Supplier Coalition Formation in Buyer-Oriented Markets

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Modern buyer-oriented markets present participating suppliers with the opportunity to form coalitions. One of the attractive features of coalition formation is that it enables a group of suppliers to compete at the auction for production of a certain product package even if the individual suppliers cannot do so separately. Leaving the communicational difficulties between the coalition members out of the consideration, the current study focuses on the proposed Fare Opportunistic Profit Sharing mechanism (FOPS) as a mechanism of coalition coordination. The bidding strategies of the coalition members are discussed for two different cases: auctioning of a single product, and auctioning of a multiple product package.

I. Introduction

The e-Commerce markets present interesting and relatively new topics for research. One of their typical features is the greater number of suppliers as compared to the number of buyers. The *descending auction* is one of many forms of E-Commerce auctions. In this type of auction, the buyer places the information on the package he wants to buy and his maximum price for the package. The suppliers compete for the right to produce the package. If the package contains a set of the different products, then to produce this package, a supplier has to be able to produce all elements of the package, and has to have competitive prices. By forming *coalitions*, suppliers who originally were left out of the auction are now able to bid. [1] For example, consider the case where General Motors, the buyer, is auctioning the right to produce for it a certain model of the car seat. The seat itself contains many different parts, which could be produced by different

[1] Example 1: Coalition has only two players: $Supplier_1$ offers products {A, B, D, E, K},and $Supplier_2$ offers products {B, C, E, G}. Therefore, this Coalition can bid for all possible subsets of the product set {A, B, C, D, E, G, K}.

suppliers and later assembled either by the one of the producing suppliers or by a separate assembler. Generally, the larger the package is, the easier it is to see the involvement of the multiple suppliers into production of the parts for the package with subsequent assembly.

Therefore, by forming a coalition, suppliers hope to increase their market share, and as a result, to increase their profits. Increased competition between the suppliers leads to a lower price on the package. This is the major incentive for buyers to encourage formation of supplier coalitions.

Coalition formation is widely studied in Game Theory, but emerging platform for business-to-business (B2B) transactions require a review of the rules of coalition formation specific to the electronic market. Here are the two problems that will be addressed in this study:

1) One-Buyer vs. a supplier coalition at the auction of a single product;
2) One-Buyer, vs. a supplier coalition at the auction of the package of different products.

II. Background Information.

2.1 Auction Mechanism.

This chapter introduces the definitions of the auction model, as well as other related concepts that will be used throughout the discussion.

The *Second-price, sealed-bid ascending auction* model was first introduced by William Vickrey [1961]. It can be easily adapted to fit the buyer-oriented market by replacing an *ascending auction* with a *descending* one[2].

Definition 1. *Sealed-bid auction* is an auction in which all bidders submit sealed bids and the best offer wins the auction. All bids are private and committed.

Definition 2. A *second-price ascending auction* is a sealed-bid auction in which the bidder who placed the lowest bid wins the auction at the second lowest bid price.

Definitions 1 and 2 outline the rules of the auction under consideration. The buyer is interested in acquiring a particular package. The auction starts by the buyer announcing complete information about the package (i.e. dimensions, materials used, special features, etc.) that is important to insure supplier's knowledge of all characteristics of the product. Before the auction, the buyer names the maximum price h is willing to pay for the package. The suppliers are given some time to evaluate whether they can produce such a package, and how much it is going to cost them. At the beginning of the auction, all the suppliers and the supplier coalitions that have decided to participate in the auction will place their bids. To insure privacy of the auction participants, none of the information about the prices becomes public, and only the winning price is announced. At the auction, the buyer's price is automatically lowered by a fixed amount until only a single bidder is left. At the end of the auction, the buyer and the supplier that has won the auction sign a contract for production of the package. It is easy to see that the price rewarded to the supplier is, in fact, the second lowest price placed at the auction by the bidders.

[2] *Descending* auction is also called the *reversed* auction

2.2 Properties of the Supplier Coalition Mechanism.

The basic requirements for a *supplier coalition mechanism*[3] in the electronic market proposed by M. Jin and D. Wu [2001] are listed below. They argued that a proper *profit distribution mechanism*[4] among the coalition members is the key to a successful coalition formation. The following is a list of requirements for a desirable coalition mechanism:

Individual Rationality: Each member of the coalition has to have an expected profit higher then the expected profit from participating in the same market alone.

Information Privacy: No player has to be required to reveal his cost structure or other private information. Players cannot be expected to provide truthful information about their costs.

Observability and Controllability: Information that is utilized by the coalition can only be obtained by observations since each member of the coalition has a right to keep his information private. At the same time, the market controls the actions taken by the mechanism, and a punishment could be imposed on coalition members that have violated the market rules.

Social Welfare Compatibility: The mechanism has to insure that all products produced by the coalition are going to be produced by the members with the lowest cost of the products.

Competitiveness: All players in the market have to have incentive to reduce the cost in order to compete with other players. Therefore, the proposed coalition formation

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[3] *Supplier coalition mechanism* – set of the regulations that outline the way for the coalition members to submit their bids in order to evaluate minimal bid, which coalition should submit during an auction, and profit sharing mechanism.

[4] *Profit sharing mechanism* defines the rules for coalition members to split profit created by coalition.

mechanism should not encourage formation of a *grand coalition* whose goal is to acquire monopoly of the market.

Financial Independence: The proposed coalition formation mechanism has to ensure financial independence of the coalition from the market, i.e., no external subsidy should be provided to any coalition formed by the mechanism.

A coalition has a high chance of winning the auction if generally prices in the coalition are competitive. However, if some members of the coalition fell that they do not receive all appropriate profit as result of the participation in the coalition, they are most likely will leave such coalition. *Unstable* coalition is the coalition that losses members with generally competitive low prices. Whether or not a newly formed coalition will be stable depends on both

external environment (political situation, market situation) and

infrastructure (profit sharing mechanism).

While the external environment is difficult to influence, it is imperative that the proposed fare opportunistic profit sharing (FOPS) does not destabilize the formed coalition.

III. Literature Review.

Profit Sharing in Labor-Managed Economies.

The profit-sharing mechanisms have been a subject of several studies. The authors found that the primary focus of the majority of profit sharing related research publication was on profit distribution within a company. These studies clearly show that the conclusive results are difficult to obtain. The major problem appears to be that profit sharing process depends on the number of intangible factors. These factors are often difficult or impossible to implement in a mathematical model.

Below is a summary of investigations most closely related to the present study.

The equivocal theoretical prescriptions concerning profit-sharing methodologies - particularly the contrasting approaches of equal and performance-based allocations - were reflected in the findings. In Burrows and Black (1998), having unverified information on profit sharing ration, the authors had to conclude that profit-reward systems could have a great impact on partners' incentives.

Another interesting result was found in Heon Jun (1989) paper on the workers forming a joint union. The sharing rule has to be decided through a costly bargaining process. Followed result stated that when people are acting according to their own self-interest, the resulting sharing rule is favorable to both parties only when they are in similar bargaining positions against the third party. This result can be easily verified for the special case of a single product and multiple suppliers, which is the subject of the current investigation. In this case, the expected profit of each party greatly depends on amount of information available to the bidder about the rest of the market players. In the same paper, it was also argued that coalitions are difficult to form when there is a significant amount of asymmetry among participating members. It is have to be mentioned, that the paper by Heon Jun (1998) described above was written under the assumption of "complete information".

In the present study, it was observed that under "complete information" assumption in coalition with multiple players producing the same product, the problem with "free-riders" is unavoidable. Weitzman and Kruse (1990) considered a labor-management problem as well. They argued the repetition of the profit sharing “games” overtime solves “free-rider” problem. In Kim (1998), the following explanation was given: the workers learn to expend the effort together; at the same time often workers gain a better position than managers to monitor each other. In certain situations, the workers can sanction violators of profit-sharing games more effectively than managers.

Results of Seongsu Kim (1998) suggest that profit sharing as a part of the labor contract has an insignificant effect on the profits of the company. Profit sharing is found to increase production level, but also increases the labor costs. This explains why profit sharing has insignificant effect on the employers’ profitability.

Cooperation of the companies sharing one market.

Out of those considered, the articles described in this section are the most closely related to the present study. The focus of the articles is on the alliances formed by companies sharing one market.

Radner (1986) considered a partnership game, where each player’s utility depends on the other players’ action through a commonly observed consequence (e.g. output, profit, price). Different strategies of such alliances have been considered for one-period game, and optimal equilibria of such games have been proved to exist. If a partnership game is repeated infinitely, then efficient combination of one-period actions can be sustained as Nash equilibria of the supergame even if players do not have complete information on the other players and can only observe the resulting consequences.

The article by Morasch (1998) is one of few publications that analyze incentives of firms to form strategic alliances. The author considered an oligopoly market, where a strategic alliance aims to influence the behavior in product market competition by means of a strategic contract between alliance members. Morasch considered an intermediate good production joint venture and has shown that the contractual terms about transfer prices and profit sharing may be used to influence the market competition. Appropriate terms of alliance allow its members to behave together as a “Stackelberg cartel” in relation to the rest of the industry. Members of the alliance are assumed to agree to equally share in the resulting profits or losses of the joint venture. The model assumes linear Cournot oligopoly and identical costs of the production that leads to the following results: as long as the number of firms in the industry does not exceed five, only one alliance will form. Those explicit results depend on the specific assumptions of the model. The more general conclusions are as follows: in small oligopolies, it is relatively likely that the alliance formation process yields a single alliance comprising all or at least almost all firms. In industries with larger number of competitors, such an alliance structure will no longer be feasible. Resulting formation of competing alliances will usually result in a more competitive behavior than in the initial equilibrium.

In the article by Salant et al. (1983), it was shown that only cartels or mergers, which comprise more than 80% of the oligopoly industry, lead to higher profits for the member firms.

Coordination of firms operating in different markets.

The articles analyzed in this section address the effects of Multimarket contracts on the degree of cooperation that firms can sustain in a setting of repeated competition.

