(x)\right) dx \end{aligned} \tag{61}$$

Since $C(\theta, Q)$ is an affine function of θ,

- $C^{'}(x, Q) = K$ for all $x \in \Theta$
- $C(\theta, Q) - C(\eta, Q) = (\theta - \eta)C^{'}(x, Q)$

Simplifying the equation 61 we can show that Δ is always negative, proving that the bidding function defined by equation **??** is the symmetric equilibrium strategy for each supplier. □

Appendix C. Proof of Proposition 3.2. PROOF. We will follow a similar way to the one we use for proposition 3.1. We argue that while all but supplier j follow the strategy $\beta(\theta)$ defined by equation **??**, it is optimal for a given supplier j to follow the same strategy $\beta(\theta_j)$. In other words, we prove the proposition by showing that this bidding strategy is incentive compatible and individually rational for a given supplier.

The corresponding direct mechanism $(\mathbf{P}, \mathbf{M})$ for this mechanism can be defined:

$$\begin{aligned} P_j(\theta_j) &= (F(\theta_j)) \\ M_j(\theta_j) &= P_j(\theta_j)C(\theta_j, Q) - \int_{\theta_{min}}^{\theta_j} C^{'}(x, Q)(F(x))\, dx \end{aligned} \tag{62}$$

In order to show that the corresponding direct mechanism is incentive compatible, we need to show that $P_j(\theta_j)$ is an increasing function of θ_j. Taking the partial derivative leads to the following equation: $\frac{\partial P_j(\theta_j)}{\partial \theta_j} = (N-1)(F(\theta_j))^{(N-2)}(f(\theta_j)) \geq 0$. Hence, this proves that the corresponding direct mechanism is incentive compatible. As $M_j(\theta_{min}) = 0$ and $C^{'}(x, Q) < 0\ \forall x$, the corresponding direct mechanism is also individually rational.

It is again possible to show that this is indeed the equilibrium strategy by arguing a supplier can never be better off by lying when all the others are telling the truth. □

Appendix D. Mathematical Extension to N Suppliers. In the main body of this study, we consider two suppliers game. N suppliers game will just be a mathematical extension of the stated results, given the single-crossing point assumption. However, in real life this assumption might be too restrictive. Therefore, we work on an extension to pairwise single crossing points, resulting in arbitrary number of crossing points, as a further study.

In the direct mathematical extension, the significant difference will be on the winning probability, which then effects the equilibrium bid functions. The change will be to have $N-1$ power of all probability terms in the defined functions, as each supplier will be trying to compete $N-1$ other suppliers.

As the number of suppliers participating in the procurement auction increases, the bid decreases as a result of the reduction in the mark-up. Specifically, the submitted bid converges exponentially to the true cost as the number of participating suppliers increases.

The intuition is that with the increasing number of suppliers participating in the auction, the competition increases. This will lead to the conclusion that the probability of a lower cost supplier's participation increases and hence the suppliers reduce their mark-ups to be more competitive. Analytically, taking the first order partial derivative of $\beta(\theta_j)$ with respect to N will prove the proposition by showing that the partial derivative is indeed negative for both cases.

The modification in the short-term contracting involves more detailed extension. At the end of the first period there will be more than one entrant, and hence each of them will update her beliefs about the other entrants as:

$$\begin{aligned} F_{jk}(\theta \mid \theta_i) &= \frac{F(\theta) - F(\theta_i)}{1 - F(\theta_i)} \\ F_{jk}(\theta) &= \int_{\theta_{min}}^{min(\theta,\theta_j)} \left(\frac{F(\theta) - F(\theta_i)}{1 - F(\theta_i)} \right) \frac{f(\theta_i)}{F(\theta_j)} \, d\theta_i \end{aligned} \tag{63}$$

Hence, the first order condition equations will look like:

$$\begin{aligned} \phi_E^{'}(b) &= \frac{1 - F(\phi_E(b))}{(N-1) f(\phi_E(b))} \cdot \frac{1}{b - C(\phi_I(b), Q_1 + Q_2)} \\ \phi_I^{'}(b) &= \frac{F(\theta_f) - F(\phi_I(b))}{f(\phi_I(b))} \left(\frac{1}{b - C(\phi_E(b), Q_2)} - \frac{(N-2) f(\phi_E(b))}{1 - F(\phi_E(b))} \phi_E^{'}(b) \right) \end{aligned} \tag{64}$$

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