Bernheim and Whinston (1990) identify a number of circumstances, typically implying asymmetries between firms or markets, in which the Multimarket contracts facilitate collusion by optimizing the allocation of available enforcing power between markets. On the other hand, when firms and markets are identical and there are constant returns, Multimarket contract does not strengthen firms' ability to collude.

Spagnolo (1999) identify an additional circumstance in which Multimarket contract facilitates collusion. The author was able to show that under a special condition "multi-game contact" facilitates cooperation in a large class of interdependent supergames other than oligopolies.

Adaptive learning of market players.

This section contains the summary of the articles addressing the learning aspect of a market player's behavior.

Agastya (1999) studied the case, when the players learn adaptively how to bargain for the surplus available in the coalition. He shows that stochastically stable allocations are a subset of the core.

Mookherjee and Ray (1991) showed that learning does not reduce the viability of a market-sharing collusion between a given numbers of firms. It has to be mentioned, that the authors restricted their analysis to an oligopoly market with the identical firms.

IV. Fare Opportunistic Profit Sharing Mechanism.

4.1 Basic Assumptions.

The following are the principal rules of a FOPS mechanism:

- All members of the coalition have immediate access to the complete information about a package (or a separate item) placed at the auction by the Buyer.
- The members of the coalition do not have to announce their production costs to participate in the coalition.

- The members of the coalition can choose whether to participate at the auction or not.
- All the requirements imposed by the buyer has to be satisfied by the suppliers that won an auction. If there are fees to be charged by a buyer after the product has been delivered (i.e. late fees, etc.) the supplier responsible for such charge is also responsible for paying those fees.

After evaluating their production costs, the members of the coalition place their bids for the individual products in the auctioned package. Computer software or an independent third party then evaluates the coalition's bid for the whole package as a sum of lowest bids for the parts of the package. The coalition can participate at the auction only if the coalition members have bid for all parts of the package. If the coalition wins the auction, each member of the coalition who announced the lowest price for the particular part is obligated to produce the part at that price. Coalition's profit is equal to the difference between the price rewarded to the coalition at the end of the auction and the coalition's bid for the package.

4.2 Fare Opportunistic Profit Sharing Mechanism: Related Definitions.

The following definitions are applicable to the members of the coalition that wins the auction.

Definition 3. Profit Distribution within a supplier coalition refers to the way in which the overall profit is distributed between the coalition members. This profit is acquired by the coalition by winning the bidding at the auction.

Throughout this study we will be considering only *second-price, seal-bid simultaneous descending auction* described in Section 2.1. Because of the specifics of such auctions at the end of each auction, the winner knows the second lowest price for the package on the market. Proposed fare opportunistic profit sharing mechanism requires the organizer of the auction to supply the winner with the information about the bid placed on each part of the package with the second lowest bid price at the auction. Because of the requirement that the information be kept private, no information about the companies that placed those bids should be announced.

The following notation is applied only to the auction winning coalition, sense profit sharing comes into consideration only after coalition wins the auction:

P_i^I - the lowest price on the product *i* in the coalition.

P_i^{II} - the second lowest price on the product *i* in the coalition.

P_i^{rev} - the price of the product *i* offered by the player/coalition that had placed the second lowest price on the package.[5]

In what follows, we will need the following definitions:

Definition 4. *Pure Profit* of the product *i* is defined as $\left[\mathbf{min}\left(P_i^{rev}, P_i^{II}\right) - P_i^I\right]^+$.

Pure profit is the amount of profit that would have been received by the player with the lowest price on product *i* if product *i* would have been placed at the auction as a separate item, instead of as part of the package. This is illustrated in **Figure 1.**

Definition 5. *Additional Profit* of the product *i* is defined as

[5] If coalition has only one member bidding in the auction for the product *i* then P_i^{II} can be assumed to be equal to some very large number ($P_i^{II} \geq \max P_i^{rev}$).

$$\left[P_i^{rev} - P_i^{II}\right]^+ = \begin{cases} P_i^{rev} - P_i^{II}, & if\ P_i^{rev} > P_i^{II}; \\ 0, & if\ P_i^{rev} \le P_i^{II}. \end{cases}$$

By not competing publicly against each other, players in the coalition can generate extra profit on the product. *Additional profit* is the amount of profit that the coalition receives by having more than one player with competitive prices on the product *i* in the coalition, i.e. by having more than one player in the coalition with a price for product *i* lower than P_i^{rev}. This is also illustrated in **Figure 1.**

Definition 6. *Negative Profit* of the product *i* is defined as

$$\left[P_i^{rev} - P_i^{I}\right]^- = \begin{cases} P_i^{rev} - P_i^{I}, & if\ P_i^{rev} < P_i^{I}; \\ 0, & if\ P_i^{rev} \ge P_i^{I}. \end{cases}$$

Negative Profit can be interpreted as the losses taken by the coalition because of a non-competitive price offered by one of its members. This means that the same part of the package could be produced for less money by another coalition (namely, the coalition offering the second lowest price on the package). For illustration of the definition, see **Figure 2.**

4.3 Fare Opportunistic Profit Sharing (FOPS) Mechanism:

The proposed FOPS mechanism works in the following manner. First the pure profit, the additional profit, and the negative profit brought by each part of the package are calculated. If there are no negative profits, then the pure profit generated by product *i* is rewarded to the player who produces the product. The additional profit generated by product *i* will be equally divided among all members of the coalition that were bidding for product *i*.

When a multi-product package is considered, it is possible to have a situation in which the coalition wins auction without offering competitive prices for all parts of the package. This is illustrated in **Figure 3**. If product *i* generates negative profit, then the player producing product *i* is not rewarded any part of the total profit as a player with noncompetitive price. The negative profit in this case has to be covered ether by the additional profit or by the pure profit created by other products. The negative profit can be equally or proportionally distributed among those members of the coalition who receive a part of the total profit. In the future, we would like to compare those two methods of sharing negative profit in order to discover their positive and negative sides.[6]

V. Auctioning of a Single Product: specifics and problems.

5.1 Bidding Behavior of coalition Members under FOPS mechanism.

Coalition players will accept the proposed Fare Opportunistic Profit Sharing mechanism only if they are convinced that participation in the coalition is potentially more profitable to them than trading at auction by themselves.

The following simple scenario is a very special case of the application of the FOPS. Suppose a single product "A" is placed at auction. If the coalition has only one player that could produce product "A", then this player when participating in the auction as a member of the coalition operating under the FOPS will receive profit equal to that generated if the player had acted at the auction alone. The proof of this statement is straightforward and is not given here.

[6] To provide the incentive for coalition members to stay price-competitive additional rules may be introduced in the coalition: another player from the market can replace players who create negative profit.

The situation becomes more interesting when the coalition has more than one player who can supply product “A”. In this case, it is possible for the coalition to generate additional profit.

It is known that a given player can achieve the maximum total profit by bidding his true production cost when participating in the market alone. It will be shown in Section 5.3 that the FOPS mechanism does not enforce truth telling in the coalition. Hence, the members of the coalition may lie about their prices to increase their additional profit as much as possible. In Chapter 5 we will show that FOPS mechanism does not guaranty that the player with the lowest price on the product “A” in the coalition will always be rewarded as much as he would have receives by acting at the auction alone.

5.2 Expected Profit of a Coalition Member.

Next, we consider the expected profit of any single supplier as a member of a coalition under FOPS. This player is referred to as the *selected player*. Since in Chapter 5 we consider auction of a single product, we will not need index *i* to list all the products in the auctioned package (see Section 4.2).

The following is list of variables needed to express the expected profit of a coalition member:

P^{I} - the lowest of the product prices offered by the coalition members excluding the *selected player*;

P^{II} - the second lowest of the product prices offered by the coalition members excluding the *selected player*;

P^{tr} - the true cost of production for the *selected player*;

P^f - the price announced by the *selected player*;

P^{rev} - the price rewarded to the coalition in case of its winning the auction. It is also the lowest price announced by the players outside the coalition;

$|\alpha|$ - the number of players inside the coalition.

In the case when only a single product is auctioned, the coalition wins if and only if the price offered by the coalition for that product is the lowest price in the market. For any selected member of the coalition, his production cost P^{tr} is a known value. The number of the coalition members $|\alpha|$ is known as well. To calculate expected profit, each selected player will have to estimate the distributions for variables P^I, P^{II} and P^{rev}, since all information about the cost structure of the coalition members never reviled. Adapting our notation, we can express the expected profit the same way Jin and Wu (2001) did in their work as:

$$
\begin{aligned}
&E(profit) = profit^I * \mathbf{Pr}(player\ has\ Lowest\ price\ in\ coalition) * \mathbf{Pr}(win, P^f) + \\
&+ profit^{II} * \mathbf{Pr}(player\ has\ Second\ lowest\ price\ in\ coalition) * \mathbf{Pr}(win, P^I) + \\
&+ profit^{III} * \mathbf{Pr}(player\ has\ price\ higher\ then\ Second\ lowest\ price\ in\ coalition) * \mathbf{Pr}(win, P^{II}), \\
&where\ \mathbf{Pr}(win, P^k) = \mathbf{Pr}(coalition\ wins\ by\ placing\ bid\ P^k), \\
&profit^I = \mathbf{max}\left(\frac{P^{rev} - P^I}{|\alpha|}, 0\right) + \mathbf{min}(P^{rev}, P^I) - P^{tr}, \\
&profit^{II} = \frac{P^{rev} - P^f}{|\alpha|}, \\
&profit^{III} = \frac{P^{rev} - P^{II}}{|\alpha|}.
\end{aligned}
$$

In estimating the expected profit of the selected player we make the following assumptions:

1) $\mathbf{Pr}\left(a \le P^{I} \le b\right) = 1$;[7]

2) $P^{II} \in \left(P^{I}, b\right]$, $\mathbf{Pr}\left(P^{II} = p_2 \middle| P^{I} = p_1, p_1 < p_2 \le b\right) = \dfrac{1}{b - p_1}$;[8]

3) $P^{rev} \in [a, b], \forall p_{rev} \in [a, b]$, $\mathbf{Pr}\left(P^{rev} = p_{rev}\right) = \dfrac{1}{b - a + 1}$;[9]

where $b - a \ge 4$, $a \in Z_+$, $b \in Z_+$.

Expected profit of a selected player as a member of the coalition under FOPS mechanism is equal to:

$$Expected\ \ profit = Term\ I + Term\ II + Term\ III + Term\ IV\ ,$$

where *Terms I* is expected profit of the selected player if he has the lowest price in the coalition and coalition created only pure profit;[10]

Terms II is expected profit of the selected player if he has the lowest price in the coalition and coalition created pure and additional profit;

Terms III is expected profit of the selected player if he has the second lowest price in the coalition and coalition created pure and additional profit;

Terms IV is expected profit of the selected player if he has bid price above the second lowest price in the coalition and coalition created pure and additional profit.

Mathematically Terms I-IV defined as:

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[7] We would like to have no restrictions on the form of the P^{I} distribution function.

[8] P^{II} assumed to be random distribution function with dependants on the P^{I} distribution function.

[9] Since we are interested in the market with many suppliers, oligopoly and monopoly would not be considered here. By definition of the rewarded price to calculate expected rewarded price we would have to know (or estimate) price distribution function for each player in the market. Since market is large it is unlikely that selected player would have accurate estimation of the price distribution function for each supplier in the market. In view of such difficulties we assumed that P^{rev} has discrete uniform distribution on interval [a, b].

[10] If $P^{I}=P^{f}<P^{rev}$ then we assume that this auction was not successful and it has been canceled, and new auction will be running in different units. For example, 1 old unit =10 new units.

$$Term\ I = \begin{cases} Term1, & if\ a \le P^f \le (b-1) \\ 0, & otherwise \end{cases}$$

$$Term\ II = \begin{cases} Term2, & if\ a \le P^f \le (b-2) \\ 0, & otherwise \end{cases}$$

$$Term\ III = \begin{cases} Term3, & if\ (a+1) \le P^f \le (b-1) \\ 0, & otherwise \end{cases}$$

$$Term\ IV = \begin{cases} Term4, & if\ (a+1) \le P^f \le (b-1) \\ 0, & otherwise \end{cases}$$

$$Term1 = \sum_{p_{rev}=P^f+1}^{b} \sum_{p_1=p_{rev}}^{b} \left(p_{rev} - P^{tr}\right) * \Pr\left(P^{rev} = p_{rev}\right) * \Pr\left(P^I = p_1\right)$$

$$Term2 = \sum_{p_1=P^f+1}^{b-1} \sum_{p_{rev}=p_1+1}^{b} \left(\left(p_1 - P^{tr}\right) + \frac{\left(p_{rev} - p_1\right)}{|\alpha|} \right) * \Pr\left(P^{rev} = p_{rev}\right) * \Pr\left(P^I = p_1\right)$$

$$Term3 = \sum_{p_{rev}=P^f+1}^{b} \sum_{p_1=a}^{P^f-1} \sum_{p_2=P^f}^{b} \frac{\left(p_{rev} - P^f\right)}{|\alpha|} * \Pr\left(P^{rev} = p_{rev}\right) * \Pr\left(P^I = p_1\right) * \Pr\left(P^{II} = p_2\right)$$

$$Term4 = \sum_{p_1=a}^{P^f-2} \sum_{p_2=p_1+1}^{P^f-1} \sum_{p_{rev}=p_2+1}^{b} \frac{\left(p_{rev} - p_2\right)}{|\alpha|} * \Pr\left(P^{rev} = p_{rev}\right) * \Pr\left(P^I = p_1\right) * \Pr\left(P^{II} = p_2\right).$$

Unfortunately, this expression cannot be simplified. It is evident that the analytical comparison of the expected profits for a coalition member and a Single supplier is impossible for the general case.

Generally, the evaluation of the *selected player's* expected profit as a member of the coalition, as well as his expected profit as a single prayer in a market, has to be calculated numerically. Based on these estimates, the selected player will decide whether or not to participate in the auction as a coalition member.

5.3 Case-by-Case Analysis of Bidding Behavior of a Coalition Member.

When the *selected player*, announces his price, it is easy to see that the player has no reason to announce a price higher then his true production cost ($P^f > P^{tr}$).

To prove this we will assume that $P^f \geq P^{tr}$:

Case (1): $P^{tr} < P^{rev} < P^I$

The *selected player's* profit when bidding his true cost, $P^f = P^{tr}$, is equal to:

$P^{rev} - P^{tr}$.

- $P^{tr} < P^f < P^{rev}$

  The *selected player's* profit when bidding P^f is equal to:

  $P^{rev} - P^f + \left(P^f - P^{tr}\right) = P^{rev} - P^{tr}$.

- $P^{tr} < P^{rev} < P^f$

  The *selected player's* profit when bidding P^f is equal to zero, since coalition will not win the auction ($P^{rev} < P^I, P^{rev} < P^f$).

Result shows that placing bid above the *selected player's* true cost ($P^f > P^{tr}$) is undesirable in case (1).

Case (2): $P^{tr} < P^I < P^{rev}$

The *selected player's* profit when bidding his true cost, $P^f = P^{tr}$, is equal to:

$$\frac{\left(P^{rev} - P^I\right)}{|\alpha|} + \left(P^I - P^{tr}\right).$$

- $P^{tr} < P^f < P^I < P^{rev}$

  The *selected player's* profit when bidding P^f is equal to:

  $$\frac{\left(P^{rev} - P^I\right)}{|\alpha|} + \left(P^I - P^f\right) + \left(P^f - P^{tr}\right) = \frac{\left(P^{rev} - P^I\right)}{|\alpha|} + \left(P^I - P^{tr}\right)$$

- $P^{tr} < P^{I} < P^{f} < P^{rev}$, $P^{f} < P^{II}$

The *selected player's* profit when bidding P^{f} is equal to:

$$\frac{\left(P^{rev} - P^{f}\right)}{|\alpha|}.$$

Because $P^{f} > P^{I}$, and $P^{I} - P^{tr} > 0$:

$$\frac{\left(P^{rev} - P^{f}\right)}{|\alpha|} < \frac{\left(P^{rev} - P^{I}\right)}{|\alpha|} + \left(P^{I} - P^{tr}\right)$$

- $P^{tr} < P^{I} < P^{II} < P^{rev}$, $P^{II} < P^{f}$

The *selected player's* profit when bidding P^{f} is equal to:

$$\frac{\left(P^{rev} - P^{II}\right)}{|\alpha|}.$$

Because $P^{II} > P^{I}$, and $P^{I} - P^{tr} > 0$:

$$\frac{\left(P^{rev} - P^{II}\right)}{|\alpha|} < \frac{\left(P^{rev} - P^{I}\right)}{|\alpha|} + \left(P^{I} - P^{tr}\right)$$

- $P^{tr} < P^{I} < P^{rev}, P^{rev} < P^{II}, P^{rev} < P^{f}$

The *selected player's* profit when bidding P^{f} is equal to zero, because $P^{II} > P^{rev}, P^{f} > P^{rev}$.

Result shows that placing bid above the *selected player's* true cost ($P^{f} > P^{tr}$) is undesirable in case (2).

Case (3): $P^{I} < P^{tr} < P^{rev}, P^{tr} < P^{II}$

The *selected player's* profit when bidding his true cost, $P^{f} = P^{tr}$, is equal to:

$$\frac{\left(P^{rev} - P^{tr}\right)}{|\alpha|}.$$

- $P^{I} < P^{tr} < P^{f} < P^{rev}, P^{f} < P^{II}$

The *selected player's* profit when bidding P^{f} is equal to:

$$\frac{\left(P^{rev} - P^{f}\right)}{|\alpha|}$$

Because $P^{f} > P^{tr}$:

$$\frac{\left(P^{rev} - P^{f}\right)}{|\alpha|} < \frac{\left(P^{rev} - P^{tr}\right)}{|\alpha|}$$

- $P^{I} < P^{tr} < P^{II} < P^{rev}, P^{II} < P^{f}$

The *selected player's* profit when bidding P^{f} is equal to:

$$\frac{\left(P^{rev} - P^{II}\right)}{|\alpha|}$$

Because $P^{II} > P^{tr}$:

$$\frac{\left(P^{rev} - P^{II}\right)}{|\alpha|} < \frac{\left(P^{rev} - P^{tr}\right)}{|\alpha|}$$

- $P^{I} < P^{tr} < P^{rev} < P^{f}, P^{rev} < P^{II}$

The *selected player's* profit when bidding P^{f} is equal to zero, because $P^{II} > P^{rev}, P^{f} > P^{rev}$, therefore no additional profit has been created by the coalition.

Result shows that placing bid above the *selected player's* true cost ($P^{f} > P^{tr}$) is undesirable in case (3).

Case (4): $P^{I} < P^{II} < P^{rev}, P^{II} < P^{tr}$

The *selected player's* profit when bidding his true cost, $P^{f} = P^{tr}$, is equal to:

$$\frac{\left(P^{rev} - P^{II}\right)}{|\alpha|}.$$

- $P^{I} < P^{II} < P^{rev}, P^{II} < P^{tr} < P^{f}$

  The *selected player's* profit when bidding P^{f} is equal to:

  $$\frac{\left(P^{rev} - P^{II}\right)}{|\alpha|}$$

Result shows that placing bid above the *selected player's* true cost ($P^{f} > P^{tr}$) does not increase *selected player's* profit in case (4).

Now let us try to find the conditions under which the *selected player* could have an incentive to name a price less then his true cost ($P^{f} < P^{tr}$).

Several situations are analyzed below.

Case (5): $P^{f} < P^{I} < P^{tr} < P^{rev}, P^{tr} < P^{II}$

If *selected player* tells his true production cost and coalition wins, then as a player with second lowest price, the *selected player* receives the profit of:

$$\frac{\left(P^{rev} - P^{tr}\right)}{|\alpha|}.$$

- If *selected player* announces price $P^{f} < P^{tr}$ and coalition wins, then the player receives the following amount of profit:

  $\frac{\left(P^{rev} - P^{I}\right)}{|\alpha|} + \left(P^{I} - P^{tr}\right)$, where in this case $\left(P^{I} - P^{tr}\right)$<0.

It is easy to see that $\frac{\left(P^{rev} - P^{I}\right)}{|\alpha|} + \left(P^{I} - P^{tr}\right) < \frac{\left(P^{rev} - P^{tr}\right)}{|\alpha|}, \quad for\ \forall|\alpha| > 1$.

Therefore, the *selected player* has no incentive to lie in case (5).

Case (6): $P^{f} < P^{tr} < P^{I} < P^{rev}$

If *selected player* tells his true production cost and coalition wins, then as a player with the lowest price in the coalition, the *selected player* receives the profit of:

$$\frac{\left(P^{rev} - P^{I}\right)}{|\alpha|} + \left(P^{I} - P^{tr}\right).$$

- If *selected player* announces price $P^{f} < P^{tr}$ and coalition wins, then the profit received by *selected player* is:

  $$\frac{\left(P^{rev} - P^{I}\right)}{|\alpha|} + \left(P^{I} - P^{tr}\right).$$

Therefore, the *selected player* will not benefit from lying in case (6).

Case (7): $P^{f} < P^{I} < P^{II} < P^{rev}, P^{II} < P^{tr}$

If *selected player* tells his true production cost and coalition wins, then as a player with not a competitive price in the coalition, the *selected player* receives:

$$\frac{\left(P^{rev} - P^{II}\right)}{|\alpha|}.$$

- If *selected player* announces price $P^{f} < P^{tr}$ and coalition wins, then the *selected player* receives the profit of:

  $\frac{\left(P^{rev} - P^{I}\right)}{|\alpha|} + \left(P^{I} - P^{tr}\right)$, where in this case $\left(P^{I} - P^{tr}\right)$<0.

$$\frac{\left(P^{rev} - P^{I}\right)}{|\alpha|} + \left(P^{I} - P^{tr}\right) < \frac{\left(P^{rev} - P^{II}\right)}{|\alpha|}, \quad for\ \forall|\alpha| > 1.$$

Therefore, the *selected player* is better off by telling the truth in case (7) as well.

In all the cases described above, the coalition would have won the bidding even without the *selected player's* participation.

Now let us consider the situation when the coalition would have lost, if not for the participation of the *selected player*.

Case (8): $P^f < P^{tr} < P^{rev} < P^I$

If *selected player* tells his true production cost and coalition wins, then as the only player with competitive price in the coalition, the *selected player* receives the profit of:

$P^{rev} - P^{tr}$.

- If *selected player* announces price $P^f < P^{tr}$ and coalition wins, then the *selected player* receives the profit of:

  $P^{rev} - P^{tr}$.

Therefore, the *selected player* will not benefit from lying in case (8).

Case (9): $P^f < P^{rev} < P^{tr}$, $P^{rev} < P^I$

If *selected player* tells his true production cost, then coalition loses the bidding. Therefore, the amount of profit received by the *selected player* is equal to zero

- If *selected player* announces price $P^f < P^{tr}$ and coalition wins, then the *selected player* receives the profit of:

  $P^{rev} - P^{tr}$.

  It is a negative value; the player suffers a loss by receiving less money for the product than his true production cost.

Therefore, the *selected player* should tell the truth in case (9).

This proves that *selected player* has no incentive to announce price lower then his production cost in order to become a player with the lowest price in the coalition.

In the following situations the *selected player* becomes the player with the second lowest price.

Case (10): $P^{I} < P^{f} < P^{tr} < P^{rev}, P^{tr} < P^{II}$

If *selected player* tells his true production cost and coalition wins, then the *selected player* receives a profit of:

$$\frac{\left(P^{rev} - P^{tr}\right)}{|\alpha|}.$$

- If *selected player* announces price $P^{f} < P^{tr}$ and coalition wins, then the *selected player* receives the following profit:

$$\frac{\left(P^{rev} - P^{f}\right)}{|\alpha|}.$$

In this case $\frac{\left(P^{rev} - P^{f}\right)}{|\alpha|} > \frac{\left(P^{rev} - P^{tr}\right)}{|\alpha|}, \quad for\ \forall|\alpha| \geq 1$, which provides the *selected player* with an incentive to lie about his production cost.

Case (11): $P^{I} < P^{f} < P^{II} < P^{rev}, P^{II} < P^{tr}$

If *selected player* tells his true production cost and coalition wins, then the *selected player* receives a profit of:

$$\frac{\left(P^{rev} - P^{II}\right)}{|\alpha|}.$$

- If *selected player* announces price $P^{f} < P^{tr}$ and coalition wins, then the *selected player* receives a profit of:

$$\frac{\left(P^{rev} - P^{f}\right)}{|\alpha|}.$$

Clearly, $\frac{\left(P^{rev} - P^{f}\right)}{|\alpha|} > \frac{\left(P^{rev} - P^{II}\right)}{|\alpha|}$ for $\forall|\alpha| \geq 1$.

Therefore, in Case (11), the *selected player* has incentive to announces price below his true production cost ($P^f < P^{tr}$).

Case (12): $P^I < P^{II} < P^{rev}, P^{II} < P^f$

If *selected player* tells his true production cost and coalition wins, then the *selected player* receives a profit of:

$$\frac{\left(P^{rev} - P^{II}\right)}{|\alpha|}.$$

- If *selected player* announces price $P^f < P^{tr}$ and coalition wins, then as the player with not a competitive price, the *selected player* receives a profit of:

$$\frac{\left(P^{rev} - P^{II}\right)}{|\alpha|}.$$

In this case, the amount of profit of the *selected player* no longer depends on P^f. Hence, the selected player does not benefit from supplying the false information about his true production cost.

It can be concluded that if the *selected player* has the information about P^I, P^{II}, and knows that $P^{tr} > P^I$, he will benefit by bidding price lower than his production cost. However, in reality, the information is kept private most of the time. Therefore, the *selected player* has to estimate his opponent's production cost, and try not to become the lowest price player by announcing price lower than his true cost. After case-by-case analysis we would like to prove that under proposed FOPS mechanism if the *selected player* does not have the lowest production cost in the coalition, his best bid would lower then his production cost. In fact, in chapter 6 we will show that bid placed by such *selected player* should as close to P^I as possible.

VI. Analytical Study of the Selected Player's Behavior in Two Special Cases.

As result of numerical analysis, two interesting rules for selected player's behavior have emerged.

Statement 1: If there exists $d \in (a,b]$, $d \in Z_+$ such that $\mathbf{Pr}(P^I \geq d) = 1$, and the selected player's true cost (P^{tr}) is strictly less than d, then the optimal (maximizing expected profit) bid for this player is to report $P^f = P^{tr}$ or $P^f = P^{tr} - 1$.

<u>Proof:</u>

First let us show that for $P^{tr} < d$: $Expected\ \ profit(P^{tr}) = Expected\ \ profit(P^{tr} - 1)$.

Term III, and Term IV are equal to zero, since $\mathbf{Pr}(P^I < d) = 0$, and therefore $\mathbf{Pr}(P^I < P^{tr}) = 0$.

$$Expected\ \ profit(P^{tr}) = \sum_{p_{rev}=P^{tr}+1}^{b}\left(\sum_{p_1=p_{rev}}^{b}(p_{rev} - P^{tr}) * \mathbf{Pr}(P^{rev} = p_{rev}) * \mathbf{Pr}(P^I = p_1)\right) +$$
$$\sum_{p_1=P^{tr}+1}^{b-1}\left(\sum_{p_{rev}=P^1+1}^{b}\left((p_1 - P^{tr}) + \frac{(p_{rev} - p_1)}{|\alpha|}\right) * \mathbf{Pr}(P^{rev} = p_{rev}) * \mathbf{Pr}(P^I = p_1)\right).$$

$$Expected\ \ profit(P^{tr} - 1) = \sum_{p_{rev}=P^{tr}}^{b}\left(\sum_{p_1=p_{rev}}^{b}(p_{rev} - P^{tr}) * \mathbf{Pr}(P^{rev} = p_{rev}) * \mathbf{Pr}(P^I = p_1)\right) +$$
$$+ \sum_{p_1=P^{tr}}^{b-1}\left(\sum_{p_{rev}=P^1+1}^{b}\left((p_1 - P^{tr}) + \frac{(p_{rev} - p_1)}{|\alpha|}\right) * \mathbf{Pr}(P^{rev} = p_{rev}) * \mathbf{Pr}(P^I = p_1)\right).$$

The second terms in these expressions are equal: the significant sub-range for P^I is $[d,b]$, therefore the terms in expected profit for $P^I = P^{tr}$ are equal to zero. Furthermore,

$$Expected\ \ profit(P^{tr}) - Expected\ \ profit(P^{tr}-1) =$$

$$= \sum_{p_{rev}=P^{tr}+1}^{b} \left(\sum_{p_1=p_{rev}}^{b} (p_{rev}-P^{tr}) * \mathbf{Pr}(P^{rev}=p_{rev}) * \mathbf{Pr}(P^{I}=p_1) \right) -$$

$$- \sum_{p_{rev}=P^{tr}}^{b} \left(\sum_{p_1=p_{rev}}^{b} (p_{rev}-P^{tr}) * \mathbf{Pr}(P^{rev}=p_{rev}) * \mathbf{Pr}(P^{I}=p_1) \right) =$$

$$= \sum_{p_1=p_{rev}}^{b} (p_{rev}-P^{tr}) * \mathbf{Pr}(P^{rev}=p_{rev}) * \mathbf{Pr}(P^{I}=p_1) \Bigg|_{p_{rev}=P^{tr}} = 0.$$

Now let us show that $Expected\ \ profit(P^{tr}) > Expected\ \ profit(P^{tr}-n),\ n \geq 2,\ \ n \in Z_+$.

$$Expected\ \ profit(P^{tr}-n) = \sum_{p_{rev}=P^{tr}-n+1}^{b} \left(\sum_{p_1=p_{rev}}^{b} (p_{rev}-P^{tr}) * \mathbf{Pr}(P^{rev}=p_{rev}) * \mathbf{Pr}(P^{I}=p_1) \right) +$$

$$+ \sum_{p_1=P^{tr}-n+1}^{b-1} \left(\sum_{p_{rev}=P^{1}+1}^{b} \left((p_1-P^{tr}) + \frac{(p_{rev}-p_1)}{|\alpha|} \right) * \mathbf{Pr}(P^{rev}=p_{rev}) * \mathbf{Pr}(P^{I}=p_1) \right).$$

Similarly, it can be shown that:

$$Expected\ \ profit(P^{tr}) - Expected\ \ profit(P^{tr}-n) =$$

$$= - \sum_{p_{rev}=P^{tr}-n+1}^{P^{tr}-1} \left(\sum_{p_1=p_{rev}}^{b} (p_{rev}-P^{tr}) * \Pr(P^{rev}=p_{rev}) * \Pr(P^{I}=p_1) \right) =$$

$$= \sum_{p_{rev}=P^{tr}-n+1}^{P^{tr}-1} \left(\sum_{p_1=p_{rev}}^{b} (P^{tr}-p_{rev}) * \Pr(P^{rev}=p_{rev}) * \Pr(P^{I}=p_1) \right) > 0$$

, because $P^{tr} - (P^{tr}-n+1) = n-1 \geq 1$ for $\forall n \geq 2$.

♦

Statement 2: If there is exists $c \in [a,b),\ c \in Z_+$ such that $\mathbf{Pr}(P^{I} \leq c) = 1$ and selected player's true cost (P^{tr}) is strictly greater than *c*, then the optimal (maximizing expected profit) bid for this player is to report $P^{f} \leq (c+1)$.

Proof:

Let us show that for $\forall b \geq x > c$ **:** $Expected\ profit(x+1) > Expected\ profit(x+2)$**.**

$$Expected\ \ profit(x+1) - Expected\ \ profit(x+2) =$$

$$= \sum_{p_1=a}^{c} \left(\sum_{p_{rev}=x+2}^{b} \frac{(p_{rev}-x)-(p_{rev}-(x+1))}{|\alpha|} * \mathbf{Pr}\left(P^{rev}=p_{rev}\right) * \mathbf{Pr}\left(P^{I}=p_1\right) * \mathbf{Pr}\left(P^{II}=p_2\right) \right) \Bigg|_{p_2=x+1}$$

It can be simplified further:

$$Expected\ \ profit(x+1) - Expected\ \ profit(x+2) =$$

$$= \sum_{p_1=a}^{c} \left(\sum_{p_{rev}=x+2}^{b} \frac{1}{|\alpha|} * \mathbf{Pr}\left(P^{rev}=p_{rev}\right) * \mathbf{Pr}\left(P^{I}=p_1\right) * \mathbf{Pr}\left(P^{II}=p_2\right) \right) \Bigg|_{p_2=x+1} > 0.$$

♦

VII. Numerical Studies.

The numerical study was performed before theoretical study in order to gain intuition about what can influence the expected profit and at what level. Expected profit of the selected player with true cost P^{tr} while bidding P^{f} has to be evaluated numerically according to formula described in chapter 5.2. There are only two parameters known to the selected player: his production cost P^{tr} and the size of the coalition $|\alpha|$. Since we have already made the assumptions on probabilistic distribution for P^{II} and P^{rev} in chapter 5.2 there now only three parameters left to operate with: a, b, and P^{I}. Interval $[a,b]$ has to contain all possible P^{rev} and P^{I} at the considered auction. Selected player would also have to make assumption about the form of probability distribution of P^{I} on interval $[a,b]$. Then for any chosen P^{f} selected player can evaluate terms I, II, III, IV (see chapter 5.2) to calculate his expected profit for the bid P^{f}. By doing such calculation for all possible $P^f \in [a,b]$, selected player can choose *optimal bid* – the bid P^* that maximizes his expected profit.

To find out if any results of our calculation for specific interval $[a,b]$ could be considered general we looked at two different cases:

1) probability distribution function of P^I defined on interval $[1,40]$;

2) probability distribution function of P^I defined on interval $[1,20]$

where the shape of probability distribution of P^I used in second case with respect to interval $[1,20]$ was the same to the shape of probability distribution of P^I used in first case with respect to interval $[1,40]$. Results (see **Figure 4**) have shown that for a given coalition size, expected profit changes proportionally to the change in the interval length, $k = \frac{b_{new} - a_{new}}{b_{old} - a_{old}}$. Optimal bid of the player is also changed with respect to the same parameter *k* (see **Figure 4**). Therefore, any results found for the specific interval $[a_{old}, b_{old}]$ (such as expected profit, optimal response) can be used for any other interval $[a_{new}, b_{new}]$ after multiplication by scaling parameter *k*, as long as probability distribution of P^I scaled appropriately along with the interval.

In addition, the expected profit depends on the probability distribution of P^I for the given interval $[a,b]$. A number of different distributions where considered in order to capture such dependence (see **Figure 5**). Results indicate that the closer to the uniform distribution of P^I is, the less additional profit is generated by coalition; therefore, the selected player's profit is closer to the amount this player expects to get by bidding at the auction by himself. It is also obvious that the less competitive selected player (P^{tr} is closer to ***b***, than to ***a***) will expect to see more profit as result of joining the coalition (see **Figure 6**).

Another parameter that has large influence on the expected profit of the selected player is a size of the coalition $|\alpha|$. Depending on the probability distribution of P^I, influence of the coalition size varies (see **Figure 7**). In general, influence of the coalition size is especially significant for competitive P^I and low P^{tr}. The "optimal" coalition consists of just two players. As size of the coalition grows, the expected profit of the selected player approaches the expected profit of the same player participating in the auction along. It has to be mentioned that in Figure 7 while size of the coalition was changed, probability distribution of P^I stayed unchanged from case to case.

Behavior of the selected player is affected by the same factors as the expected profit. **Figure 8** demonstrates influence of the probability distribution of P^I on the selected player's behavior. Illustration of statements 1 and 2 from chapter IV can be observed in **Figure 9**. Size of the coalition has direct influence on the amount of the additional profit, expected by the selected player. As an amount of the expected additional profit decreases, the selected player's incentive to lie about products cost also decreases.

VIII. Auctioning a Package of the Multiple Products: Specifics and Problems.

In a case when a multiple products package is auctioned, each member of the coalition that participates in the auction by bidding for the product *i* is still going to be rewarded according to the amount of the total profit associated with this product, as in the case of a single product auction. The major difference of the two cases is that in the single product case the winner of the auction is always the supplier who has the lowest bid for the product, while in the case of multiple product package the winner of the auction does not necessary have the lowest bid for each part of the package.

By analogy with the more simple single product case, it is generally not possible to find a closed form expression of the expected profit for the coalition member bidding on the multiple products package. The number of random variables is dramatically increased because of the necessity to account for a possibility of negative profit from each of the products in the package.

It is proposed to employ a computer simulation to investigate the effects of the fare opportunistic profit sharing mechanism in the multiple products package case. There are few possible ways to share pure, additional and negative profit among coalition members in case of the multiple products package auctioned:

1) If total additional profit is less than total negative profit, then split negative profit equally among all members of the coalition. If total additional profit is greater than total negative profit, then additional profit has to be used completely to cover negative profit created by coalition, and the rest of the negative profit has to be split between coalition members that will receive pure profit proportionally to amount of the pure profit received.
2) Players awarded randomly selected amounts of the total profit created by coalition.

IX. Conclusions:

We have shown that in case of the single product been auctioned, FOPS mechanism does not enforce truth telling. In the cases, when selected player's production cost is clearly below the lowest possible bid in the coalition, selected player's optimal bid is equal to his production cost. In all other situations selected player's optimal bid is always below his production cost. If the selected player knows maximum possible lowest bid at the auction, he will never bid above it.

By analyzing the bidding behavior of the coalition members in auction of a single product, we conclude that the coalition member who expects to lose the auction to another member of the coalition has the incentive to announce a price below his true production cost for the product, intentionally increasing the additional profit associated with this product, and lowering the amount of the pure profit created by the product. It is possible that the player with the lowest bid for the product will receive a lower profit by bidding for that single product as a member of the coalition comparing to the profit received if participating in the auction alone. This is the case when adding another player to the coalition dramatically changes probability distribution of the lowest price in the coalition. Optimal size of the coalition for any given probability distribution function of P^1 is two. Increase of the coalition size leads to decrease of the expected profit of coalition member down to his expected profit as a single player.

Further investigation of the bidding behavior of the coalition members has to be performed for a case of the auctioning of a multiple products package. It is expected that the way of distributing the additional and negative profits in a coalition is going to have a great influence on the bids placed by the coalition members.

A computer simulation can also provide the information on the stability of the coalition formed under such fare opportunistic profit sharing mechanism, and whether it will lead to a *grand coalition*.[11] In other words, the possibility of the fare opportunistic

[11] Grand coalition - coalition that includes all the suppliers in the market

profit sharing mechanism to promote monopoly of the coalition in the market can also be investigated numerically.

Profit sharing mechanisms proposed in chapter VIII for the auction of the multiple products have to bee compared to each other since in one case it is theoretically possible to estimate expected profit of the selected player, and therefore theoretically possible to find optimal bid that maximizes expected profit over time. While second proposed profit sharing mechanism we expect to promote truth telling among coalition members. Running computer simulation for the those two profit sharing mechanisms would allow us to see if having auction of the multiple products will destroy selected player's incentive to bid price lower then his production cost, as well as to evaluate if forming coalition in fact more profitable to its members then participation in the auctions by themselves.

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Figure 1: Illustrations for the definitions of the Total Profit, the Pure Profit and the Additional Profit for the case of the auction of a Single Product.
(a) There is only one player with a competitive price.
(B) There are at least two players with a competitive price.

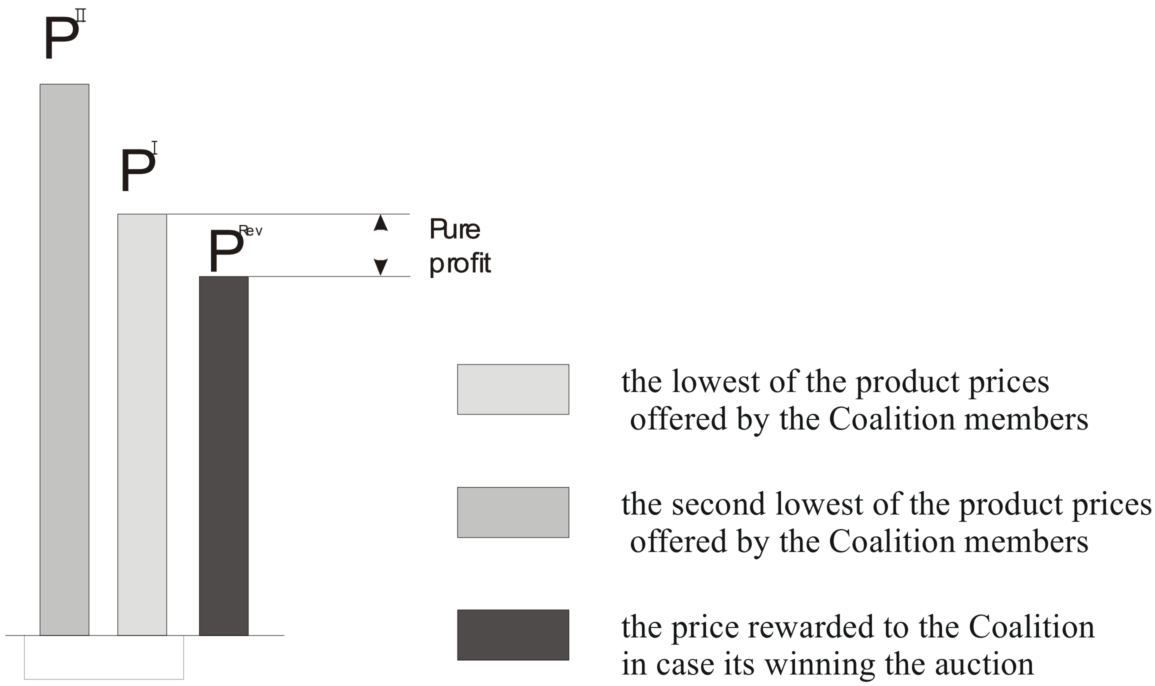


Figure 2: Illustrations for the definition of the Negative Profit.

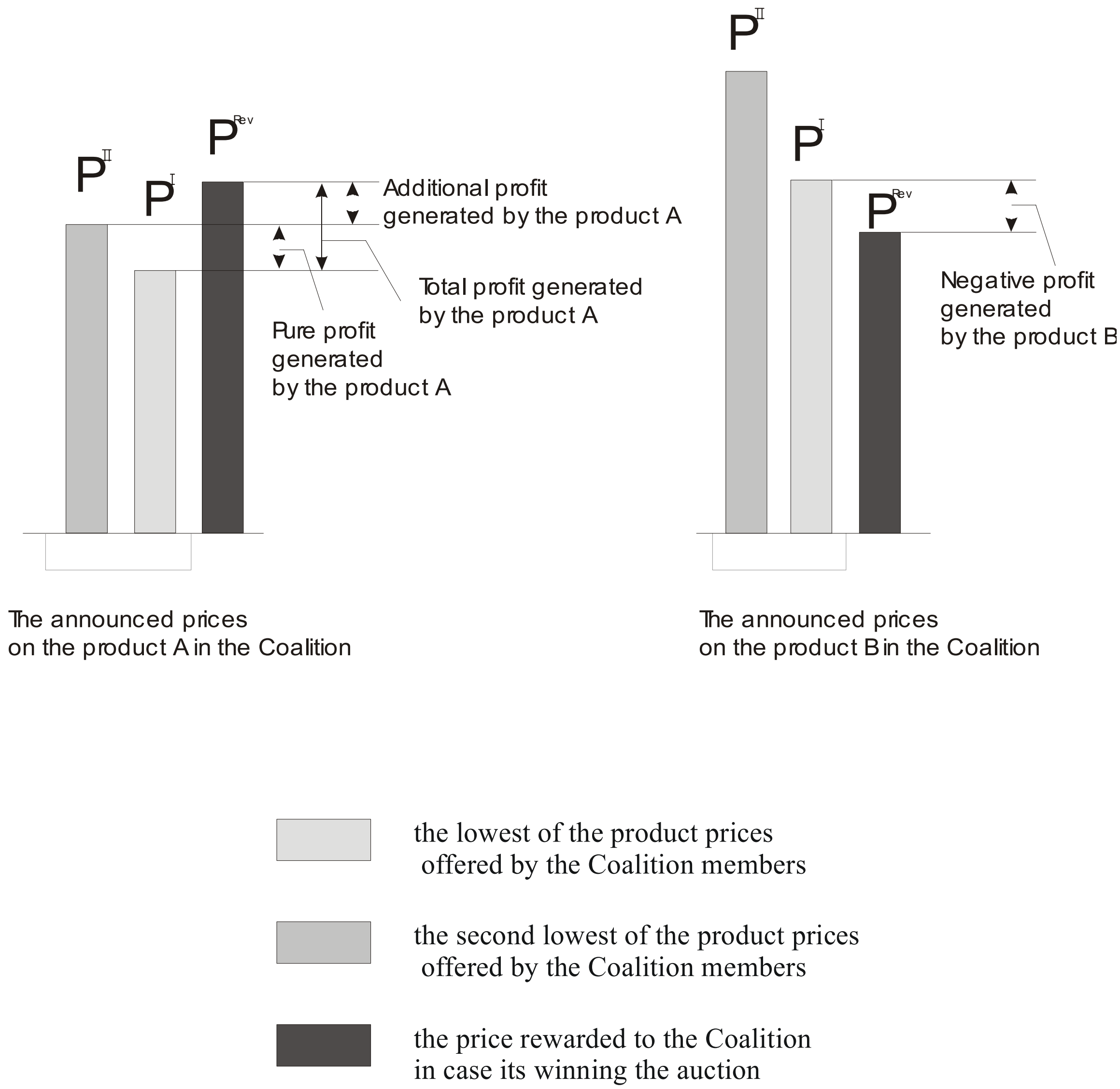
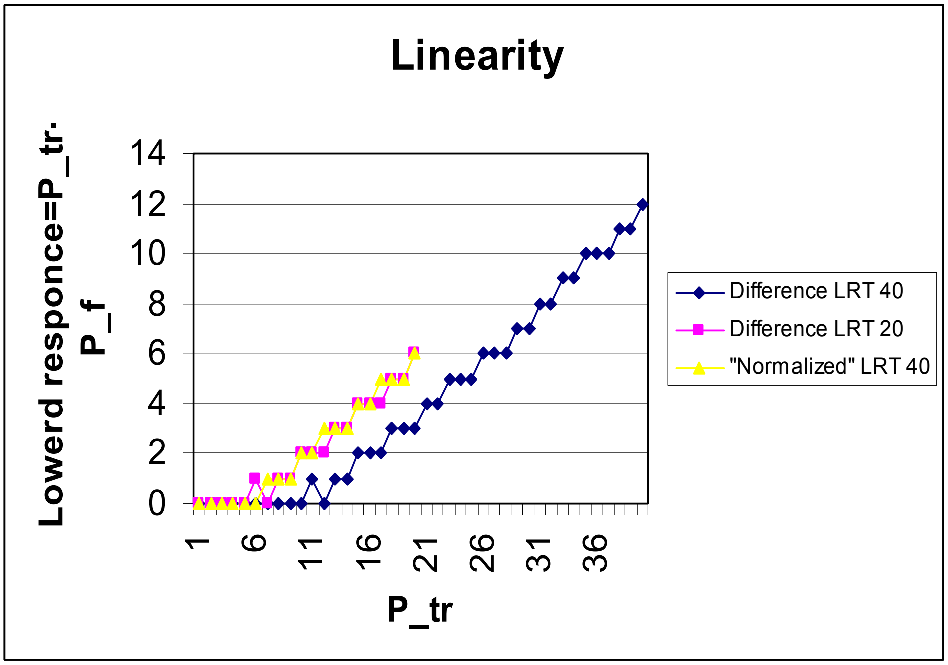


Figure 3: Illustrations for the situation when the Coalition wins the auction of the Multiple Product Package {A, B} having a noncompetitive price of the product B..


Linearity
Lowerd responce=P_tr-P_f
P_tr
Difference LRT 40
Difference LRT 20
"Normalized" LRT 40
0
2
4
6
8
10
12
14
1
6
11
16
21
26
31
36

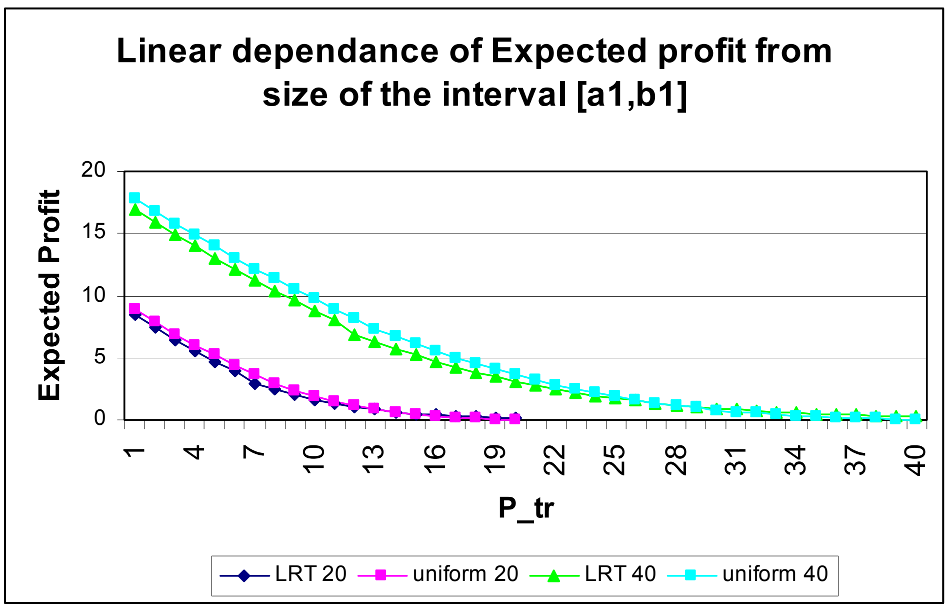

Linear dependance of Expected profit from size of the interval [a1,b1]
Expected Profit
P_tr
LRT 20
uniform 20
LRT 40
uniform 40
0
5
10
15
20
1
4
7
10
13
16
19
22
25
28
31
34
37
40

Figure 4:

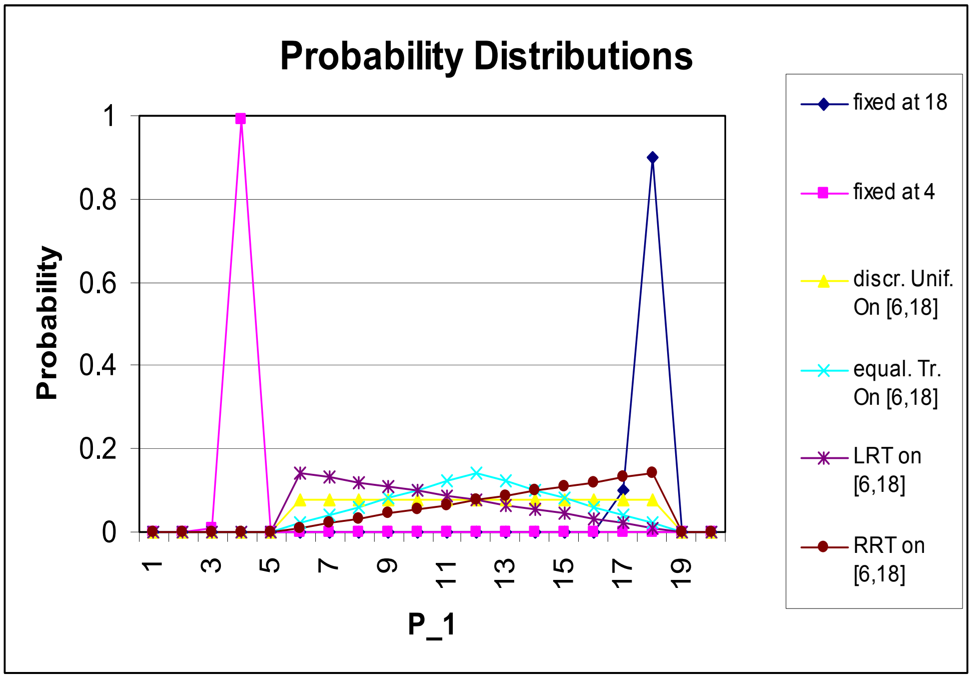
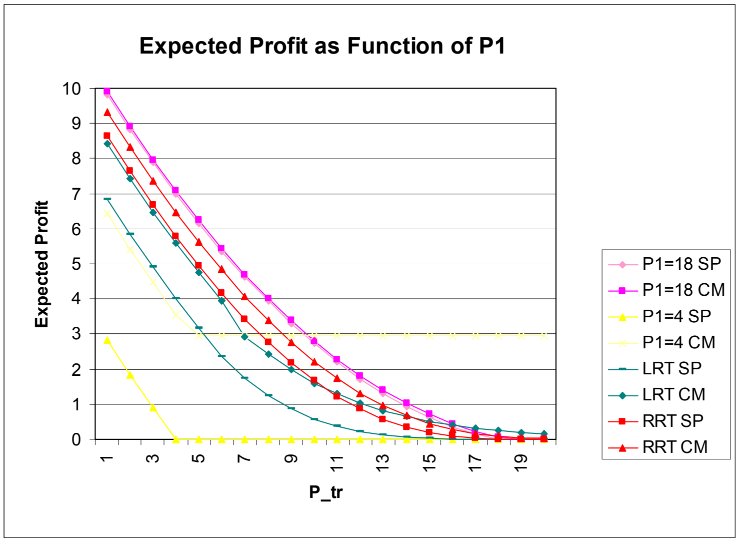
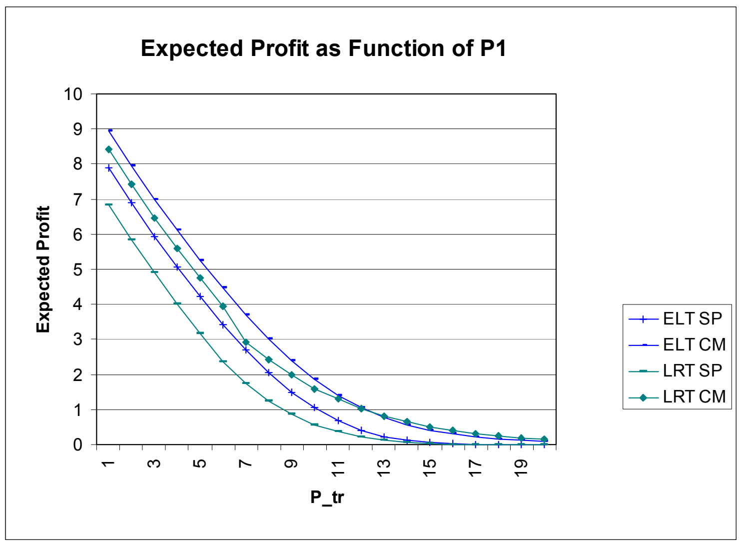

Probability Distributions
Probability
1
0.8
0.6
0.4
0.2
0
1
3
5
7
9
11
13
15
17
19
P_1
fixed at 18
fixed at 4
discr. Unif. On [6,18]
equal. Tr. On [6,18]
LRT on [6,18]
RRT on [6,18]

Figure 5:


Expected Profit as Function of P1
Expected Profit
P_tr
P1=18 SP
P1=18 CM
P1=4 SP
P1=4 CM
LRT SP
LRT CM
RRT SP
RRT CM


Expected Profit as Function of P1
Expected Profit
P_tr
ELT SP
ELT CM
LRT SP
LRT CM

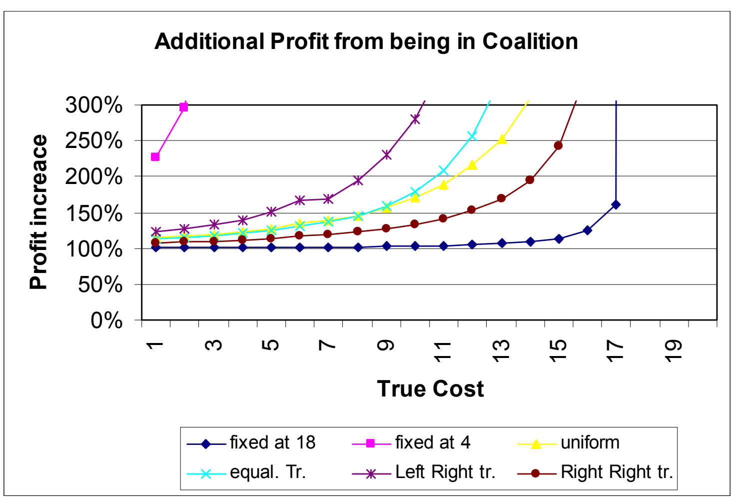
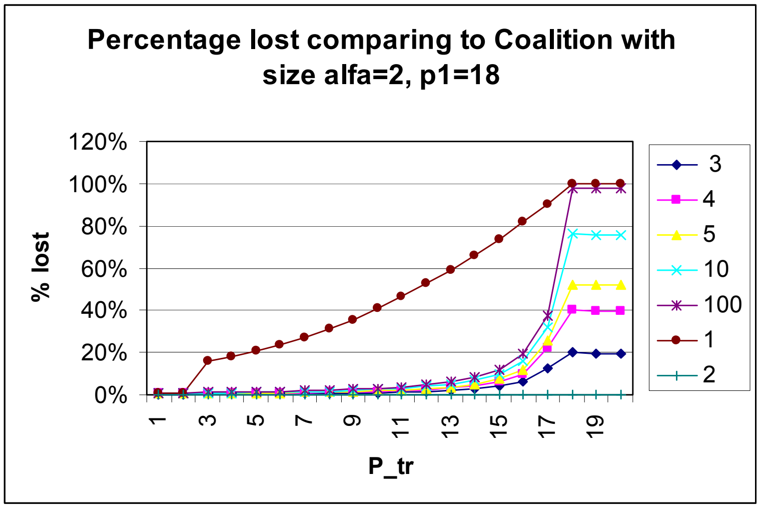
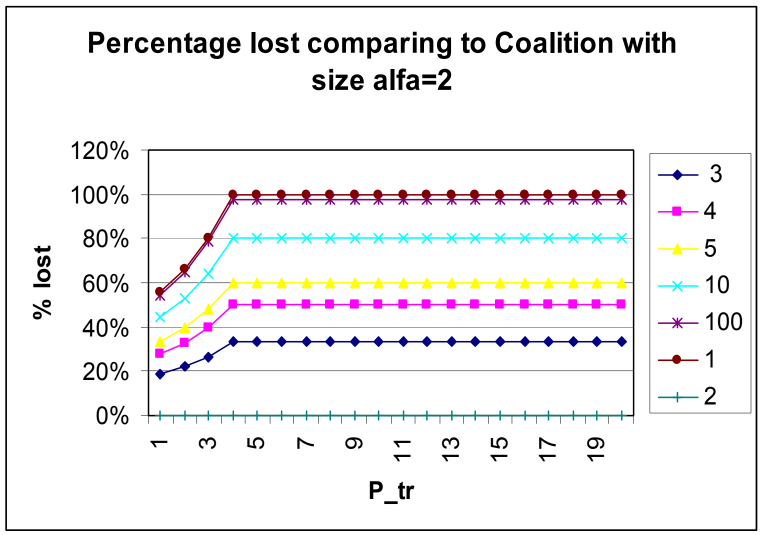

Additional Profit from being in Coalition
Profit increace
True Cost
fixed at 18
fixed at 4
uniform
equal. Tr.
Left Right tr.
Right Right tr.

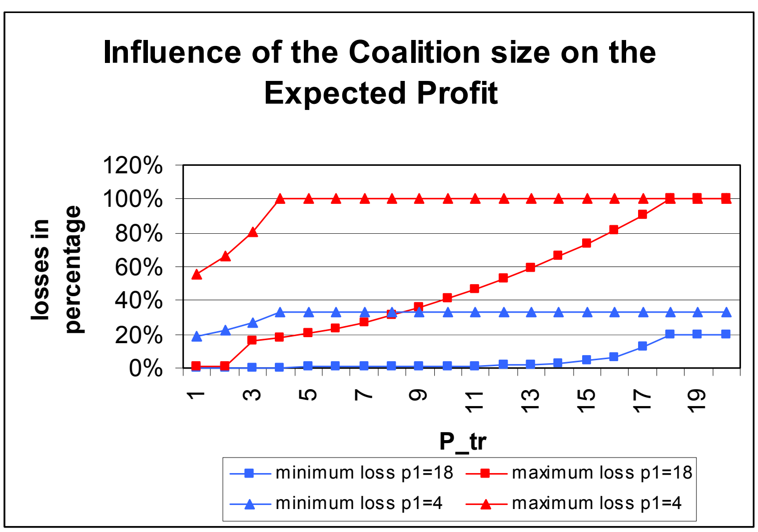
Figure 6:



Percentage lost comparing to Coalition with size alfa=2, p1=18
120%
100%
80%
60%
40%
20%
0%
% lost
1
3
5
7
9
11
13
15
17
19
P_tr
3
4
5
10
100
1
2

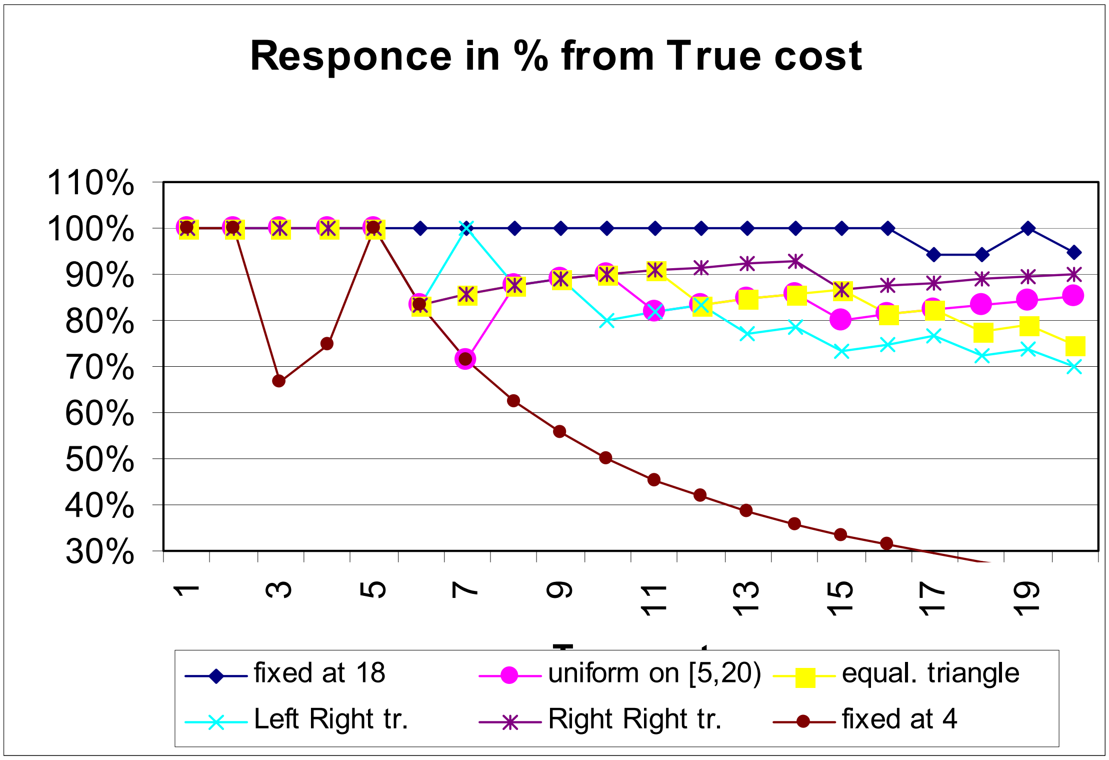


Percentage lost comparing to Coalition with size alfa=2
120%
100%
80%
60%
40%
20%
0%
% lost
1
3
5
7
9
11
13
15
17
19
P_tr
3
4
5
10
100
1
2



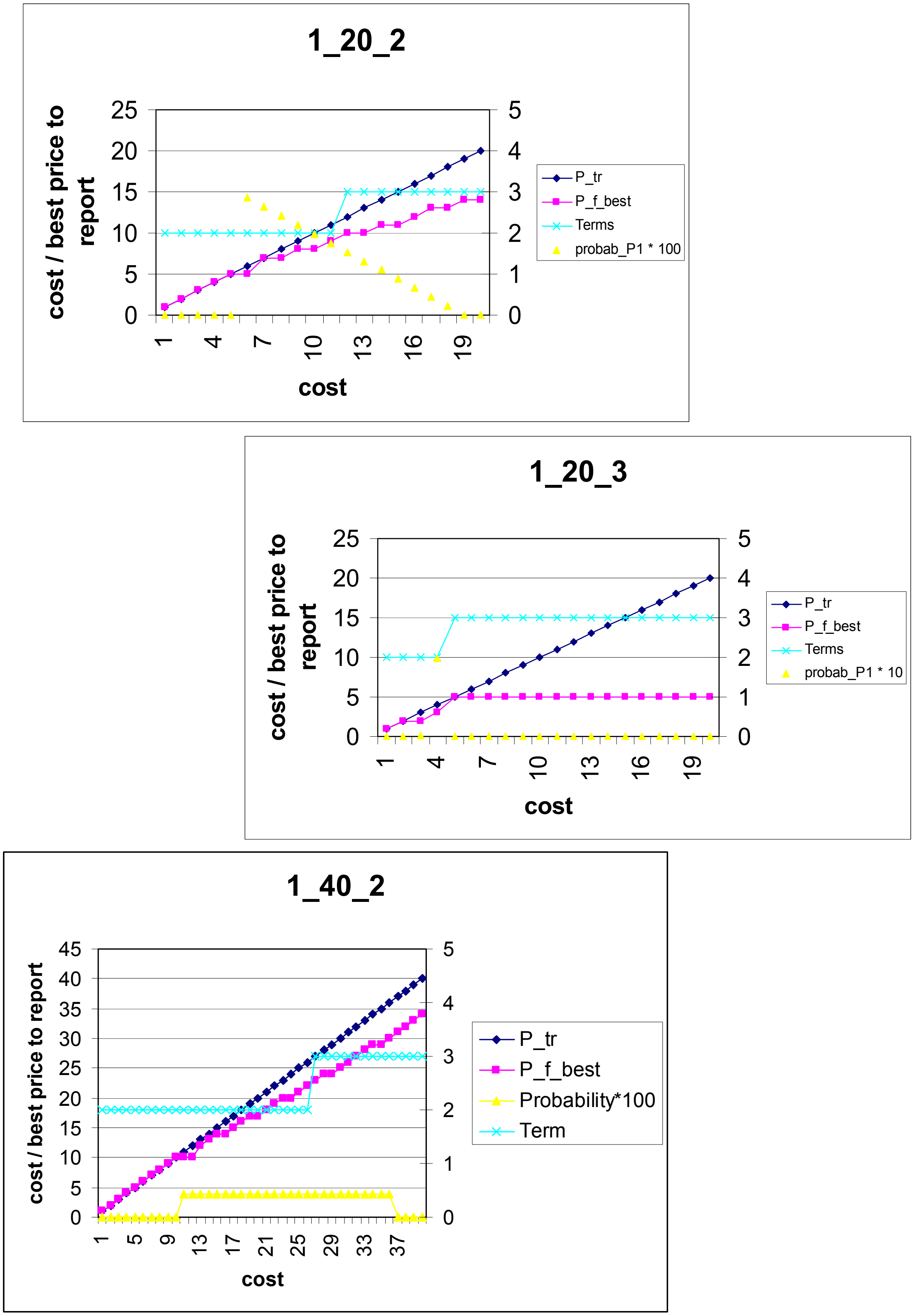
Influence of the Coalition size on the Expected Profit
120%
100%
80%
60%
40%
20%
0%
losses in percentage
1
3
5
7
9
11
13
15
17
19
P_tr
minimum loss p1=18
maximum loss p1=18
minimum loss p1=4
maximum loss p1=4

Figure 7:



Responce in % from True cost
110%
100%
90%
80%
70%
60%
50%
40%
30%
1
3
5
7
9
11
13
15
17
19
fixed at 18
uniform on [5,20)
equal. triangle
Left Right tr.
Right Right tr.
fixed at 4

Figure 8:



1_20_2
cost / best price to report
cost
P_tr
P_f_best
Terms
probab_P1 * 100
1_20_3
cost / best price to report
cost
P_tr
P_f_best
Terms
probab_P1 * 10
1_40_2
cost / best price to report
cost
P_tr
P_f_best
Probability*100
Term

Figure 9: