Curriculum Vita - Linghai Zhang
Department of Mathematics
Lehigh University

Home address: 109 Wyndham Drive, Allentown, Pennsylvania 18104

Office address: 17 Memorial Drive East, Bethlehem, PA 18015 USA

Email: liz5@lehigh.edu. Cell Phone: 484-505-0189

A. Biographical Information
I: Education

Bachelor: Department of Mathematics, Beijing Normal University. 1982 - 1986

Master: Institute of Applied Physics and Computational Mathematics. 1986 - 1989

Ph.D: Department of Mathematics, The Ohio State University. 1994 - 1999

II: Positions Held

Researcher: Institute of Applied Physics and Computational Mathematics. 1989 - 1994

Dunham Jackson Assistant Professor: University of Minnesota. 1999 - 2002

Assistant Professor, Department of Mathematics, Lehigh University. 2002 - 2008

Associate Professor, Department of Mathematics, Lehigh University. 2008 - 2020

Professor: Department of Mathematics, Lehigh University. 2020 - now

III: Research Interests

(1) Properties of Global Solutions of Fluid Dynamics Equations

(2) Existence and Stability of Traveling Waves of Equations in Mathematical Neuroscience

(3) Representations of Bounded Smooth Traveling Waves of Various Nonlinear Evolution Equations

B: List of Publications

I: Book Chapters

1 Solutions to Some Open Problems in n-Dimensional Fluid Dynamics. *Progress in Nonlinear Analysis Research*. Editor: Erik T. Hoffmann. Nova Science Publishers Inc. New York. ISBN: 978-1-60456-359-7. 2009. Pages 69-135.

2 Traveling Waves Arising from Synaptically Coupled Neuronal Networks. *Advances in Mathematics Research*, Volume **10**. Editor: Albert R. Baswell. Nova Science Publishers Inc. New York. ISBN: 978-1-60876-265-1. 2010. Pages 53-204.

3 Particular solutions of nonlinear ordinary differential equations and applications. *Nonlinear systems: Design, Applications and Analysis*. Editor: Christos K. Volos. Nova Science Publishers, Inc. 2017. 37-60.

II: Journal Papers

1 Initial value problem for a nonlinear parabolic equation with singular integral-differential term. *Acta Mathematicae Applicatae Sinica, English Series*, **8**(1992), 367-376.

2 On weak solution of the mixed nonlinear Schrödinger equations. *Chinese Advances in Mathematics*, **21**(1992), 495-497. (with Shaobin Tan)

3 Decay of solutions of the multidimensional generalized Kuramoto-Sivashinsky system. *IMA Journal of Applied Mathematics*, **50**(1993), 29-42.

4 Periodic boundary problem and Cauchy problem for the fluid dynamic equation in geophysics. *Journal of Partial Differential Equations*, **6**(1993), 173-192. (with Yulin Zhou and Boling Guo)

5 Decay of solutions to 2-dimensional Navier-Stokes equations. *Chinese Advances in Mathematics*, **22**(1993), 469-472.

6 A generalized third-order Benjamin-Ono equation. *Acta Mathematica Scientia*, **13**(1993), 473-479. (In Chinese)

7 Decay of solutions of a higher order multidimensional nonlinear Korteweg-de Vries-Burgers system. *Proceedings of the Royal*

Society of Edinburgh, Section A: Mathematics, **124**(1994), 263-271.

8 Decay of solutions of generalized Benjamin-Bona-Mahony equations. *Acta Mathematica Sinica, New Series*, **10**(1994), 428-438.

9 Asymptotic property for the solution to the generalized Korteweg-de Vries equation. *Acta Mathematicae Applicatae Sinica, English Series*, **10**(1994), 377-386.

10 Initial value problem for a nonlinear singular integro-differential equation. *Acta Mathematica Scientia, English Edition*, **14**(1994), 261-271.

11 Initial value problem for a nonlinear evolution system with singular integral differential terms. *Journal of Partial Differential Equations*, **7**(1994), 64-76.

12 Cauchy problem for a nonlinear singular integral-differential evolution system. *Applied Mathematics, A Journal of Chinese Universities, Series B*, **9**(1994), 45-53.

13 Initial value problem for a generalized Korteweg-de Vries equation with singular integral-differential terms. *Northeastern Mathematical Journal*, **10**(1994), 41-46.

14 On a weak solution of the mixed nonlinear Schrödinger equations. *Journal of Mathematical Analysis and Applications*, **182**(1994), 409-421. (with Shaobin Tan)

15 Decay estimates for the solutions of some nonlinear evolution equations. *Journal of Differential Equations*, **116**(1995), 31-58.

16 Sharp rate of decay of solutions to 2-dimensional Navier-Stokes equations. *Communications in Partial Differential Equations*, **20**(1995), 119-127.

17 Decay of solution of generalized Benjamin-Bona-Mahony-Burgers equations in n-space dimensions. *Nonlinear Analysis, Theory, Methods and Applications*, **25**(1995), 1343-1369.

18 Decay of solutions to magnetohydrodynamics equations in two space dimensions. *Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences*, **449**(1995), 79-91. (with Boling Guo)

19 Decay estimates for solutions of initial value problems for the generalized nonlinear Korteweg-de Vries equation. *Chinese Annals of Mathematics, Series A*, **16**(1995), 22-32. (In Chinese)

20 Decay estimates for the solution of the initial value problem to the generalized nonlinear Korteweg-de Vries equation. *Chinese Journal of Contemporary Mathematics*, **16**(1995), 1-14.

21 Decay estimations of the solution of a nonlinear pseudoparabolic equation. *Acta Mathematica Scientia, English Edition*, **15**(1995), 326-334. (with Longjun Shen)

22 Decay of solution of a parabolic equation in 2-space dimensions. *Journal of Partial Differential Equations*, **8**(1995), 126-134. (with Boling Guo)

23 On the solutions of the coupled nonlinear parabolic equations. *Journal of Partial Differential Equations*, **9**(1996), 71-83. (with Longjun Shen)

24 Decay of global solutions of two nonlinear evolution equations. *Acta Mathematicae Applicatae Sinica, English Series*, **13**(1997), 23-32. (with Boling Guo)

25 L^2 uniform stability of solutions to a generalized Korteweg-de Vries equation arising from plasma physics. *Chinese Advances in Mathematics*, **26**(1997), 537-544.

26 Long time uniform stability of solutions of magnetohydrodynamics equations. *Taiwanese Journal of Mathematics*, **1**(1997), 39-46.

27 Uniform stability and asymptotic behavior of solutions of 2-dimensional magnetohydrodynamics equations. *Chinese Annals of Mathematics, Series B*, **19**(1998), 35-58.

28 L^2 decay rate of solution to Cauchy problem for the fluid dynamic equation in geophysics with dissipation. *Nonlinear Partial Differential Equations and Applications*. (1998), 222-230. (with Linge Yang and Haiyang Huang)

29 Long time uniform stability for solutions of n-dimensional Navier-Stokes equations. *Quarterly of Applied Mathematics*, **57**(1999), 283-315.

30 Uniform stability for solutions of n-dimensional Navier-Stokes equations. *Bulletin of the Institute of Mathematics, Academia Sinica*, **27**(1999), 265-315.

31 Long-time uniform stability of solution to magnetohydrodynamics equation. *Applied Mathematics, A Journal of Chinese Universities, Series B*, **14**(1999), 45-50. (with Boling Guo and Haiyang Huang)

32 Local Lipschitz continuity of a nonlinear bounded operator induced by a generalized Benjamin-Ono-Burgers equation. *Nonlinear Analysis, Theory and Methods*, **39**(2000), 379-402.

33 Evans functions and asymptotic stability of traveling wave solutions. *Chinese Annals of Mathematics, Series B*, **22**(2001), 343-360.

34 On traveling waves of a generalized bistable equation. *Acta Mathematicae Applicatae Sinica, English Series*, **17**(2001), 286-288.

35 Long-time asymptotic behaviors of solutions of n-dimensional dissipative partial differential equations. *Discrete and Continuous Dynamical Systems, Series A*, **8**(2002), 1025-1042.

36 Stability and regularity of suitably weak solutions of n-dimensional magnetohydrodynamics equations. *Journal of Partial Differential Equations*, **16**(2003), 82-96.

37 On stability of traveling wave solutions in synaptically coupled neuronal networks. *Differential and Integral Equations*, **16**(2003), 513-536.

38 Existence, uniqueness and exponential stability of traveling wave solutions of some integral differential equations arising from neuronal networks. *Journal of Differential Equations*, **197**(2004), 162-196.

39 Dissipation and decay estimates. *Acta Mathematicae Applicatae Sinica, English Series*, **20**(2004), 59-76.

40 Exponential stability of traveling pulse solutions of a singularly perturbed system of integral differential equations aris-

ing from excitatory neuronal networks. *Acta Mathematicae Applicatae Sinica, English Series*, **20**(2004), 283-308.

41 L^p estimates of solutions of some partial differential equations. *Nonlinear Analysis, Theory and Methods*, **56**(2004), 147-155.

42 On the modified Navier-Stokes equations in n-dimensional spaces. *Bulletin of the Institute of Mathematics, Academia Sinica*, **32**(2004), 185-193.

43 Eigenvalue functions in excitatory-inhibitory neuronal networks. *Journal of Partial Differential Equations*, **17**(2004), 329-350.

44 Positive steady states of an elliptic system arising from biomathematics. *Nonlinear Analysis, Real World Applications*, **6**(2005), 83-110.

45 Traveling waves of a singularly perturbed system of integral-differential equations arising from neuronal networks. *Journal of Dynamics and Differential Equations*, **17**(2005), 489-522.

46 Steady-state solutions in nonlocal neuronal networks. *Dynamics in Partial Differential Equations*, **2**(2005), 71-100.

47 Dynamics of neuronal waves. *Mathematische Zeitschrift*, **255**(2007), 283-321.

48 How do synaptic coupling and spatial temporal delay influence traveling waves in nonlinear nonlocal neuronal networks? *SIAM Journal on Applied Dynamical Systems*, **6**(2007), 597-644.

49 Solutions to some open problems in fluid dynamics. *Chinese Annals of Mathematics, Series B*, **29**(2008), 179-198.

50 New results of a general n-dimensional incompressible Navier-Stokes equations. *Journal of Differential Equations*, **245**(2008), 3470-3502.

51 Analysis of speeds of neuronal waves. *Chinese Advances in Mathematics*, **38**(2009), 19-35.

52 Speeds of traveling waves of some integral differential equations. *Japan Journal of Industrial and Applied Mathematics*, **27**(2010), 347-373. (with Eiji Yanagida).

53 Influence of sodium currents on speeds of traveling wave fronts in synaptically coupled neuronal networks. *Physica D*, **239**(2010), 9-32. (with Ping-Shi Wu and Melissa Anne Stoner)

54 Influence of neurobiological mechanisms on speeds of traveling wave fronts in mathematical neuroscience. *Discrete and Continuous Dynamical Systems, Series B*, **16**(2011), 1003-1037. (with Ping-Shi Wu and Melissa Anne Stoner)

55 Explicit traveling wave solutions of five kinds of nonlinear evolution equations. *Journal of Mathematical Analysis and Applications*, **379**(2011), 91-124.

56 Explicit traveling wave solutions of nonlinear evolution equations. *Chinese Annals of Mathematics*, **32B**(2011), 929-964.

57 Evans functions and bifurcations of standing wave solutions in delayed synaptically coupled neuronal networks. *Journal of Applied Analysis and Computation*, **2**(2012), 213-240.

58 Standing wave solutions in nonhomogeneous delayed synaptically coupled neuronal networks. *Journal of Partial Differential Equations*, **25**(2012), 295-329. (with Melissa Anne Stoner)

59 Existence and uniqueness of a traveling wave front of a model equation in synaptically coupled neuronal networks. *Journal of Applied Analysis and Computation*, **3**(2013), 145-167.

60 Distributed nonlocal feedback delays may destabilize fronts in neural fields, distributed transmission delays do not. *Journal of Mathematical Neuroscience*, **9**(2013). Article ID:3, pages 1-21. (with Axel Hutt)

61 Traveling wave solutions of nonlinear scalar integral differential equations arising from synaptically coupled neuronal networks. *Journal of Applied Analysis and Computation*, **4**(2014), 1-68. (with Axel Hutt)

62 Wave speed analysis of traveling wave fronts in delayed synaptically coupled neuronal networks. *Discrete and Continuous Dynamical Systems, Series A*, **34**(2014), 2405-2450.

63 Evans functions and instability of a standing pulse solution of a nonlinear system of reaction diffusion equations. *Annals of Applied Mathematics*, **32**(2016), 79-101.

64 The improved Fourier splitting method and decay estimates of the global solutions of the Cauchy problems for nonlinear systems of fluid dynamics equations. *Annals of Applied Mathematics*, **32**(2016), 396-417.

65 Evans functions and bifurcations of standing wave fronts of a nonlinear system of reaction diffusion equations. *Journal of Applied Analysis and Computation*, **6**(2016), 515-530.

66 Evans functions for multiple standing pulse solutions of a nonlinear system of reaction diffusion equations. *Journal of Shanghai Normal University*, (2016), 265-285.

67 Decay estimates with sharp rates of global solutions of nonlinear systems of fluid dynamics equations. *Discrete and Continuous Dynamical Systems, Series S*, **9**(2016), 2181-2200.

68 Global smooth solution of a two-dimensional nonlinear singular system of differential equations arising from geostrophics. *Journal of Differential Equations*, **262**(2017), 3980-4020. (with Boling Guo, Yongqian Han, Daiwen Huang and Dongfen Bian)

69 Evans functions and bifurcations of nonlinear waves of some reaction diffusion equations. *Journal of Differential Equations*, **263**(2017), 3627-3686.

70 New proofs of the decay estimate with sharp rate of the global weak solution of the n-dimensional incompressible Navier-Stokes equations. *Annals of Applied Mathematics*, **34**(2018), 416-438.

71 Properties of solutions of n-dimensional incompressible Navier-Stokes equations. *Annals of Applied Mathematics*, **35**(2019), 392-448.

72 The exact limits and improved decay estimates for all order derivatives of the global weak solutions to a two-dimensional incompressible dissipative quasi-geostrophic equation. *Journal of Nonlinear Modeling and Analysis*, **5**(2023), 146-202.

73 The exact limits and improved decay estimates for all order derivatives of global weak solutions of three incompressible

fluid dynamics equations. *Journal of Nonlinear Modeling and Analysis*, **5**(2023), 803-903.

74 The Benjamin-Ono-Burgers equation: new ideas and new results. *Journal of Nonlinear Modeling and Analysis*, **6**(2024), 841-872.

75 The exact limits and improved decay estimates for all order derivatives of the global weak solutions of n-dimensional incompressible Navier-Stokes equations. *Acta Mathematicae Applicatae Sinica, English Series*, **41**(2025), 27-83.

76 A general Korteweg-de Vries Burgers equation: novel ideas and novel results. *Journal of Nonlinear Modeling and Analysis*, **7**(2025), 334-382.

77 The exact limits of all order derivatives of the global smooth solution of a general Korteweg-de Vries-Burgers equation. *Discrete and Continuous Dynamics Systems, Series S*, **18**(2025), 3846-3895.

Remarks

(A) I will open a New Research Direction in Applied Mathematics in the near future. I will couple together novel ideas, the nonlinear diffusion, the linear diffusion, and the relationships between the nonlinear diffusion and the linear diffusion, as well as several classical ideas to establish the uniform energy estimates for all order derivatives of the global solutions of a few n-dimensional incompressible equations (including the n-dimensional incompressible magnetohydrodynamics equations, the n-dimensional incompressible Navier-Stokes equations and other similar incompressible equations). This way we will be able to prove the existence of the global smooth solutions of several important fluid dynamics equations. They are very important, very difficult open problems in applied mathematics, national defence, engineering and industry. These model equations have strong physical backgrounds and play important roles in industry, engineering and applied mathematics.

(B) Influences of my research to the Ph.D program in applied mathematics: I have developed a completely different way (much easier to understand, to consume, to absorb and to apply for graduate students and young researchers) to accomplish the exact limits and improved decay estimates with sharp rates for all order derivatives of the global solutions of a broad class of equations involving diffusions. I have saved a few research topics for my future Ph.D students. I would like to list the topics here for my personal record.

(1) the Korteweg-de Vries-Burgers equation and the Benjamin-Ono-Burgers equation in one-dimensional space.

(2) the two-dimensional incompressible dissipative quasi-geostrophic equation.

(3) the incompressible magnetohydrodynamics equations, incompressible Navier-Stokes equations, and the Benjamin-Bona-Mahony-Burgers equations in n-dimensional space.

(C) I have published many papers in academic journals after I came to Lehigh in 2002. All of the papers were refereed anonymously before accepted for publication. At least twenty papers have been highly recognized and cited by other experts.

(D) I was the main author of the joint papers: [18], [21], [22], [23], [24], [53], [54], [58], [61], [68]. I was responsible for the main body (the main results, the development of the main ideas, the mathematical analysis, the proofs, and the structure).

(E) I was a co-author of the joint papers [14], [52], [60]. I contributed at least half of the mathematical analysis.

(F) I was a minor author of the joint papers [2], [4], [28], [31]. I made some contributions to the development of the main ideas and the mathematical analysis.

C. Honors and Awards

Franz Summer Research Award: $4000. Lehigh University. 2003.

D. Research Funding and Training Grants

Research Grant (with Michael Burger and Ping-Shi Wu): $120,000. Title: Influence of neurobiological mechanisms on wave speeds of nerve impulses in synaptically coupled neuronal networks. Howard Hughs Medical Institute (Biosystems Dynamics Summer Institute of Lehigh University). 2007 - 2008.

Faculty Research Grant: $2500. Lehigh University. 2005.

Faculty Research Grant: $2500. Lehigh University. 2007.

Faculty Research Grant: $3000. Lehigh University. 2012.

Faculty Research Grant: $6000. Lehigh University. 2014.

Faculty Grant for International Connection: $2800. Lehigh University. 2012.

Reidler Research Foundation: $23,000. Title: Modeling propagation of nerve impulses arising from synaptically coupled neuronal networks. 2003 - 2005.

These research grants helped me to travel, to present my results in conferences, to visit experts, to collaborate, to improve the quality and the publications of my mathematics papers.

E. Editorial Board Membership

1 Associate Editor of International Journal of Mathematical Physics since July 2018.

2 Editorial Board Membership of SCIREA Journal of Mathematics since April, 2019.

3 Guest Co-Editor of the Special Issue "Nonlinear Partial Differential Equations in Mathematics and Physics" for *Abstract and Applied Analysis* in 2014.

F. Scholarly Presentations

I: Invited Conference Presentations

Representing Lehigh University, I have been invited to give talks in many different countries (including Andorra, Australia, Canada, China, Germany, Japan and US).

Workshop on Nonlocal Integral Differential Equations in Mathematics and Biology. Mathematical Biosciences Institute, The Ohio State University. March 6 - 8, 2003. Lecture: Asymptotic stability of traveling pulse solutions arising from neuronal networks.

The Fifth International Congress on Industrial and Applied Mathematics. Sydney, Australia. July 7 - 11, 2003. Lecture: Evans functions and stability of traveling wave solutions of nonlinear systems of integral differential equations.

The Fourth World Congress of Nonlinear Analysts. Orlando, Florida. June 30 - July 7, 2004. Lecture: Stability of traveling pulse solutions in synaptically coupled neuronal networks.

Mini-Symposium on Nonlinear Dynamics. Department of Mathematics and Statistics, York University, Canada. August 3 - 4, 2005. Lecture: Traveling waves of a singularly perturbed system of integral-differential equations arising from neuronal networks.

International Conference on Applied Mathematics and Interdisciplinary Research. Nankai Institute of Mathematics, Nankai University, China. June 12 - 15, 2006. Lecture: Maximum principle of Evans functions and its applications to stability analysis of nonlinear waves.

Conference on Mathematical Neuroscience (A Satellite Activity of the International Congress of Mathematicians 2006 - NEURO-MATH 2006) (an activity of the project: Shaping new directions in Mathematics for Science and Society) at Sant Julia de Loria, Principat d'Andorra. September 1 - 4, 2006. Lecture: How do synaptic coupling and spatial temporal delay influence traveling waves in synaptically coupled neuronal networks?

International Conference on Nonlinear Waves - Theory and Applications. Research Center for Applied Mathematics, Tsinghua University, China. June 9 - 12, 2008. Lectures: I Solutions to some open problems in n-dimensional fluid dynamics. II Dynamics of nonlinear traveling waves in synaptically coupled neuronal networks.

The Eighth AIMS Conference on Dynamical Systems, Differential Equations and Applications. Dresden University of Technology, Germany. May 25 - 28, 2010. Lectures I: Neuronal Waves. II: How do neurobiological mechanisms influence the speeds of traveling waves in mathematical neuroscience?

The Eighth Annual Conference on Frontiers in Applied and Computational Mathematics. New Jersey Institute of Technology, New Jersey. June 9 - 11, 2011. Lecture: Traveling wave solutions of integral differential equations arising from synaptically coupled neuronal networks.

International Conference on Partial Differential Equations and Mathematical Physics. Jiangsu University, Zhenjiang, Jiangsu, China. May 25 - May 28, 2012. Lecture: Some recent research results on integral differential equations, ordinary differential equations and partial differential equations.

International Mini-Workshop on Dynamical Systems and Nonlinear Waves. June 1 - June 5, 2012. Zhejiang Normal University, Jinhua, Zhejiang, China. Lecture I: Traveling wave solutions of integral differential equations arising from delayed synaptically coupled neuronal networks. Lecture II: Evans functions and stability of traveling wave solutions of some nonlinear integral differential equations arising from synaptically coupled neuronal networks.

International Mini-Workshop on Interdisciplinary Research. Department of Mathematics, Yunnan Normal University. May 28, 2014. Lecture: Traveling pulse solutions of a nonlinear singularly perturbed system arising from mathematical neuroscience.

The Eighth International Conference on Recent Advances in Applied Dynamical Systems. June 2-4, 2014. Guilin University of Electronic Technology, Guilin, China. Lecture: Harmonic functions, Hopf lemma, Evans functions and stability of traveling pulse solutions of nonlinear singularly perturbed systems of differential equations.

American Mathematical Society Sectional meeting, November 14-15, 2015. Rutgers University. New Brunswick, New Jersey. Lecture: Existence and stability of infinitely many traveling pulse solutions of nonlinear singularly perturbed system of integral differential equations arising from synaptically coupled neuronal networks.

International Conference on Nonlinear Systems of Fluid Dynamics Equations and Applications. Tsinghua Sanya International Mathematics Forum, China. December 19-22, 2015. Lecture: The global smooth solution of a two-dimensional nonlinear singular system of differential equations arising from geostrophics.

January 11, 2016. Workshop on Nonlinear PDE at City University of Hong Kong. Lecture: The global smooth solution of a two-dimensional nonlinear singular system of differential equations arising from geostrophics.

The International Conference on Nonlinear Mathematical Analysis with Applications. August 12-14, 2016. Longyan University, Fujian, China. Lecture: Global smooth solution of a two-dimensional nonlinear singular system of differential equations arising from geostrophics.

PDE Model and Nonlinear Waves for Fluids and Plasma Workshop. December 25-29, 2017. Tsinghua Sanya International Mathematics Forum. China. Lecture: Several properties of the global smooth solutions of a nonlinear singular system of differential equations arising from geostrophics.

International Conference on Integrable Systems and Differential Equations in Mathematical Physics. Jiangsu University, Zhenjiang. June 14-16, 2019. Lecture: Properties of solutions of n-dimensional incompressible Navier-Stokes equations.

Conference on Theory and Applications of Partial Differential Equations (Celebration of the 90th birthday of Professor Boling Guo). Xiamen, China. October 25 - 26, 2025. Talk via ZOOM: The mathematical influences of physical mechanisms on global weak solutions of equations in incompressible fluid dynamics.

F. Scholarly Presentations

II: Invited Colloquium and Seminar Presentations

Department of Mathematics, University of British Columbia, Canada. May 22, 2003. Lecture: On the stability of traveling wave solutions of integral-differential equations arising from synaptically coupled neuronal networks.

Department of Mathematics, Rutgers University. February 22, 2005. Lecture: Traveling wave solutions and bifurcations in nonlocal neuronal networks.

Department of Mathematics, Lehigh University. September 28, 2005. Colloquium Lecture: Dynamics of nonlinear waves in synaptically coupled neuronal networks.

School of Mathematical Sciences, Beijing University. June 17, 2006. Lecture: Solutions to some open problems in fluid dynamics.

Mathematical Biosciences Institute, The Ohio State University. October 17, 2006. Lecture: Dynamics of neuronal waves.

Department of Mathematical Sciences, Carnegie Mellon University. November 28, 2006. Lecture: Exact limits of global solutions of some nonlinear dissipative partial differential equations.

Department of Mathematics, Lehigh University. September 19, 2007. Colloquium Lecture: Recent research activities in mathematical neuroscience and dynamical systems.

School of Mathematical Sciences, Beijing University. June 4, 2008. Lecture: Sharp rates of decay and exact limits of solutions of some dissipative partial differential equations.

Research Center for Applied Mathematics, Tsinghua University, China. June 9, 2009. Lectures: I: Traveling waves of integral differential equations arising from synaptically coupled neuronal networks. II: Exact limits of global strong solutions of some nonlinear dissipative partial differential equations arising from n-dimensional fluid dynamics.

Department of Mathematics, Lehigh University. January 25, 2012. Lecture: Bounded explicit traveling wave solutions of some nonlinear evolution equations.

Department of Mathematics, Shanghai Jiaotong University, May 29, 2012. Lecture: Some recent research results on integral differential equations, ordinary differential equations and partial differential equations.

Department of Mathematics, East China Normal University, May 29, 2012. Lecture: Some recent research results on integral differential equations, ordinary differential equations and partial differential equations.

Department of Mathematics, Shanghai Normal University, May 30, 2012. Lecture: Some recent research results on integral differential equations, ordinary differential equations and partial differential equations.

Department of Mathematics, Tsinghua University, June 7, 2012. Lecture: Traveling wave solutions of integral differential equations arising from delayed synaptically coupled neuronal networks.

Dynamical System Seminar, Institute for Mathematics and its Applications, The University of Minnesota. January 24, 2013. Lecture: Recent results in mathematical neuroscience and nonlinear systems of fluid dynamical equations.

Shanghai Jiaotong University. March 18, 2013. Lecture: Traveling pulse solutions of a nonlinear singularly perturbed system arising from mathematical neuroscience.

Capital Normal University. March 26, 2013. Lecture: Traveling pulse solutions of a nonlinear singularly perturbed system of reaction diffusion equations with a Heaviside step function.

Beijing Normal University. April 11, 2013. Lecture: Traveling pulse solutions of a nonlinear singularly perturbed system of reaction diffusion equations with a Heaviside step function.

Tsinghua University. April 12, 2013. Lecture: Traveling pulse solutions of a nonlinear singularly perturbed system arising from mathematical neuroscience.

Department of Mathematics, Lehigh University, September 4, 2013. Colloquium Lecture: Evans functions and stability of traveling wave solutions of differential equations.

Department of Mathematics, Yunnan Normal University. May 27, 2014. Lecture: Evans functions and exponential stability of traveling wave solutions of nonlinear integral differential equations arising from synaptically coupled neuronal networks.

Division of Applied Mathematics, Beijing Institute of Applied Physics and Computational Mathematics. June 19, 2014. Lecture: The global smooth solutions of n-dimensional nonlinear systems of fluid dynamics equations.

Research Center for Applied Mathematics, Tsinghua University, China. June 20, 2014. Lecture: The existence, decay estimates and exact limits of the global smooth solutions of n-dimensional nonlinear systems of fluid dynamics equations.

Lehigh University. September 8, 2015. Lecture: Recent progress in nonlinear systems of fluid dynamics equations, mathematical neuroscience and dynamical systems.

Department of Applied Mathematics, Hong Kong Polytechnic University, December 28, 2015. Lecture: Evans functions and bifurcations of standing wave fronts of a nonlinear system of reaction diffusion equations.

Department of Mathematics, South China Institute of Technology. December 30, 2015. Lecture: Evans functions and bifurcations of standing wave fronts of a nonlinear system of reaction diffusion equations.

Department of Mathematics, South China Normal University, December 31, 2015. Lecture: Stability of traveling wave solutions of nonlinear reaction diffusion equations.

Beijing Institute of Technology. August 24, 2016. Lecture: Properties of the global smooth solution of a two-dimensional nonlinear singular system of differential equations.

Colloquium talk at the Department of Mathematics, Lehigh University. December 7, 2016. Lecture: Global smooth solution of a two-dimensional nonlinear singular system of differential equations arising from geostrophics.

Division of Applied Mathematics, Beijing Institute of Applied Physics and Computational Mathematics. August 24, 2017. Properties of global solutions of n-dimensional incompressible fluid dynamics equations

Department of Applied Mathematics, Hong Kong Polytechnic University. January 3, 2018. Lecture: Evans functions and bifurcations of nonlinear waves of some reaction diffusion equations

Shanghai Normal University, June 1, 2018. Talk: Existence and stability of a traveling wave front of a reaction diffusion equation.

Fudan University. June 8, 2018. Talk: Properties of global smooth solutions of nonlinear evolution equations with dissipation.

Department of Mathematics, Lehigh University. October 10, 2018. Colloquium talk: Exact limits of the global weak solutions of n-dimensional incompressible Navier-Stokes equations.

Department of Mathematics, Jiaxing University. June 21, 2019. Talk: Several new properties of global weak solutions of n-dimensional incompressible Navier-Stokes equations.

Department of Mathematics, Lehigh University. September 25, 2019. Colloquium talk: New properties of global weak solutions of n-dimensional magnetohydrodynamics equations.

Department of Mathematics, Henan University. June 24, 2021. Lecture: The exact limits and improved decay estimates with sharp rates for all order derivatives of the global weak solutions to nonlinear evolution equations with diffusions.

Department of Mathematics, Lehigh University. January 25, 2023. Colloquium lecture: The exact limits and improved decay estimates for all order derivatives of global weak solutions of incompressible fluid dynamics equations.

Department of Mathematics, Jinan University, Guangzhou, China, December 22, 2023. Seminar Talk: Various properties of the global weak solutions of n-dimensional incompressible magnetohydrodynamics equations.

Department of Mathematics, Lehigh University. April 2, 2025. Colloquium lecture: The influence of physical mechanisms on global solutions of nonlinear evolution equations with dissipations.

G. Teaching and Research Advising

I: Courses Taught

(Courses for Undergraduate Students)

Mathematics 21 - 22 - 23, Mathematics 75 - 76

Mathematics 43: Survey of Linear Algebra

Mathematics 205: Linear Methods

Mathematics 295: Introduction to Dynamical Systems in Mathematical Biology

Mathematics 301: Principles of Mathematical Analysis

Mathematics 320: Ordinary Differential Equations

Mathematics 341: Mathematical Models and Their Formulation

Mathematics 405 - 406: Partial Differential Equations

Mathematics 435: Introduction to Functional Analysis

Mathematics 450: Dynamical Systems and Mathematical Biology

Mathematics 450: Existence and Stability of Traveling Waves

Mathematics 450: Nonlinear Partial Differential Equations

Mathematics 450: Topics in Applied Mathematics

Mathematics 450: Topics in Evans Functions

Mathematics 450: Topics in Mathematical Neurobiology

Mathematics 450: Topics in Traveling Pulse Solutions

G. Teaching and Research Advising
II: Advising Students, Postdocs and Visitors

1 Melissa Anne Stoner received Ph.D in 2011. The dissertation is: Existence and stability of standing and traveling wave solutions arising from synaptically coupled neuronal networks. Currently Melissa is an Associate Professor in Salisbury University.

2 Alan Dyson received Ph.D in 2019. The dissertation is: Traveling wave solutions of nonlinear integral differential equations arising from synaptically coupled neuronal networks. Currently Alan is a tenure track Assistant Professor in Lycoming College.

3 Mashael Alshammari received Ph.D in 2024. The dissertation is: Properties of the global solutions of a two-dimensional incompressible dissipative quasi-geostrophic equation.

4 Hang Zhao received Ph.D in 2024. The dissertation is: Some topics in partial differential equations: exact limits and diffusive limit. (Lei Wu was a co-advisor)

5 In 2014 - 2016, I supervised Nan Lu as a post doctoral. Nan Lu received his Ph.D from Georgia Institute of Technology.

6 From September 1, 2011 to August 31, 2012, I supervised an international visitor: Lijun Zhang, Professor of Mathematics, Zhejiang Sci-Tech University.

7 From November 1, 2011 to October 31, 2012, I supervised an international visitor: Haihong Liu, Professor of Mathematics, Yunnan Normal University.

8 From July 7, 2018 to October 5, 2018, I supervised an international visitor: Ruifeng Zhang, Professor of Mathematics, Henen University.

9 In the summer of 2006, I supervised three undergraduate students to do research in mathematical neuroscience - speed analysis of traveling waves arising from synaptically coupled neuronal networks.

10 In the summer of 2007, Michael Burger and I supervised a team of students to conduct research in mathematical neuroscience.

11 In the summer of 2008, Michael Burger, Ping-Shi Wu and I supervised a team of students to conduct research in mathematical neuroscience.

12 In the summer of 2010, I supervised Neil Whitman Dexter to conduct research in mathematical neuroscience.

13 In Spring 2020, invited several undergraduate students from my classes (Fall 2019, Mathematics 205) to make a strong research group to do research: Explicit Representations of Bounded Smooth Solutions of Nonlinear Evolution Equations.

H. Services

I performed many services for the Department of Mathematics, the College of Arts and Sciences, Lehigh University, and the mathematical society.

1 University Educational Policy Committee (CAS representative), 2016 - 2019

2 College of Arts and Sciences Policy Committee, 2016 - 2019

3 College of Arts and Sciences Nomination Committee: 2019 - 2022. Chair of the Nominations Committee: 2021 - 2022

4 Hiring Search Committee member for many times (2002 - 2003, 2013 - 2014, 2017 - 2018, 2021 - 2022, 2025)

5 Calculus Committee member (Spring 2007 to Fall 2012)

6 Graduate Committee member (Fall 2003 to Spring 2006)

7 Advisor of undergraduate mathematics majors (Fall 2011 to Fall 2013)

8 Advisor of undergraduate non-mathematics majors, since Fall 2005

9 Advisor of undergraduate applied mathematics majors since Fall 2013

10 Advisor of undergraduate applied mathematics minors since Fall 2013

11 Course coordinators of Mathematics 205, Mathematics 22 and Mathematics 23 for many times

12 Making, proctoring and grading comprehensive examinations and qualifying examinations for many times

13 Advisor of undergraduate pure mathematics minors (Fall 2011 to Fall 2013)

14 Advisor of a team of students in Mathematics Contest in Modelling (Wang Shuai, Xu Duo, Liu Yanxi) in Spring 2018

15 Library representative (Fall 2004 to Spring 2006)

16 Overall Coordinator for the Conference for Undergraduates Considering Graduate School in Mathematics. Title: Making the Most of Mathematics Graduate School. April 22, 2006. Department of Mathematics, Lehigh University

17 Organizer of Seminar in Applied Mathematics (Fall 2003 to Spring 2006).

18 Main organizer of the international conference: Nonlinear Systems of Fluid Dynamics Equations and Applications. December 19 - 22, 2015

19 Organizer of the Special Session on Nonlinear Waves in Differential Equations at the American Mathematical Society's Sectional Meeting at Rutgers University. November 14 - 15, 2015

20 Co-organizer of the mini-symposium on "Neuronal and Biological Dynamical Systems" at the Fifth International Congress on Industrial and Applied Mathematics. Sydney, Australia. July 7 - 11, 2003

21 Beginning January 2021, I am responsible for transferring Credits for Courses in Mathematics (except for probability and statistics).

22 I was the main organizer of the Seminar in Applied Mathematics in 2023. I invited 9 international experts in applied mathematics to give talks

23 I was the Committee Chair in Qualifying Exams (Differential Equations) for many times in 2020 - 2024

24 I made, proctored and graded the Comprehensive exams in 2024 and 2025

I am doing and will continue high quality, valuable services for the Department of Mathematics, the College of Arts and Sciences, Lehigh University and the mathematical society.

2023 - 2025 Professional Activity Report

Linghai Zhang - Department of Mathematics

ONE: RESEARCH

A - Papers Published:

(1) The exact limits and improved decay estimates for all order derivatives of the global weak solutions to a two-dimensional incompressible dissipative quasi-geostrophic equation. *Journal of Nonlinear Modeling and Analysis*, **5**(2023), 146-202.

(2) The exact limits and improved decay estimates for all order derivatives of global weak solutions of three incompressible fluid dynamics equations. *Journal of Nonlinear Modeling and Analysis*, **5**(2023), 803-903.

(3) The Benjamin-Ono-Burgers equation: new ideas and new results. *Journal of Nonlinear Modeling and Analysis*, **6**(2024), 841-872.

(4) The exact limits and improved decay estimates for all order derivatives of the global weak solutions of n-dimensional incompressible Navier-Stokes equations. *Acta Mathematicae Applicatae Sinica, English Series*, **41**(2025), 27-83.

(5) A general Korteweg-de Vries Burgers equation: novel ideas and novel results. *Journal of Nonlinear Modeling and Analysis*, **7**(2025), 334-382.

(6) The exact limits of all order derivatives of the global smooth solution of a general Korteweg-de Vries-Burgers equation. *Discrete and Continuous Dynamics Systems, Series S*, **18**(2025), 3846-3895.

B - Talks Given:

(1) Department of Mathematics, Lehigh University. January 25, 2023. Colloquium lecture: The exact limits and improved decay estimates for all order derivatives of global weak solutions of incompressible fluid dynamics equations.

(2) Department of Mathematics, Jinan University, Guangzhou, China, December 22, 2023. Seminar Talk: Various properties of the global weak solutions of n-dimensional incompressible magnetohydrodynamics equations.

(3) Department of Mathematics, Lehigh University. April 2, 2025. Colloquium lecture: The influence of physical mechanisms on global solutions of nonlinear evolution equations with dissipations.

C - Conference Attended:

Conference on Theory and Applications of Partial Differential Equations (Celebration of the 90th birthday of Professor Boling Guo). Xiamen, China. October 25 - 26, 2025. Talk via ZOOM: The mathematical influences of physical mechanisms on global weak solutions of equations in incompressible fluid dynamics.

D - Papers Reviewed: Each year, I review a few papers in my area for academic journals.

TWO: TEACHING

A - Classes Taught: I taught two classes in every semester in each of the last three years.

2023 Spring: 23 *. Summer: 22, 205. Fall: 43, 405 (* means two sections)

2024 Spring: 205 *. Summer: 23, 205. Fall: 205 *.

2025 Spring: 205, 320. Summer: 22, 23. Fall: 205 *.

Over many years, I wrote many valuable supplemental materials to teach classes, including Mathematics 21 - 22 - 23, Mathematics 205 (Linear Methods), Mathematics 320 (Ordinary Differential Equations), Mathematics 405 (Partial Differential Equations), Mathematics 435 (Introduction to Functional Analysis). These PDF documents are more valuable than many textbooks because I found that majority students in my classes use them (rather than the textbooks) to study, to review, to solve homework assignments, and to prepare for exams.

THREE: SERVICES

A - Ph.D Students:

(1) Mashael Alshammari received a Ph.D in mathematics in 2024. The title of the thesis: Properties of global solutions of a two-dimensional incompressible dissipative quasi-geostrophic equation.

(2) Hang Zhao received a Ph.D in mathematics in 2024. The title of the thesis: Some aspects in partial differential equations: exact limits and diffusive limits. (Professor Lei Wu was a co-advisor)

B - Transferring Credits for Many Mathematics Classes

I have been transferring credits for many mathematics courses (except Probability and Statistics). This is absolutely a heavy duty job - on average I receive about 10 - 20 messages everyday in March, April, May, June, each year. I have to review, to visit certain websites whenever necessary, to comment on, to answer, to approve various requests. Everyday, I have to read large amounts of information online to finish the job.

C - Seminar in Applied Mathematics I was the main organizer of the seminar in 2023.

D - Coordinators of Mathematics 23 and Mathematics 205:

(1) Coordinator of Math 23 in Spring 2023.

(2) Coordinator of Math 205 in Spring 2024.

(3) Coordinator of Math 205 in Autumn 2025.

E - Writing, Proctoring and Grading Qualifying Exam in Differential Equations

(1) Committee Chair (writing, proctoring and grading) of the Qualifying Exam in Differential Equations in January 2024.

(2) Qualifying Exam in Differential Equations in 2025 - 2026.

F - Committee Chair of Hiring:

Committee Chair for interviewing a special candidate (Zhimeng Ouyang) in January 2025.

G - Miscellaneous Things Done During 2025 Topology Hiring:

(1) Picked up a candidate from Bethlehem Hotel.

(2) Brought a candidate to lunch.

(3) Did many other miscellaneous things related to this hiring.

H - Advisor of Minors in Applied Mathematics I have been the advisor for many years before Megan Cream took it over.

Summary of research done after tenure promotion in 2008:

* By coupling together existing ideas and methods (including the Fourier transformation, the Parseval's identity, Lebesgue's dominated convergence theorem and Gagliardo-Nirenberg's interpolation inequality) and results (the existence of the global weak solutions) and new ideas (appropriate decomposition of spatial and temporal spaces), I have invented a new method to accomplish the exact limits of the global weak solutions of some nonlinear systems of fluid dynamics equations, including the n-dimensional incompressible Navier-Stokes equations and the n-dimensional magnetohydrodynamics equations.

* I discovered a few special structures in a two-dimensional nonlinear singular system of differential equations arising from geostrophics. I made complete use of the special structures to accomplish the existence and uniqueness of the global smooth solution. The existence and uniqueness of the global smooth solution have been open for at least half a century.

* To establish the stability of multiple traveling pulse solutions of nonlinear singularly perturbed systems of integral differential equations arising from synaptically coupled neuronal networks, to accomplish the stability of multiple traveling pulse solutions of nonlinear singularly perturbed systems of reaction diffusion equations arising from mathematical neuroscience, I have constructed Evans functions for the pulse solutions and have built valuable relationships between the Evans function of a traveling wave front, the Evans function of a traveling wave back and the Evans function of the multiple pulse solution. Moreover, I have established new properties of the Evans functions by

coupling together a global strong maximum principle for harmonic functions defined in unbounded open simply connected domains, Hopf lemma and many other important ideas and I have given applications of the new property.

* By coupling together the Fourier transformation, the Plancherel's identity, some elementary uniform energy estimate and a very general Gronwall's inequality, I have generated a very different method to improve the Fourier splitting method to simplify the proof of the decay estimates with sharp rates of the global weak solutions of the n-dimensional incompressible Navier-Stokes equations and the n-dimensional magnetohydrodynamics equations.

The Research Statement
Past Research Results

Since I came to Lehigh University as a tenure track assistant professor in August 2002, I have been doing research in three directions in applied mathematics.

1 Properties of the Global Weak Solutions of the Cauchy Problems for the n-Dimensional Incompressible Navier-Stokes Equations, the n-Dimensional Magnetohydrodynamics Equations and a Two-Dimensional Nonlinear Singular System of Differential Equations

The first research direction is about the properties of the global weak solutions of the Cauchy problems for the n-dimensional incompressible Navier-Stokes equations and for the n-dimensional magnetohydrodynamics equations. In particular, I study decay estimates with sharp rates, stability estimates and exact limits of the global weak solutions. Moreover, I also study the properties of the global solutions of other nonlinear systems of fluid dynamics equations (such as a two-dimensional nonlinear singular system of differential equations arising from geostrophics).

1.1 Project 1: The Exact Limits of the Global Weak Solutions of the Cauchy Problems for the n-Dimensional Incompressible Navier-Stokes Equations and the n-Dimensional Magneto-hydrodynamics Equations

Consider the Cauchy problem for the n-dimensional incompressible Navier-Stokes equations

$$\frac{\partial}{\partial t}\mathbf{u} - \alpha\triangle\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f}(\mathbf{x},t), \quad \nabla\cdot\mathbf{u} = 0, \nabla\cdot\mathbf{f} = 0,$$
$$\mathbf{u}(\mathbf{x},0) = \mathbf{u}_0(\mathbf{x}), \quad \nabla\cdot\mathbf{u}_0 = 0.$$

Let the initial function $\mathbf{u}_0 \in L^1(\mathbb{R}^n)\cap L^2(\mathbb{R}^n)$ and the external force $\mathbf{f} \in L^1(\mathbb{R}^n\times\mathbb{R}^+)\cap L^1(\mathbb{R}^+, L^2(\mathbb{R}^n))$, $\mathbf{f} \in L^1(\mathbb{R}^n)$, for each fixed $t > 0$. Suppose that there exist real scalar functions $\phi_{kl} \in C^1(\mathbb{R}^n)\cap L^1(\mathbb{R}^n)$ and $\psi_{kl} \in C^1(\mathbb{R}^n\times\mathbb{R}^+)\cap L^1(\mathbb{R}^n\times\mathbb{R}^+)$, such that

$$\mathbf{u}_0(\mathbf{x}) = \left(\sum_{l=1}^{n}\frac{\partial}{\partial x_l}\phi_{1l}(\mathbf{x}), \sum_{l=1}^{n}\frac{\partial}{\partial x_l}\phi_{2l}(\mathbf{x}), \cdots, \sum_{l=1}^{n}\frac{\partial}{\partial x_l}\phi_{nl}(\mathbf{x})\right),$$
$$\mathbf{f}(\mathbf{x},t) = \left(\sum_{l=1}^{n}\frac{\partial}{\partial x_l}\psi_{1l}(\mathbf{x},t), \sum_{l=1}^{n}\frac{\partial}{\partial x_l}\psi_{2l}(\mathbf{x},t), \cdots, \sum_{l=1}^{n}\frac{\partial}{\partial x_l}\psi_{nl}(\mathbf{x},t)\right).$$

These assumptions are not unusual. They are motivated by the following structures of the nonlinear functions $(\mathbf{u}\cdot\nabla)\mathbf{u}$ and ∇p. Note that

$$(\mathbf{u}\cdot\nabla)\mathbf{u} = \left(\sum_{l=1}^{n}\frac{\partial}{\partial x_l}(u_k u_l)\right),$$

and

$$\nabla p(\mathbf{x},t) = \nabla(-\triangle)^{-1}\sum_{k=1}^{n}\sum_{l=1}^{n}\frac{\partial^2}{\partial x_k\partial x_l}[u_k(\mathbf{x},t)u_l(\mathbf{x},t)].$$

They are also motivated by the facts that the integrals of the initial function and the integral of the external force are equal to zero, that is

$$\int_{\mathbb{R}^n} \mathbf{u}_0(\mathbf{x})\mathrm{d}\mathbf{x} = \mathbf{0}, \qquad \int_{\mathbb{R}^n} \mathbf{f}(\mathbf{x},t)\mathrm{d}\mathbf{x} = \mathbf{0},$$

for all $t > 0$. Note that the initial function and the external force are divergence free. Upon performing the Fourier transformation to the equations $\nabla \cdot \mathbf{u}_0 = 0$ and $\nabla \cdot \mathbf{f} = 0$, we get

$$\xi \cdot \widehat{\mathbf{u}}_0(\xi) = 0, \qquad \xi \cdot \widehat{\mathbf{f}}(\xi,t) = 0,$$

for all $\xi \in \mathbb{R}^n$. Let $\xi = \varepsilon\widehat{\mathbf{u}}_0(\mathbf{0})$, where $0 < \varepsilon \ll 1$ is a sufficiently small positive constant. Now it is easy to see why the integrals are equal to zero.

Suppose that there exist the following integrals

$$\int_0^\infty (1+t)^{2+n/2} \int_{\mathbb{R}^n} |\mathbf{f}(\mathbf{x},t)|^2 \mathrm{d}\mathbf{x}\mathrm{d}t < \infty,$$
$$\int_0^\infty (1+t)^{2m+2+n/2} \int_{\mathbb{R}^n} |\triangle^m \mathbf{f}(\mathbf{x},t)|^2 \mathrm{d}\mathbf{x}\mathrm{d}t < \infty.$$

Theorem 1. Let $\mathbf{u} = \mathbf{u}(\mathbf{x},t)$ be the global weak solution of the n-dimensional incompressible Navier-Stokes equations corresponding to $\mathbf{u}_0 = \mathbf{u}_0(\mathbf{x})$ and $\mathbf{f} = \mathbf{f}(\mathbf{x},t)$. Then there hold the following exact limits

$$\begin{aligned}
&\lim_{t\to\infty}\left\{(1+t)^{1+n/2}\int_{\mathbb{R}^n}|\mathbf{u}(\mathbf{x},t)|^2\mathrm{d}\mathbf{x}\right\}\\
=\ &\frac{1}{(2\pi)^n}\int_{\mathbb{R}^n}|\eta|^2\exp(-2\alpha|\eta|^2)\mathrm{d}\eta\\
\cdot\ &\left\{\sum_{k=1}^n\sum_{l=1}^n\left[\int_{\mathbb{R}^n}\phi_{kl}(\mathbf{x})\mathrm{d}\mathbf{x}+\int_0^\infty\int_{\mathbb{R}^n}\psi_{kl}(\mathbf{x},t)-u_k(\mathbf{x},t)u_l(\mathbf{x},t)\mathrm{d}\mathbf{x}\mathrm{d}t\right]^2\right\},\\
&\lim_{t\to\infty}\left\{(1+t)^{2+n/2}\int_{\mathbb{R}^n}|\nabla\mathbf{u}(\mathbf{x},t)|^2\mathrm{d}\mathbf{x}\right\}\\
=\ &\frac{1}{(2\pi)^n}\int_{\mathbb{R}^n}|\eta|^4\exp(-2\alpha|\eta|^2)\mathrm{d}\eta\\
\cdot\ &\left\{\sum_{k=1}^n\sum_{l=1}^n\left[\int_{\mathbb{R}^n}\phi_{kl}(\mathbf{x})\mathrm{d}\mathbf{x}+\int_0^\infty\int_{\mathbb{R}^n}\psi_{kl}(\mathbf{x},t)-u_k(\mathbf{x},t)u_l(\mathbf{x},t)\mathrm{d}\mathbf{x}\mathrm{d}t\right]^2\right\},\\
&\lim_{t\to\infty}\left\{(1+t)^{2m+1+n/2}\int_{\mathbb{R}^n}|\triangle^m\mathbf{u}(\mathbf{x},t)|^2\mathrm{d}\mathbf{x}\right\}\\
=\ &\frac{1}{(2\pi)^n}\int_{\mathbb{R}^n}|\eta|^{4m+2}\exp(-2\alpha|\eta|^2)\mathrm{d}\eta\\
\cdot\ &\left\{\sum_{k=1}^n\sum_{l=1}^n\left[\int_{\mathbb{R}^n}\phi_{kl}(\mathbf{x})\mathrm{d}\mathbf{x}+\int_0^\infty\int_{\mathbb{R}^n}\psi_{kl}(\mathbf{x},t)-u_k(\mathbf{x},t)u_l(\mathbf{x},t)\mathrm{d}\mathbf{x}\mathrm{d}t\right]^2\right\},\\
&\lim_{t\to\infty}\left\{(1+t)^{2m+2+n/2}\int_{\mathbb{R}^n}|\nabla\triangle^m\mathbf{u}(\mathbf{x},t)|^2\mathrm{d}\mathbf{x}\right\}\\
=\ &\frac{1}{(2\pi)^n}\int_{\mathbb{R}^n}|\eta|^{4m+4}\exp(-2\alpha|\eta|^2)\mathrm{d}\eta\\
\cdot\ &\left\{\sum_{k=1}^n\sum_{l=1}^n\left[\int_{\mathbb{R}^n}\phi_{kl}(\mathbf{x})\mathrm{d}\mathbf{x}+\int_0^\infty\int_{\mathbb{R}^n}\psi_{kl}(\mathbf{x},t)-u_k(\mathbf{x},t)u_l(\mathbf{x},t)\mathrm{d}\mathbf{x}\mathrm{d}t\right]^2\right\}.
\end{aligned}$$

Theorem 2. Let $\mathbf{u}=\mathbf{u}(\mathbf{x},t)$ be the global weak solution of the n-dimensional incompressible Navier-Stokes equations corresponding to the initial function $\mathbf{u}_0 = \mathbf{u}(\mathbf{x})$ and the external force $\mathbf{f} = \mathbf{f}(\mathbf{x}, t)$. Let $\mathbf{v} = \mathbf{v}(\mathbf{x}, t)$ be the global solution of the Cauchy problem for the heat equation corresponding to the same initial function $\mathbf{u}_0 = \mathbf{u}_0(\mathbf{x})$ and external force $\mathbf{f} = \mathbf{f}(\mathbf{x}, t)$

$$\frac{\partial}{\partial t}\mathbf{v} - \alpha\triangle\mathbf{v} = \mathbf{f}(\mathbf{x}, t), \quad \nabla \cdot \mathbf{v} = 0, \nabla \cdot \mathbf{f} = 0,$$
$$\mathbf{v}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x}), \quad \nabla \cdot \mathbf{u}_0 = 0.$$

Then there hold the following exact limits

$$\lim_{t\to\infty}\left\{(1+t)^{1+n/2}\int_{\mathbb{R}^n}|\mathbf{u}(\mathbf{x},t)-\mathbf{v}(\mathbf{x},t)|^2\mathrm{d}\mathbf{x}\right\}$$
$$=\frac{1}{(2\pi)^n}\int_{\mathbb{R}^n}|\eta|^2\exp(-2\alpha|\eta|^2)\mathrm{d}\eta$$
$$\cdot\left\{\sum_{k=1}^{n}\sum_{l=1}^{n}\left[\int_0^\infty\int_{\mathbb{R}^n}u_k(\mathbf{x},t)u_l(\mathbf{x},t)\mathrm{d}\mathbf{x}\mathrm{d}t\right]^2\right\},$$
$$\lim_{t\to\infty}\left\{(1+t)^{2+n/2}\int_{\mathbb{R}^n}|\nabla[\mathbf{u}(\mathbf{x},t)-\mathbf{v}(\mathbf{x},t)]|^2\mathrm{d}\mathbf{x}\right\}$$
$$=\frac{1}{(2\pi)^n}\int_{\mathbb{R}^n}|\eta|^4\exp(-2\alpha|\eta|^2)\mathrm{d}\eta$$
$$\cdot\left\{\sum_{k=1}^{n}\sum_{l=1}^{n}\left[\int_0^\infty\int_{\mathbb{R}^n}u_k(\mathbf{x},t)u_l(\mathbf{x},t)\mathrm{d}\mathbf{x}\mathrm{d}t\right]^2\right\},$$
$$\lim_{t\to\infty}\left\{(1+t)^{2m+1+n/2}\int_{\mathbb{R}^n}|\triangle^m[\mathbf{u}(\mathbf{x},t)-\mathbf{v}(\mathbf{x},t)]|^2\mathrm{d}\mathbf{x}\right\}$$
$$=\frac{1}{(2\pi)^n}\int_{\mathbb{R}^n}|\eta|^{4m+2}\exp(-2\alpha|\eta|^2)\mathrm{d}\eta$$
$$\cdot\left\{\sum_{k=1}^{n}\sum_{l=1}^{n}\left[\int_0^\infty\int_{\mathbb{R}^n}u_k(\mathbf{x},t)u_l(\mathbf{x},t)\mathrm{d}\mathbf{x}\mathrm{d}t\right]^2\right\},$$
$$\lim_{t\to\infty}\left\{(1+t)^{2m+2+n/2}\int_{\mathbb{R}^n}|\nabla\triangle^m[\mathbf{u}(\mathbf{x},t)-\mathbf{v}(\mathbf{x},t)]|^2\mathrm{d}\mathbf{x}\right\}$$
$$=\frac{1}{(2\pi)^n}\int_{\mathbb{R}^n}|\eta|^{4m+4}\exp(-2\alpha|\eta|^2)\mathrm{d}\eta$$
$$\cdot\left\{\sum_{k=1}^{n}\sum_{l=1}^{n}\left[\int_0^\infty\int_{\mathbb{R}^n}u_k(\mathbf{x},t)u_l(\mathbf{x},t)\mathrm{d}\mathbf{x}\mathrm{d}t\right]^2\right\}.$$

Theorem 3. Let $\mathbf{u}_k=\mathbf{u}_k(\mathbf{x},t)$ be the global weak solution of the n-dimensional incompressible Navier-Stokes equations corresponding to the initial function $\mathbf{u}_{0k} = \mathbf{u}_{0k}(\mathbf{x})$ and the external force $\mathbf{f}_k = \mathbf{f}_k(\mathbf{x}, t)$, where $k = 1, 2$. Then there hold the following exact limits

$$\begin{aligned}
& \lim_{t\to\infty}\left\{(1+t)^{1+n/2}\int_{\mathbb{R}^n}|\mathbf{u}_2(\mathbf{x},t)-\mathbf{u}_1(\mathbf{x},t)|^2\mathrm{d}\mathbf{x}\right\} \\
= & \frac{1}{(2\pi)^n}\int_{\mathbb{R}^n}|\eta|^2\exp(-2\alpha|\eta|^2)\mathrm{d}\eta \\
\cdot & \left\{\sum_{k=1}^n\sum_{l=1}^n\left[\int_{\mathbb{R}^n}\phi_{2kl}(\mathbf{x})-\phi_{1kl}(\mathbf{x})\mathrm{d}\mathbf{x}+\int_0^\infty\int_{\mathbb{R}^n}\psi_{2kl}(\mathbf{x},t)-\psi_{1kl}(\mathbf{x},t)\mathrm{d}\mathbf{x}\mathrm{d}\right.\right. \\
& \lim_{t\to\infty}\left\{(1+t)^{2+n/2}\int_{\mathbb{R}^n}|\nabla[\mathbf{u}_2(\mathbf{x},t)-\mathbf{u}_1(\mathbf{x},t)]|^2\mathrm{d}\mathbf{x}\right\} \\
= & \frac{1}{(2\pi)^n}\int_{\mathbb{R}^n}|\eta|^4\exp(-2\alpha|\eta|^2)\mathrm{d}\eta \\
\cdot & \left\{\sum_{k=1}^n\sum_{l=1}^n\left[\int_{\mathbb{R}^n}\phi_{2kl}(\mathbf{x})-\phi_{1kl}(\mathbf{x})\mathrm{d}\mathbf{x}+\int_0^\infty\int_{\mathbb{R}^n}\psi_{2kl}(\mathbf{x},t)-\psi_{1kl}(\mathbf{x},t)\mathrm{d}\mathbf{x}\mathrm{d}\right.\right. \\
& \lim_{t\to\infty}\left\{(1+t)^{2m+1+n/2}\int_{\mathbb{R}^n}|\triangle^m[\mathbf{u}_2(\mathbf{x},t)-\mathbf{u}_1(\mathbf{x},t)]|^2\mathrm{d}\mathbf{x}\right\} \\
= & \frac{1}{(2\pi)^n}\int_{\mathbb{R}^n}|\eta|^{4m+2}\exp(-2\alpha|\eta|^2)\mathrm{d}\eta \\
\cdot & \left\{\sum_{k=1}^n\sum_{l=1}^n\left[\int_{\mathbb{R}^n}\phi_{2kl}(\mathbf{x})-\phi_{1kl}(\mathbf{x})\mathrm{d}\mathbf{x}+\int_0^\infty\int_{\mathbb{R}^n}\psi_{2kl}(\mathbf{x},t)-\psi_{1kl}(\mathbf{x},t)\mathrm{d}\mathbf{x}\mathrm{d}\right.\right. \\
& \lim_{t\to\infty}\left\{(1+t)^{2m+2+n/2}\int_{\mathbb{R}^n}|\nabla\triangle^m[\mathbf{u}_2(\mathbf{x},t)-\mathbf{u}_1(\mathbf{x},t)]|^2\mathrm{d}\mathbf{x}\right\} \\
= & \frac{1}{(2\pi)^n}\int_{\mathbb{R}^n}|\eta|^{4m+4}\exp(-2\alpha|\eta|^2)\mathrm{d}\eta \\
\cdot & \left\{\sum_{k=1}^n\sum_{l=1}^n\left[\int_{\mathbb{R}^n}\phi_{2kl}(\mathbf{x})-\phi_{1kl}(\mathbf{x})\mathrm{d}\mathbf{x}+\int_0^\infty\int_{\mathbb{R}^n}\psi_{2kl}(\mathbf{x},t)-\psi_{1kl}(\mathbf{x},t)\mathrm{d}\mathbf{x}\mathrm{d}\right.\right.
\end{aligned}$$

Consider the Cauchy problems for the n-dimensional magnetohydrodynamics equations

$$\begin{aligned}
&\frac{\partial}{\partial t}\mathbf{u} - \frac{1}{\mathrm{RE}}\triangle\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} - (\mathbf{A}\cdot\nabla)\mathbf{A} + \nabla P = \mathbf{f}(\mathbf{x},t),\\
&\frac{\partial}{\partial t}\mathbf{A} - \frac{1}{\mathrm{RM}}\triangle\mathbf{A} + (\mathbf{u}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{u} = \mathbf{g}(\mathbf{x},t),\\
&\nabla\cdot\mathbf{u} = 0, \qquad \nabla\cdot\mathbf{f} = 0, \qquad \nabla\cdot\mathbf{A} = 0, \qquad \nabla\cdot\mathbf{g} = 0,\\
&\mathbf{u}(\mathbf{x},0) = \mathbf{u}_0(\mathbf{x}), \mathbf{A}(\mathbf{x},0) = \mathbf{A}_0(\mathbf{x}), \nabla\cdot\mathbf{u}_0 = 0, \nabla\cdot\mathbf{A}_0 = 0.
\end{aligned}$$

There hold very similar exact limits for the global weak solutions of the magnetohydrodynamics equations.

1.2 Project 2: The Existence and Uniqueness of the Global Smooth Solution of the Cauchy Problems for a Two-Dimensional Nonlinear Singular Systems of Differential Equations Arising from Geostrophics

Consider the Cauchy problems for the two-dimensional nonlinear singular system of differential equations arising from geostrophics

$$\begin{aligned}
&\frac{\partial}{\partial t}[\gamma(\psi_1 - \psi_2) - \triangle\psi_1] + \alpha(-\triangle)^\rho\psi_1 + \beta\frac{\partial\psi_1}{\partial x} + J(\psi_1, \gamma(\psi_1 - \psi_2) - \triangle\psi_1)\\
&\frac{\partial}{\partial t}[\gamma\delta(\psi_2 - \psi_1) - \triangle\psi_2] + \alpha(-\triangle)^\rho\psi_2 + \beta\frac{\partial\psi_2}{\partial x} + J(\psi_2, \gamma\delta(\psi_2 - \psi_1) - \triangle\psi\\
&\psi_1(x,y,0) = \psi_{01}(x,y), \qquad \psi_2(x,y,0) = \psi_{02}(x,y).
\end{aligned}$$

In this system, $\alpha > 0$, $\gamma > 0$, $\delta > 0$ and $\rho > 0$ are positive constants, $\beta \neq 0$ is a real nonzero constant, representing the Reynolds number. Moreover, ψ_1 and ψ_2 represent stream functions of top layer and bottom layer in convective fluids. The constants D_1 and D_2 represent the depths of the top layer and the bottom layer, respectively. Furthermore, the Jacobian determinant is defined by $J(p,q) = \frac{\partial}{\partial x}p\frac{\partial}{\partial y}q - \frac{\partial}{\partial y}p\frac{\partial}{\partial x}q$, for all continuously differentiable functions $p, q \in C^1(\mathbb{R}^2)$. Recently, my collaborators and I established the existence and uniqueness of the global smooth solution - a very difficult problem in geostrophics and applied mathematics - in the paper: Boling Guo, Yongqian Han, Daiwen Huang, Dongfen Bian and Linghai Zhang, Global smooth solution of a two-dimensional nonlinear singular system of differential equations arising from geostrophics. *Journal of Differential Equations*, **262**(2017), 3980-4020. The singularity generated by the linear parts, the strong couplings of the nonlinear functions and the fractional order of the derivatives make the existence and uniqueness very difficult, that is why it has been open for at least half a century. There are five authors in the paper and I am the corresponding author. It is the first author (Professor Boling Guo) who gave me this problem to collaborate. He is one of my advisors for my Master's degree at Beijing Institute of Applied Physics and Computational Mathematics in Beijing many years ago. My main role is to rigorously develop the mathematical analysis, write the main body of the paper, as well as the Introduction and the Concluding Remarks. By hard working and very careful observations, I found that there exists three special structures in the nonlinear singular system of equations. Here are the special structures of the nonlinear system: the system is symmetric and the terms $\gamma(\psi_1 - \psi_2) - \triangle\psi_1$ and $\gamma\delta(\psi_2 - \psi_1) - \triangle\psi_2$ appear both in the linear part and the nonlinear part. The advantage of the special structure: When making energy estimates, namely, when multiplying the first equation by $\gamma(\psi_1 - \psi_2) - \triangle\psi_1$ and the second equation by $\gamma\delta(\psi_2 - \psi_1) - \triangle\psi_2$, integrating the results with respect to (x, y) over $\mathbb{R}^2$ and then coupling the results together, the integrals with the Jacobian determinant vanish:

$$\int_{\mathbb{R}^2}[\gamma(\psi_1-\psi_2)-\triangle\psi_1]J(\psi_1,\gamma(\psi_1-\psi_2)-\triangle\psi_1)\mathrm{d}x\mathrm{d}y=0,$$
$$\int_{\mathbb{R}^2}[\gamma\delta(\psi_2-\psi_1)-\triangle\psi_2]J(\psi_2,\gamma\delta(\psi_2-\psi_1)-\triangle\psi_2)\mathrm{d}x\mathrm{d}y=0.$$

By making complete use of the special structures, by making use of an unusual energy estimate method, by coupling together traditional ideas, methods and techniques, I established the uniform energy estimates for all order derivatives of the global weak solution to accomplish the existence and uniqueness of the global smooth solution. The middle three authors (academic siblings) played minor roles, they basically established the elementary energy estimates. They checked all the details in the mathematical analysis of my part and corrected a non-substantial mistake. There are some influence of the theoretical results in astrophysics, atmospheric science, physical oceanography and in applied mathematics. Currently, my collaborators and I are solving other open problems of this system.

1.3 Project 3: The $L^\lambda - L^\mu$ Regularity of the Global Weak Solutions of the n-Dimensional Incompressible Navier-Stokes Equations and the n-Dimensional Magnetohydrodynamics Equations

Consider the Cauchy problem for the following n-dimensional incompressible Navier-Stokes equations

$$\frac{\partial}{\partial t}\mathbf{u}-\triangle\mathbf{u}+(\mathbf{u}\cdot\nabla)\mathbf{u}+\nabla p=0,\quad \nabla\cdot\mathbf{u}=0,$$
$$\mathbf{u}(\mathbf{x},0)=\mathbf{u}_0(\mathbf{x}),\quad \nabla\cdot\mathbf{u_0}=0.$$

It is well known that if the initial function $\mathbf{u}_0\in L^1(\mathbb{R}^n)\cap L^2(\mathbb{R}^n)$, then there exists a global weak solution to the Cauchy problem:

$\mathbf{u} \in L^\infty(\mathbb{R}^+, L^2(\mathbb{R}^n)) \cap L^2(\mathbb{R}^+, H^1(\mathbb{R}^n))$. By making complete use of the decay estimates with sharp rates, I have accomplished the following results for the global weak solution of the Cauchy problem for the n-dimensional incompressible Navier-Stokes equations.

Theorem . Let the initial function $\mathbf{u}_0 \in L^1(\mathbb{R}^n) \cap H^2(\mathbb{R}^n)$. Suppose that there holds the following condition for the global weak solution

$$\mathbf{u} \in L^\mu(\mathbb{R}^+; L^\lambda(\mathbb{R}^n)),$$

for some positive constants $\lambda > n \geq 3$ and $\mu > 2$, $\frac{n}{\lambda} + \frac{2}{\mu} = 1$, then

$$\mathbf{u} \in \left[\bigcap_{\lambda \leq r < \infty} L^{2r/(r-n)}(\mathbb{R}^+; L^r(\mathbb{R}^n)) \right] \bigcap \left[\bigcap_{2 \leq s < \infty} L^\infty(\mathbb{R}^+; L^s(\mathbb{R}^n)) \right].$$

This implies that $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$ is a global smooth solution. There holds the following solution representation

$$\begin{aligned}
\mathbf{u}(\mathbf{x}, t) &= \frac{1}{(4\pi\alpha t)^{n/2}} \int_{\mathbb{R}^n} \exp\left[-\frac{|\mathbf{x}-\mathbf{y}|^2}{4\alpha t} \right] \mathbf{u}_0(\mathbf{y}) \mathrm{d}\mathbf{y} \\
&+ \int_0^t \frac{1}{[4\pi\alpha(t-\tau)]^{n/2}} \int_{\mathbb{R}^n} \exp\left[-\frac{|\mathbf{x}-\mathbf{y}|^2}{4\alpha(t-\tau)} \right] \mathbf{f}(\mathbf{y}, \tau) \mathrm{d}\mathbf{y} \mathrm{d}\tau \\
&- \int_0^t \frac{1}{[4\pi\alpha(t-\tau)]^{n/2}} \int_{\mathbb{R}^n} \exp\left[-\frac{|\mathbf{x}-\mathbf{y}|^2}{4\alpha(t-\tau)} \right] (\mathbf{u} \cdot \nabla)\mathbf{u}(\mathbf{y}, \tau) \mathrm{d}\mathbf{y} \mathrm{d}\tau \\
&- \int_0^t \frac{1}{[4\pi\alpha(t-\tau)]^{n/2}} \int_{\mathbb{R}^n} \exp\left[-\frac{|\mathbf{x}-\mathbf{y}|^2}{4\alpha(t-\tau)} \right] \nabla p(\mathbf{y}, t) \mathrm{d}\mathbf{y} \mathrm{d}\tau.
\end{aligned}$$

Consider the n-dimensional magnetohydrodynamics equations

$$\begin{aligned}
&\frac{\partial}{\partial t}\mathbf{u} - \frac{1}{\mathrm{RE}} \triangle \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} - (\mathbf{A} \cdot \nabla)\mathbf{A} + \nabla P = \mathbf{f}(\mathbf{x}, t), \\
&\frac{\partial}{\partial t}\mathbf{A} - \frac{1}{\mathrm{RM}} \triangle \mathbf{A} + (\mathbf{u} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{u} = \mathbf{g}(\mathbf{x}, t), \\
&\nabla \cdot \mathbf{u} = 0, \qquad \nabla \cdot \mathbf{f} = 0, \qquad \nabla \cdot \mathbf{A} = 0, \qquad \nabla \cdot \mathbf{g} = 0, \\
&\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x}), \mathbf{A}(\mathbf{x}, 0) = \mathbf{A}_0(\mathbf{x}), \nabla \cdot \mathbf{u}_0 = 0, \nabla \cdot \mathbf{A}_0 = 0.
\end{aligned}$$

There holds very similar result for the global weak solution.

1.4 Project 4: The Decay Estimates with Sharp Rates of the Global Weak Solutions of the Cauchy Problems for the n-Dimensional Incompressible Navier-Stokes Equations and the n-Dimensional Magnetohydrodynamics Equations

Consider the Cauchy problem for the n-dimensional incompressible Navier-Stokes equations

$$\frac{\partial}{\partial t}\mathbf{u} - \alpha\triangle\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f}(\mathbf{x},t), \quad \nabla\cdot\mathbf{u} = 0, \nabla\cdot\mathbf{f} = 0,$$
$$\mathbf{u}(\mathbf{x},0) = \mathbf{u}_0(\mathbf{x}), \quad \nabla\cdot\mathbf{u}_0 = 0.$$

Suppose that the initial function $\mathbf{u}_0 \in L^2(\mathbb{R}^n)$ and the external force $\mathbf{f} \in L^1(\mathbb{R}^+, L^2(\mathbb{R}^n))$. Then there exists a global weak solution to the Cauchy problem: $\mathbf{u} \in L^\infty(\mathbb{R}^+, L^2(\mathbb{R}^n))$, such that $\nabla\mathbf{u} \in L^2(\mathbb{R}^+, L^2(\mathbb{R}^n))$. This is well known.

Theorem . Suppose that there exist real scalar functions $\phi_{kl} \in C^1(\mathbb{R}^n) \cap L^1(\mathbb{R}^n)$ and $\psi_{kl} \in C^1(\mathbb{R}^n \times \mathbb{R}^+) \cap L^1(\mathbb{R}^n \times \mathbb{R}^+)$, such that

$$\mathbf{u}_0(\mathbf{x}) = \left(\sum_{l=1}^n \frac{\partial}{\partial x_l}\phi_{1l}(\mathbf{x}), \sum_{l=1}^n \frac{\partial}{\partial x_l}\phi_{2l}(\mathbf{x}), \cdots, \sum_{l=1}^n \frac{\partial}{\partial x_l}\phi_{nl}(\mathbf{x})\right),$$
$$\mathbf{f}(\mathbf{x},t) = \left(\sum_{l=1}^n \frac{\partial}{\partial x_l}\psi_{1l}(\mathbf{x},t), \sum_{l=1}^n \frac{\partial}{\partial x_l}\psi_{2l}(\mathbf{x},t), \cdots, \sum_{l=1}^n \frac{\partial}{\partial x_l}\psi_{nl}(\mathbf{x},t)\right).$$

Suppose that there exists the following integral

$$\int_0^\infty (1+t)^{2+n/2}\int_{\mathbb{R}^n} |\mathbf{f}(\mathbf{x},t)|^2 \mathrm{d}\mathbf{x}\mathrm{d}t < \infty.$$

Then there holds the following decay estimate with sharp rate

$$(1+t)^{1+n/2}\int_{\mathbb{R}^n} |\mathbf{u}(\mathbf{x},t)\mathrm{d}\mathbf{x} \le C,$$

for all $t > 0$, where the positive constant $C > 0$ is independent of $\mathbf{u}$ and $(\mathbf{x}, t)$.

The decay estimate with sharp rate of the global weak solution has been proved before. My main contribution is to simplify the main ideas and steps in the mathematical analysis. By coupling together the Fourier transformation, the Parseval's identity and a general Gronwall's inequality, I simplified and improved the Fourier splitting method in a recent paper: Decay estimates with sharp rates of global solutions of nonlinear systems of fluid dynamics equations. *Discrete and Continuous Dynamical Systems, Series S*, **9**(2016), 2181-2200. The new method is widely applicable and is easier to use to establish the decay estimates with sharp rates for the global weak solutions to a large class of nonlinear evolution equations with dissipation in higher-dimensional spaces. This method was developed in the middle 1980's to study the long time asymptotic behaviours of the global weak solutions of nonlinear evolution equations with dissipation, such as the n-dimensional incompressible Navier-Stokes equations and the n-dimensional magnetohydrodynamics equations. A major influence of the new method is that the exact limits of the global weak solutions may be accomplished.

Consider the magnetohydrodynamics equations

$$\begin{aligned}
&\frac{\partial}{\partial t}\mathbf{u} - \frac{1}{\mathrm{RE}}\triangle\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} - (\mathbf{A}\cdot\nabla)\mathbf{A} + \nabla P = \mathbf{f}(\mathbf{x}, t),\\
&\frac{\partial}{\partial t}\mathbf{A} - \frac{1}{\mathrm{RM}}\triangle\mathbf{A} + (\mathbf{u}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{u} = \mathbf{g}(\mathbf{x}, t),\\
&\nabla\cdot\mathbf{u} = 0, \qquad \nabla\cdot\mathbf{f} = 0, \qquad \nabla\cdot\mathbf{A} = 0, \qquad \nabla\cdot\mathbf{g} = 0,\\
&\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x}), \mathbf{A}(\mathbf{x}, 0) = \mathbf{A}_0(\mathbf{x}), \nabla\cdot\mathbf{u}_0 = 0, \nabla\cdot\mathbf{A}_0 = 0.
\end{aligned}$$

There holds very similar decay estimate with sharp rate for the global weak solution.

Summary

Consider the n-dimensional incompressible Navier-Stokes equations

$$\begin{aligned} \frac{\partial}{\partial t}\mathbf{u} - \alpha\triangle\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f}(\mathbf{x},t), \quad \nabla\cdot\mathbf{u} = 0, \nabla\cdot\mathbf{f} = 0,\\ \mathbf{u}(\mathbf{x},0) = \mathbf{u}_0(\mathbf{x}), \quad \nabla\cdot\mathbf{u}_0 = 0, \end{aligned}$$

and n-dimensional magnetohydrodynamics equations

$$\begin{aligned} &\frac{\partial}{\partial t}\mathbf{u} - \frac{1}{\mathrm{RE}}\triangle\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} - (\mathbf{A}\cdot\nabla)\mathbf{A} + \nabla P = \mathbf{f}(\mathbf{x},t),\\ &\frac{\partial}{\partial t}\mathbf{A} - \frac{1}{\mathrm{RM}}\triangle\mathbf{A} + (\mathbf{u}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{u} = \mathbf{g}(\mathbf{x},t),\\ &\nabla\cdot\mathbf{u} = 0, \qquad \nabla\cdot\mathbf{f} = 0, \qquad \nabla\cdot\mathbf{A} = 0, \qquad \nabla\cdot\mathbf{g} = 0,\\ &\mathbf{u}(\mathbf{x},0) = \mathbf{u}_0(\mathbf{x}), \mathbf{A}(\mathbf{x},0) = \mathbf{A}_0(\mathbf{x}), \nabla\cdot\mathbf{u}_0 = 0, \nabla\cdot\mathbf{A}_0 = 0. \end{aligned}$$

We have studied several important properties of the global weak solutions of these equations, including the exact limits of the derivatives of any order, regularity of the global weak solutions and the decay estimates with sharp rates. We made complete use of the elementary uniform energy estimates and we made good use of a general Gronwall's inequality to improve the Fourier splitting method to simplify the proof of the decay estimates with sharp rates. For the two-dimensional nonlinear singular system of differential equations arising from geostrophics, we solved a long time open problem - we accomplished the existence and uniqueness of the global smooth solution of the Cauchy problem for the system of equations.

2 Traveling Wave Solutions of Nonlinear Integral Differential Equations and Nonlinear Reaction Diffusion Equations

The second research direction is the existence and stability of traveling wave fronts of nonlinear scalar integral differential equations, the existence and stability of traveling pulse solutions of nonlinear singularly perturbed systems of integral differential equations arising from synaptically coupled neuronal networks, mathematical analysis of wave speeds of the traveling wave fronts, bifurcations of nonlinear waves of nonlinear singularly perturbed systems of reaction diffusion equations arising from mathematical neuroscience.

2.1 Project 5: Existence and Stability of Traveling Wave Fronts of Nonlinear Scalar Integral Differential Equations - Part A

First of all, consider the following nonlinear scalar integral differential equation

$$\frac{\partial u}{\partial t} + u = \alpha \int_{\mathbb{R}} K(x-y) H(u(y,t) - \theta) \mathrm{d}y,$$

and the nonlinear scalar integral differential equation with spatial temporal delay due to finite speed of action potential along axons

$$\frac{\partial u}{\partial t} + u = \alpha \int_{\mathbb{R}} K(x-y) H\left(u\left(y, t - \frac{1}{c}|x-y|\right) - \theta\right) \mathrm{d}y,$$

where $\alpha > 0$, $\beta > 0$ and $\theta > 0$ are positive constants, such that $0 < 2\theta < \alpha$. Moreover, K represents synaptic couplings between neurons, H represents the Heaviside step function. I have accomplished the existence, uniqueness and stability of a traveling wave front for each equation. In particular, I studied how the wave shape, wave speed and wave stability vary as the synaptic coupling and the model parameters change. The synaptic coupling may be of pure excitation, lateral inhibition or lateral excitation. I have introduced two very important concepts: the speed index function and the stability index function. These functions are very useful in the study of the existence and stability of the traveling wave fronts as well as mathematical analysis of the wave speeds. One very interesting point is that I can define the stability index function through the speed index function. By using this relationship, the stability of the traveling wave front can be accomplished.

For the nonlinear scalar integral differential equation with lateral inhibition or lateral excitation, the mathematical analysis of the existence and stability of the traveling wave front is very difficult and technical. The rigorous mathematical analysis had not been established before. Solid applications of these results to applied mathematics and computational neuroscience are based on the mathematical analysis.

Theorem . There exists a unique solution (a unique wave speed $\nu_0 > 0$) to the speed equation

$$\alpha \int_{-\infty}^{0} \exp\left(\frac{x}{\nu}\right) K(x)\mathrm{d}x = \frac{\alpha}{2} - \theta,$$

and there exists a unique solution (a unique wave speed $\mu_0 > 0$) to the speed equation

$$\alpha \int_{-\infty}^{0} \exp\left(\frac{c-\mu}{c\mu}\right) K(x)\mathrm{d}x = \frac{\alpha}{2} - \theta.$$

Moreover, there holds the following important relationship

$$\frac{1}{\mu_0} = \frac{1}{\nu_0} + \frac{1}{c}.$$

There exists a unique traveling wave front $U = U(z)$ to the first nonlinear scalar integral differential equation, where $z = x + \nu_0 t$, and there exists a unique traveling wave front $U = U(z)$ to the second nonlinear scalar integral differential equation, where $z = x + \mu_0 t$. Both fronts satisfy

$$U < \theta \text{ on } (-\infty, 0), U(0) = \theta, \qquad U'(0) > 0, U > \theta \text{ on } (0, \infty),$$

and the boundary conditions

$$\lim_{z\to-\infty} U(z) = 0, \qquad \lim_{z\to\infty} U(z) = \alpha, \qquad \lim_{z\to\pm\infty} U'(z) = 0.$$

The traveling wave fronts are spectrally stable.

2.2 Project 5: Existence and Stability of Traveling Wave Fronts of Nonlinear Scalar Integral Differential Equations - Part B

Consider the following nonlinear scalar integral differential equations arising from synaptically coupled neuronal networks

$$\begin{aligned}\frac{\partial u}{\partial t} + u &= \alpha \int_0^\infty \xi(c) \left[\int_{\mathbb{R}} K(x-y) H\left(u\left(y, t - \frac{1}{c}|x-y|\right) - \theta\right) \mathrm{d}y\right] \mathrm{d}c \\ &+ \beta \int_0^\infty \eta(\tau) \left[\int_{\mathbb{R}} W(x-y) H(u(y, t-\tau) - \Theta) \mathrm{d}y\right] \mathrm{d}\tau,\end{aligned}$$

where $\alpha > 0$, $\beta > 0$, $\theta > 0$ and $\Theta > 0$ are positive constants, K and W represent synaptic couplings between neurons, ξ and η represent probability density functions, H represents the Heaviside step function. We define the sign function $s = s(x)$ by: $s(x) = -1$ for all $x < 0$, $s(0) = 0$ and $s(x) = 1$ for all $x > 0$.

Theorem . There exist exactly three traveling wave fronts to the nonlinear scalar integral differential equation.
(I) Let $0 < \theta < \Theta$. There exists a unique solution (μ_1, Z_1) to the

following system of speed equations

$$\begin{aligned}
&\alpha \int_0^\infty \xi(c) \left[\int_{-\infty}^0 \exp\left(\frac{c-\mu}{c\mu}x\right) K(x)\mathrm{d}x\right] \mathrm{d}c \\
+\ &\beta \int_0^\infty \eta(\tau) \left[\int_{-\mu\tau-Z}^0 W(x)\mathrm{d}x\right] \mathrm{d}\tau \\
+\ &\beta \exp\left(\frac{Z}{\mu}\right) \int_0^\infty \eta(\tau)e^\tau \left[\int_{-\infty}^{-\mu\tau-Z} \exp\left(\frac{x}{\mu}\right) W(x)\mathrm{d}x\right] \mathrm{d}\tau = \frac{\alpha+\beta}{2} - \theta,
\end{aligned}$$

and

$$\begin{aligned}
&\alpha \int_0^\infty \xi(c) \left[\int_{-\infty}^0 \exp\left(\frac{x}{\mu}\right) \frac{c}{c+s(x+Z)\mu} K\left(\frac{c(x+Z)}{c+s(x+Z)\mu}\right) \mathrm{d}x\right] \mathrm{d}c \\
-\ &\alpha \int_0^\infty \xi(c) \left[\int_0^{cZ/(c+s(Z)\mu)} K(x)\mathrm{d}x\right] \mathrm{d}c \\
+\ &\beta \int_0^\infty \eta(\tau) \left[\int_{-\mu\tau}^0 W(x)\mathrm{d}x\right] \mathrm{d}\tau \\
+\ &\beta \int_0^\infty \eta(\tau)e^\tau \left[\int_{-\infty}^{-\mu\tau} \exp\left(\frac{x}{\mu}\right) W(x)\mathrm{d}x\right] \mathrm{d}\tau = \frac{\alpha+\beta}{2} - \Theta.
\end{aligned}$$

Let $\theta = \Theta$. There exists a unique solution $\mu_1 > 0$ to the following speed equation

$$\begin{aligned}
&\alpha \int_0^\infty \xi(c) \left[\int_{-\infty}^0 \exp\left(\frac{c-\mu}{c\mu}x\right) K(x)\mathrm{d}x\right] \mathrm{d}c \\
+\ &\beta \int_0^\infty \eta(\tau) \left[\int_{-\mu\tau}^0 W(x)\mathrm{d}x\right] \mathrm{d}\tau \\
+\ &\beta \int_0^\infty \eta(\tau)e^\tau \left[\int_{-\infty}^{-\mu\tau} \exp\left(\frac{x}{\mu}\right) W(x)\mathrm{d}x\right] \mathrm{d}\tau = \frac{\alpha+\beta}{2} - \theta.
\end{aligned}$$

The first traveling wave front $U = U_{\text{front}-1}(z)$ is given by

$$\begin{aligned}
U(z) &= \alpha \int_0^\infty \xi(c) \left[\int_{-\infty}^{cz/(c+s(z)\mu_1)} K(x)\mathrm{d}x \right] \mathrm{d}c \\
&- \alpha \int_0^\infty \xi(c) \left[\int_{-\infty}^{z} \exp\left(\frac{x-z}{\mu_1}\right) \frac{c}{c+s(x)\mu_1} K\left(\frac{cx}{c+s(x)\mu_1}\right) \mathrm{d}x \right] \mathrm{d}c \\
&+ \beta \int_0^\infty \eta(\tau) \left[\int_{-\infty}^{z-\mu_1\tau-Z_1} W(x)\mathrm{d}x \right] \mathrm{d}\tau \\
&- \beta \exp\left(\frac{Z_1}{\mu_1}\right) \int_0^\infty \eta(\tau)e^{\tau} \left[\int_{-\infty}^{z-\mu_1\tau-Z_1} \exp\left(\frac{x-z}{\mu_1}\right) W(x)\mathrm{d}x \right] \mathrm{d}\tau, \\
U'(z) &= \frac{\alpha}{\mu_1} \int_0^\infty \xi(c) \left[\int_{-\infty}^{z} \exp\left(\frac{x-z}{\mu_1}\right) \frac{c}{c+s(x)\mu_1} K\left(\frac{cx}{c+s(x)\mu_1}\right) \mathrm{d}x \right] \mathrm{d}c \\
&+ \frac{\beta}{\mu_1} \exp\left(\frac{Z_1}{\mu_1}\right) \int_0^\infty \eta(\tau)e^{\tau} \left[\int_{-\infty}^{z-\mu_1\tau-Z_1} \exp\left(\frac{x-z}{\mu_1}\right) W(x)\mathrm{d}x \right] \mathrm{d}\tau, \\
&\lim_{z\to-\infty} U(z) = 0, \qquad \lim_{z\to\infty} U(z) = \alpha+\beta, \qquad \lim_{z\to\pm\infty} U'(z) = 0.
\end{aligned}$$

(II) There exists a unique solution $\mu_2 > 0$ to the following speed equation

$$\alpha \int_0^\infty \xi(c) \left[\int_{-\infty}^{0} \exp\left(\frac{c-\mu}{c\mu}x\right) K(x)\mathrm{d}x \right] \mathrm{d}c = \frac{\alpha}{2} - \theta.$$

The second traveling wave front $U = U_{\text{front}-2}(z)$ is given by

$$\begin{aligned}
U(z) &= \alpha \int_0^\infty \xi(c) \left[\int_{-\infty}^{cz/(c+s(z)\mu_2)} K(x)\mathrm{d}x \right] \mathrm{d}c \\
&- \alpha \int_0^\infty \xi(c) \left[\int_{-\infty}^{z} \exp\left(\frac{x-z}{\mu_2}\right) \frac{c}{c+s(x)\mu_2} K\left(\frac{cx}{c+s(x)\mu_2}\right) \mathrm{d}x \right] \mathrm{d}c \\
U'(z) &= \frac{\alpha}{\mu_2} \int_0^\infty \xi(c) \left[\int_{-\infty}^{z} \exp\left(\frac{x-z}{\mu_2}\right) \frac{c}{c+s(x)\mu_2} K\left(\frac{cx}{c+s(x)\mu_2}\right) \mathrm{d}x \right] \mathrm{d} \\
&\lim_{z\to-\infty} U(z) = 0, \qquad \lim_{z\to\infty} U(z) = \alpha, \qquad \lim_{z\to\pm\infty} U'(z) = 0.
\end{aligned}$$

(III) There exists a unique solution $\mu_3 > 0$ to the following speed equation

$$\begin{aligned} & \beta \int_0^\infty \eta(\tau) \left[\int_{-\mu\tau}^0 W(x) \mathrm{d}x \right] \mathrm{d}\tau \\ + & \beta \int_0^\infty \eta(\tau) e^\tau \left[\int_{-\infty}^{-\mu\tau} \exp\left(\frac{x}{\mu}\right) W(x) \mathrm{d}x \right] \mathrm{d}\tau = \alpha + \frac{\beta}{2} - \Theta. \end{aligned}$$

The third traveling wave front $U = U_{\text{front}-3}(z)$ is given by

$$\begin{aligned} U(z) &= \alpha + \beta \int_0^\infty \eta(\tau) \left[\int_{-\infty}^{z-\mu_3\tau} W(x) \mathrm{d}x \right] \mathrm{d}\tau \\ & - \beta \int_0^\infty \eta(\tau) e^\tau \left[\int_{-\infty}^{z-\mu_3\tau} \exp\left(\frac{x-z}{\mu_3}\right) W(x) \mathrm{d}x \right] \mathrm{d}\tau, \\ U'(z) &= \frac{\beta}{\mu_3} \int_0^\infty \eta(\tau) e^\tau \left[\int_{-\infty}^{z-\mu_3\tau} \exp\left(\frac{x-z}{\mu_3}\right) W(x) \mathrm{d}x \right] \mathrm{d}\tau, \\ & \lim_{z\to-\infty} U(z) = \alpha, \qquad \lim_{z\to\infty} U(z) = \alpha + \beta, \qquad \lim_{z\to\pm\infty} U'(z) = 0. \end{aligned}$$

2.3 Project 6: Stability of a Traveling Pulse Solution of a Nonlinear Singularly Perturbed System of Integral Differential Equations - Part A

First of all, consider the following nonlinear singularly perturbed system of integral differential equations

$$\begin{aligned} \frac{\partial u}{\partial t} + u + w &= \alpha \int_{\mathbb{R}} K(x-y) H(u(y,t) - \theta) \mathrm{d}y, \\ \frac{\partial w}{\partial t} &= \varepsilon(u - \gamma w), \end{aligned}$$

where $\alpha > 0$, $\gamma > 0$, $\varepsilon > 0$ and $\theta > 0$ are positive constants, such that $0 < 2\theta < \alpha$, $0 < \alpha\gamma < (1+\gamma)\theta$ and $0 < \varepsilon \ll 1$. The function $u = u(x,t)$ represents the membrane potential of a neuron at position x and time t, the function $w = w(x, t)$ represents the leaking current. The positive constant $\alpha > 0$ represents the synaptic rate, the positive constant $\theta > 0$ represents the threshold for excitation, the positive constant $\gamma > 0$ represents the decay rate, the positive constant $\varepsilon > 0$ represents the ratio of the activation of Na^+ ion channels over the activation of K^+ ion channels and $0 < \varepsilon \ll 1$. The kernel function K represents the synaptic coupling, $H = H(u - \theta)$ represents the Heaviside step function, which is defined by $H(u-\theta) = 0$ for all $u < \theta$, $H(0) = \frac{1}{2}$ and $H(u-\theta) = 1$ for all $u > \theta$. Moreover, $\alpha \int_{\mathbb{R}} K(x-y)H(u(y,t)-\theta)\mathrm{d}y$ represents nonlocal interactions between neurons.

Ermentrout and Pinto established the existence and uniqueness of a fast traveling pulse solution and a slow traveling pulse solution, respectively. Note that unlike traditional traveling wave problems, there exists neither conservation law nor maximum principle in this system of integral differential equations. By linearizing the nonlinear system about the fast traveling pulse solution and by using the method of separation of variables, I obtained an eigenvalue problem associated with a linear differential operator $\mathcal{L}(\varepsilon)$. It is well known that the slow traveling pulse solution is unstable due to the existence of a positive eigenvalue. In my paper [37], by using the linearized stability criterion (the equivalence of the nonlinear stability, the linear stability and the spectral stability of the traveling pulse solution), I accomplished the nonlinear stability of the fast traveling pulse solution. By coupling together several important ideas in differential equations, real analysis, complex analysis, functional analysis and linear algebra, I constructed a complex analytic function, called the Evans function, and I established important properties of the Evans function. One very important point is that a complex number λ_0 is an eigenvalue of the associated linear differential operator $\mathcal{L}(\varepsilon)$, if and only if λ_0 is a zero of the Evans function. Then I used the Evans function to study the eigenvalues of the operator $\mathcal{L}(\varepsilon)$. The rigorous mathematical analysis demonstrates that there exists no nonzero eigenvalue in the right half plane $\{\lambda \in \mathbb{C} : \mathrm{Re}\lambda \geq 0\}$ and the neutral eigenvalue $\lambda = 0$ is algebraically simple. That is a sufficient condition to establish the nonlinear stability of the fast traveling pulse solution. This is the first result on stability of traveling pulse solutions of nonlinear singularly perturbed system of integral differential equations arising from synaptically coupled neuronal networks. This paper has been cited at least forty times by other mathematicians.

Theorem . The fast traveling pulse solution $(U, W) = (U_{\mathrm{fast}}(\varepsilon, z), W_{\mathrm{fast}}(\varepsilon,$
is stable and the slow traveling pulse solution $(U, W) = (U_{\mathrm{slow}}(\varepsilon, z), W_{\mathrm{slow}}(\varepsilon, z)$
is unstable. Both pulses satisfy the system of differential equations

$$\begin{aligned} \nu(\varepsilon)U' + U + W &= \alpha \int_{\mathbb{R}} K(z-y)H(U(y)-\theta)\mathrm{d}y, \\ \nu(\varepsilon)W' &= \varepsilon(U - \gamma W), \end{aligned}$$

and the boundary conditions

$$\lim_{z\to\pm\infty}(U(\varepsilon, z), W(\varepsilon, z)) = (0,0), \lim_{z\to\pm\infty}(U_z(\varepsilon, z), W_z(\varepsilon, z)) = (0,0).$$

Later, many mathematicians applied almost the same ideas and methods in that paper to study the stability of fast traveling pulse solutions of nonlinear singularly perturbed systems of integral differential equations. For examples, they studied the following nonlinear systems of integral differential equations

$$\begin{aligned} \frac{\partial u}{\partial t} + u + w &= \alpha \int_{\mathbb{R}} K(x-y)H\left(u\left(y, t - \frac{1}{c}|x-y|\right) - \theta\right)\mathrm{d}y, \\ \frac{\partial w}{\partial t} &= \varepsilon(u - \gamma w); \end{aligned}$$

$$\frac{\partial u}{\partial t} + u + w = \alpha \int_0^\infty \xi(c) \left[\int_{\mathbb{R}} K(x-y) H\left(u\left(y, t - \frac{1}{c}|x-y| \right) - \theta \right) \mathrm{d}y \right]$$
$$\frac{\partial w}{\partial t} = \varepsilon(u - \gamma w);$$

$$\frac{\partial u}{\partial t} + u + w = \beta \int_{\mathbb{R}} W(x-y) H(u(y, t-\tau) - \Theta) \mathrm{d}y,$$
$$\frac{\partial w}{\partial t} = \varepsilon(u - \gamma w);$$

$$\frac{\partial u}{\partial t} + u + w = \beta \int_0^\infty \eta(\tau) \left[\int_{\mathbb{R}} W(x-y) H(u(y, t-\tau) - \Theta) \mathrm{d}y \right] \mathrm{d}\tau,$$
$$\frac{\partial w}{\partial t} = \varepsilon(u - \gamma w).$$

2.4 Project 6: Stability of a Traveling Pulse Solution of a Nonlinear Singularly Perturbed System of Integral Differential Equations - Part B

Consider the nonlinear singularly perturbed system of integral differential equations arising from synaptically coupled neuronal networks

$$\frac{\partial u}{\partial t} + u + w = \alpha \int_{\mathbb{R}} K(x-y) H(u(y,t) - \theta) \mathrm{d}y + \beta \int_{\mathbb{R}} W(x-y) H(u(y,t) - \Theta) \mathrm{d}y,$$
$$\frac{\partial w}{\partial t} = \varepsilon(u - \gamma w).$$

Theorem . There exist two traveling pulse solutions: the fast traveling pulse solution $(U, W) = (U_{\text{fast}}(\varepsilon, z), W_{\text{fast}}(\varepsilon, z))$, where $\nu_{\text{fast}}(\varepsilon)$ represents the fast wave speed and $z = x + \nu_{\text{fast}}(\varepsilon)t$ represents the fast moving coordinate, and the slow traveling pulse solution $(U,W) = (U_{\rm slow}(\varepsilon,z), W_{\rm slow}(\varepsilon,z))$, where $\nu_{\rm slow}(\varepsilon)$ represents the slow wave speed and $z = x + \nu_{\rm slow}(\varepsilon)t$ represents the slow moving coordinate. The fast traveling pulse solution $(U,W) = (U_{\rm fast}(\varepsilon,z), W_{\rm fast}(\varepsilon,z))$ is stable and the slow traveling pulse solution $(U,W) = (U_{\rm slow}(\varepsilon,z), W_{\rm slow}(\varepsilon,z))$ is unstable. Both pulse satisfy the system of differential equations

$$\begin{aligned}
\nu(\varepsilon)U' + U + W &= \alpha\int_{\mathbb{R}} K(z-y)H(U(y)-\theta)\mathrm{d}y, \\
&+ \beta\int_{\mathbb{R}} W(z-y)H(U(y)-\Theta)\mathrm{d}y, \\
\nu(\varepsilon)W' &= \varepsilon(U-\gamma W),
\end{aligned}$$

and the boundary conditions

$$\lim_{z\to\pm\infty}(U(\varepsilon,z),W(\varepsilon,z)) = (0,0), \lim_{z\to\pm\infty}(U(\varepsilon,z),W(\varepsilon,z)) = (0,0).$$

2.5 Project 7: Mathematical Analysis of Wave Speeds of Traveling Wave Fronts of Nonlinear Scalar Integral Differential Equations - Part A

Consider the following nonlinear scalar integral differential equations arising from synaptically coupled neuronal networks

$$\frac{\partial u}{\partial t} + u = \alpha\int_{\mathbb{R}} K(x-y)H(u(y,t)-\theta)\mathrm{d}y,$$

$$\frac{\partial u}{\partial t} + u = \alpha\int_{\mathbb{R}} K(x-y)H\left(u\left(y, t-\frac{1}{c}|x-y|\right)-\theta\right)\mathrm{d}y,$$

$$\frac{\partial u}{\partial t} + u = \alpha\int_0^\infty \xi(c)\left[\int_{\mathbb{R}} K(x-y)H\left(u\left(y, t-\frac{1}{c}|x-y|\right)-\theta\right)\mathrm{d}y\right]\mathrm{d}c,$$

$$\frac{\partial u}{\partial t} + u = \beta\int_{\mathbb{R}} W(x-y)H(u(y,t-\tau)-\Theta)\mathrm{d}y,$$

$$\frac{\partial u}{\partial t} + u = \beta \int_0^\infty \eta(\tau) \left[\int_{\mathbb{R}} W(x-y) H(u(y, t-\tau) - \Theta) \mathrm{d}y \right] \mathrm{d}\tau,$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u \;&=\; \alpha \int_0^\infty \xi(c) \left[\int_{\mathbb{R}} K(x-y) H\left(u\left(y, t - \frac{1}{c}|x-y| \right) - \theta \right) \mathrm{d}y \right] \mathrm{d}c \\ &+\; \beta \int_0^\infty \eta(\tau) \left[\int_{\mathbb{R}} W(x-y) H(u(y, t-\tau) - \Theta) \mathrm{d}y \right] \mathrm{d}\tau, \end{aligned}$$

$$\begin{aligned} &\frac{\partial u}{\partial t} + f(u) \\ =\;& (\alpha - au) \int_0^\infty \xi(c) \left[\int_{\mathbb{R}} K(x-y) H\left(u\left(y, t - \frac{1}{c}|x-y| \right) - \theta \right) \mathrm{d}y \right] \mathrm{d}c \\ +\;& (\beta - bu) \int_0^\infty \eta(\tau) \left[\int_{\mathbb{R}} W(x-y) H(u(y, t-\tau) - \Theta) \mathrm{d}y \right] \mathrm{d}\tau. \end{aligned}$$

There exists a unique traveling wave front to each of these integral differential equations. We have applied rigorous mathematical analysis and speed index functions to study how the wave speeds of the traveling wave fronts of these equations are influenced by neurobiological mechanisms: (K, W), (ξ, η), (α, θ), (θ, Θ) and (c_0, τ_0).

Theorem . (The influence of the synaptic couplings on the wave speeds) Suppose that $0 < x_1 < x_2 < \cdots < x_{2m-1} < \infty$ and $0 < y_1 < y_2 < \cdots < y_{2n-1} < \infty$ are positive constants, where $m \geq 1$ and $n \geq 1$ are positive integers, such that

$$K_1(x) \leq K_2(x), \quad \text{on} \quad \bigcup_{k=1}^{m}(-x_{2k}, -x_{2k-1}),$$
$$K_1(x) \geq K_2(x), \quad \text{on} \quad \bigcup_{k=1}^{m}(-x_{2k-1}, -x_{2k-2}),$$
$$W_1(x) \leq W_2(x), \quad \text{on} \quad \bigcup_{l=1}^{n}(-y_{2l}, -y_{2l-1}),$$
$$W_1(x) \geq W_2(x), \quad \text{on} \quad \bigcup_{l=1}^{n}(-y_{2l-1}, -y_{2l-2}),$$

where $x_0 = 0$ and $x_{2m} = \infty$; $y_0 = 0$ and $y_{2n} = \infty$, and that

$$\int_{-x_{2k}}^{-x_{2k-2}} [K_2(x) - K_1(x)]\mathrm{d}x \geq 0, \qquad \int_{-y_{2l}}^{-y_{2l-2}} [W_2(x) - W_1(x)]\mathrm{d}x \geq 0,$$

for all positive integers $k = 1, 2, \cdots, m$ and $l = 1, 2, \cdots, n$. If $\theta = \Theta$, then

$$\mu(K_1, W_1) \leq \mu(K_2, W_2).$$

Theorem . (The influence of the probability density functions on the wave speeds) Suppose that $0 < c_1 < c_2 < \cdots < c_{2p-1} < \infty$ and $0 < \tau_1 < \tau_2 < \cdots < \tau_{2q-1} < \infty$ are positive constants, where $p \geq 1$ and $q \geq 1$ are positive integers. Suppose that ξ_1, ξ_2, η_1, η_2 are four probability density functions, such that

$$\begin{aligned}
\xi_1(c) &\geq \xi_2(c), &\quad \text{on } \bigcup_{k=1}^{p}(c_{2k-2}, c_{2k-1}),\\
\xi_1(c) &\leq \xi_2(c), &\quad \text{on } \bigcup_{k=1}^{p}(c_{2k-1}, c_{2k}),\\
\eta_1(\tau) &\leq \eta_2(\tau), &\quad \text{on } \bigcup_{l=1}^{q}(\tau_{2l-2}, \tau_{2l-1}),\\
\eta_1(\tau) &\geq \eta_2(\tau), &\quad \text{on } \bigcup_{l=1}^{q}(\tau_{2l-1}, \tau_{2l}),
\end{aligned}$$

where $c_0 = 0$ and $c_{2p} = \infty$; $\tau_0 = 0$ and $\tau_{2q} = \infty$, and that

$$\int_{c_{2k-2}}^{c_{2k}} [\xi_2(c) - \xi_1(c)]\mathrm{d}c \geq 0, \qquad \int_{\tau_{2l-2}}^{\tau_{2l}} [\eta_2(\tau) - \eta_1(\tau)]\mathrm{d}\tau \geq 0,$$

for all positive integers $k = 1, 2, \cdots, p$ and $l = 1, 2, \cdots, q$. If $\theta = \Theta$, then there holds the following estimate

$$\mu(\xi_1, \eta_1) \leq \mu(\xi_2, \eta_2).$$

Theorem . (The influence of the synaptic rate constants and the thresholds on the wave speeds) Let μ represent the wave speed of the traveling wave front.

The wave speed is an increasing function of α.

The wave speed is an increasing function of β.

The wave speed is an increasing function of c_0, if $\xi(c) = \delta(c - c_0)$.

The wave speed is an increasing function of $\Lambda \equiv \frac{1}{2} - \frac{\theta}{\alpha+\beta}$.

The wave speed is an decreasing function of θ.

The wave speed is an decreasing function of τ_0, if $\eta(\tau) = \delta(\tau - \tau_0)$.

2.6 Project 7: Mathematical Analysis of Wave Speeds of Traveling Wave Fronts of Nonlinear Scalar Integral Differential Equations - Part B

Theorem . (The upper bounds and lower bounds of the wave speeds) Suppose that the synaptic couplings K and W satisfy the condition

$$|K(x)| \le C_1 \exp(-\rho|x|), \qquad |W(x)| \le C_1 \exp(-\rho|x|) \qquad \text{on } \mathbb{R},$$

for two positive constants $C_1 > 0$ and $\rho > 0$. There hold the following estimates

$$\begin{aligned}
\mu_3 &\le \frac{1}{(\alpha+\beta)\ln\frac{\alpha+\beta}{\alpha+\beta-2\theta}} \int_{\mathbb{R}} |x|[\alpha K(x) + \beta W(x)]\mathrm{d}x, \\
\mu_3 &\ge \frac{(\alpha+\beta-2\theta)^2 c_0}{(\alpha+\beta-2\theta)^2 + 2c_0\Lambda_2^2}, \text{ for } \theta = \Theta, \\
\mu_3 &\ge \frac{(\alpha-2\theta)^2 c_0}{(\alpha-2\theta)^2 + 2c_0\Lambda_1^2}, \text{ for } \theta < \Theta, \text{ where} \\
\Lambda_1 &= \alpha\left\{\int_{-\infty}^0 |K(x)|^2 \mathrm{d}x\right\}^{1/2} + \beta\left\{\int_{-\infty}^0 |W(x)|^2 \mathrm{d}x\right\}^{1/2}, \\
\Lambda_2 &= \alpha\left\{\int_{-\infty}^0 |K(x)|^2 \mathrm{d}x\right\}^{1/2} \\
&+ \beta\left\{\int_{-\infty}^0 |W(x)|^2 \mathrm{d}x\right\}^{1/2} \left\{1 + \sqrt{2}\int_0^\infty \tau^{1/2}\eta(\tau)\mathrm{d}\tau\right\}.
\end{aligned}$$

Theorem . (The comparison of the wave speeds) Suppose that $c_0 > 0$ and $\tau_0 > 0$ are positive constants. Let $\xi_{c_0} = \delta(c - c_0)$ and $\eta_{\tau_0} = \delta(\tau - \tau_0)$ be two simple Dirac delta impulse functions on $(0, \infty)$, and let $\xi_\infty = \delta(c - \infty)$ and $\eta_0 = \delta(\tau)$ be another two Dirac delta impulse functions. Suppose that $\xi \ge 0$ and $\eta \ge 0$ are nonnegative finite probability density functions defined

on $(0,\infty)$, such that $\xi = 0$ on $[0, c_0]$ and $\xi \geq 0$ on (c_0, ∞); $\eta \geq 0$ on $[0, \tau_0]$ and $\eta = 0$ on (τ_0, ∞).
(I) Let $\mu_1(\xi)$, $\mu_1(\xi_{c_0})$ and $\mu_1(\xi_\infty)$ represent the wave speeds of the traveling wave fronts of the following three nonlinear scalar integral differential equations

$$\begin{aligned}
\frac{\partial u}{\partial t} + u &= \alpha \int_0^\infty \xi(c) \left[\int_{\mathbb{R}} K(x-y) H\left(u\left(y, t - \frac{1}{c}|x-y| \right) - \theta \right) \mathrm{d}y \right] \mathrm{d}c, \\
\frac{\partial u}{\partial t} + u &= \alpha \int_{\mathbb{R}} K(x-y) H\left(u\left(y, t - \frac{1}{c_0}|x-y| \right) - \theta \right) \mathrm{d}y, \\
\frac{\partial u}{\partial t} + u &= \alpha \int_{\mathbb{R}} K(x-y) H(u(y,t) - \theta) \mathrm{d}y,
\end{aligned}$$

respectively. Then

$$\mu_1(\xi_{c_0}) < \mu_1(\xi) < \mu_1(\xi_\infty).$$

(II) Let $\mu_2(\eta)$, $\mu_2(\eta_{\tau_0})$ and $\mu_2(\eta_0)$ represent the wave speeds of the traveling wave fronts of the following three nonlinear scalar integral differential equations

$$\begin{aligned}
\frac{\partial u}{\partial t} + u &= \beta \int_0^\infty \eta(\tau) \left[\int_{\mathbb{R}} W(x-y) H(u(y, t-\tau) - \Theta) \mathrm{d}y \right] \mathrm{d}\tau, \\
\frac{\partial u}{\partial t} + u &= \beta \int_{\mathbb{R}} W(x-y) H(u(y, t-\tau_0) - \Theta) \mathrm{d}y, \\
\frac{\partial u}{\partial t} + u &= \beta \int_{\mathbb{R}} W(x-y) H(u(y,t) - \Theta) \mathrm{d}y,
\end{aligned}$$

respectively. Then

$$\mu_2(\eta_{\tau_0}) < \mu_2(\eta) < \mu_2(\eta_0).$$

(III) Let $\mu_1(\alpha)$, $\mu_2(\beta)$ and $\mu_3(\alpha, \beta)$ represent the wave speeds of the traveling wave fronts of the following three nonlinear scalar integral differential equations

$$\begin{aligned}
\frac{\partial u}{\partial t}+u &= \alpha\int_0^\infty \xi(c)\left[\int_{\mathbb{R}} K(x-y)H\left(u\left(y,t-\frac{1}{c}|x-y|\right)-\theta\right)\mathrm{d}y\right]\mathrm{d}c,\\
\frac{\partial u}{\partial t}+u &= \beta\int_0^\infty \eta(\tau)\left[\int_{\mathbb{R}} W(x-y)H(u(y,t-\tau)-\Theta)\mathrm{d}y\right]\mathrm{d}\tau,\\
\frac{\partial u}{\partial t}+u &= \alpha\int_0^\infty \xi(c)\left[\int_{\mathbb{R}} K(x-y)H\left(u\left(y,t-\frac{1}{c}|x-y|\right)-\theta\right)\mathrm{d}y\right]\mathrm{d}c\\
&+ \beta\int_0^\infty \eta(\tau)\left[\int_{\mathbb{R}} W(x-y)H(u(y,t-\tau)-\Theta)\mathrm{d}y\right]\mathrm{d}\tau,
\end{aligned}$$

respectively. Then

$$\max\{\mu_1(\alpha),\mu_2(\beta)\}<\mu_3(\alpha,\beta).$$

2.7 Project 8: Evans Functions and Bifurcations of Nonlinear Waves of Nonlinear Singularly Perturbed Systems of Equations - Part A

Consider the following nonlinear singularly perturbed system of reaction diffusion equations arising from mathematical neurosciences

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2}+\alpha[\beta H(u-\theta)-u]-w,\\
\frac{\partial w}{\partial t} &= \varepsilon(u-\gamma w),
\end{aligned}$$

where $\alpha>0$, $\beta>0$, $\gamma>0$, $0<\varepsilon\ll 1$ and $\theta>0$ are positive constants. By hard working to overcome many difficulties, I constructed Evans functions and applied them to establish the existence, stability or instability of nonlinear waves in the recent paper: Evans functions and bifurcations of nonlinear waves of some reaction diffusion equations. *Journal of Differential Equations*,

263(2017), 3627-3686. The existence of the nonlinear waves follows from the special structures of the model equations and standard ideas, methods and techniques in differential equations. The difficulty is that we have to solve some overdetermined systems of equations to prove the existence of the standing pulse solutions. I constructed Evans functions to find the eigenvalues and eigenfunctions of several eigenvalue problems associated with several linear differential operators $\mathcal{L}$, which are obtained from the linearization of the nonlinear equations about the waves. It turns out that a complex number λ_0 is an eigenvalue of the linear differential operator $\mathcal{L}$, if and only if λ_0 is a zero of the Evans function. The stability/instability of the nonlinear waves is accomplished by coupling together the zeros of the Evans functions. The most difficult point is that the eigenvalue problems obtained by using linearization technique involve the Dirac delta impulse functions. This makes it very difficult to solve the eigenvalue problems for the eigenvalues and eigenfunctions. The main strategy to overcome the difficulty is to use the special structures of the reaction diffusion equations to reduce the eigenvalue problems to simplified differential equations. Then I applied standard theory for second order nonhomogeneous linear differential equations, in particular, the method of variation of parameters to construct bounded particular solutions of the eigenvalue problems.

The constructions and applications of the Evans functions to the stability, instability and bifurcations of the nonlinear waves of the nonlinear system of reaction diffusion equations and the nonlinear scalar reaction diffusion equations have been open for a long time, even though the existence and instability of a single standing pulse solution have been established before by using other method. This paper aims to provide positive solutions to these open problems.

Theorem . There exists an unstable standing pulse solution if $0 < 2(1+\alpha\gamma)\theta < \alpha\beta\gamma$; there exist two standing wave fronts if $2(1+\alpha\gamma)\theta = \alpha\beta\gamma$ and $\gamma^2\varepsilon > 1$; there exist two standing wave fronts if $2(1+\alpha\gamma)\theta = \alpha\beta\gamma$ and $0 < \gamma^2\varepsilon < 1$. Overall, $\frac{2(1+\alpha\gamma)\theta}{\alpha\beta\gamma} = 1$ is a bifurcation point.

Currently, my collaborators and I are proving the existence of multiple traveling pulse solutions of the system

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} + \alpha[\beta H(u-\theta) - u] - w,\\ \frac{\partial w}{\partial t} &= \varepsilon(u - \gamma w).\end{aligned}$$

2.8 Project 8: Evans Functions and Bifurcations of Nonlinear Waves of Nonlinear Singularly Perturbed System of Equations - Part B

Consider the following nonlinear singularly perturbed system of integral differential equations arising from synaptically coupled neuronal networks

$$\begin{aligned}\frac{\partial u}{\partial t} + u + w &= (\alpha - au)\int_0^\infty \xi(c)\left[\int_{\mathbb{R}} K(x-y)H\left(u\left(y, t-\frac{1}{c}|x-y|\right) - \right.\right.\\ &+ (\beta - bu)\int_0^\infty \eta(\tau)\left[\int_{\mathbb{R}} W(x-y)H(u(y,t-\tau)-\Theta)\mathrm{d}y\right]\mathrm{d}\tau,\\ \frac{\partial w}{\partial t} &= \varepsilon(u - \gamma w).\end{aligned}$$

Theorem . There exist two standing wave solutions $(U_i, W_i) = (U_i(x), W_i(x))$ to the nonlinear system, where $i = 1, 2$. There is a positive constant $\varepsilon = \varepsilon_0 > 0$, such that the standing wave solutions are stable for all $\varepsilon > \varepsilon_0$ and the standing wave solutions are unstable for all $0 < \varepsilon < \varepsilon_0$. The value $\varepsilon = \varepsilon_0$ is a bifurcation point.

We constructed Evans functions to establish the bifurcations of the standing wave solutions of the nonlinear system.

Summary

Consider the following nonlinear scalar integral differential equations

$$\frac{\partial u}{\partial t} + u = \alpha \int_{\mathbb{R}} K(x-y) H(u(y,t) - \theta) \mathrm{d}y,$$

$$\frac{\partial u}{\partial t} + u = \alpha \int_{\mathbb{R}} K(x-y) H\left(u\left(y, t - \frac{1}{c}|x-y|\right) - \theta\right) \mathrm{d}y,$$

and

$$\begin{aligned} \frac{\partial u}{\partial t} + u &= \alpha \int_0^\infty \xi(c) \left[\int_{\mathbb{R}} K(x-y) H\left(u\left(y, t - \frac{1}{c}|x-y|\right) - \theta\right) \mathrm{d}y\right] \mathrm{d}c \\ &+ \beta \int_0^\infty \eta(\tau) \left[\int_{\mathbb{R}} W(x-y)(u(y,t-\tau) - \Theta) \mathrm{d}y\right] \mathrm{d}\tau. \end{aligned}$$

Consider the following nonlinear singularly perturbed systems of integral differential equations arising from synaptically coupled neuronal networks

$$\begin{aligned} \frac{\partial u}{\partial t} + u + w &= \alpha \int_{\mathbb{R}} K(x-y) H(u(y,t) - \theta) \mathrm{d}y, \\ \frac{\partial w}{\partial t} &= \varepsilon(u - \gamma w), \end{aligned}$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u + w &= \alpha \int_{\mathbb{R}} K(x-y) H\left(u\left(y, t - \frac{1}{c}|x-y|\right) - \theta\right) \mathrm{d}y, \\ \frac{\partial w}{\partial t} &= \varepsilon(u - \gamma w), \end{aligned}$$

and

$$\begin{aligned}
\frac{\partial u}{\partial t}+u+w &= \alpha\int_0^\infty \xi(c)\left[\int_{\mathbb{R}} K(x-y)H\left(u\left(-\frac{1}{c}|x-y|\right)\right)\mathrm{d}y\right]\mathrm{d}c \\
&+ \beta\int_0^\infty \eta(\tau)\left[\int_{\mathbb{R}} W(x-y)H(u(y,t-\tau)-\Theta)\right]\mathrm{d}\tau, \\
\frac{\partial w}{\partial t} &= \varepsilon(u-\gamma w).
\end{aligned}$$

Consider the following nonlinear singularly perturbed system of reaction diffusion equations

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2}+\alpha[\beta H(u-\theta)-u]-w, \\
\frac{\partial w}{\partial t} &= \varepsilon(u-\gamma w).
\end{aligned}$$

Overall, I have accomplished the existence and stability of traveling wave fronts of the nonlinear scalar integral differential equations, the existence and stability of traveling pulse solutions of the systems of integral differential equations, conducted mathematical analysis of wave speeds and obtained the bifurcations of nonlinear waves of the reaction diffusion equations. In these problems, traditional ideas and methods cannot be applied due to certain technical restrictions and difficulties. The ideas, methods, and techniques developed in these projects may be applied to establish the existence and stability of the traveling wave solutions of more realistic model equations. The papers [37], [38], [48] in this area have been highly recognized and cited by many other mathematicians.

3 Representations of Bounded Explicit Traveling Wave Solutions of Nonlinear Evolution Equations

The third research direction is the representations of bounded explicit traveling wave solutions of nonlinear evolution equations with strong physical backgrounds (including nonlinear dispersive wave equations, nonlinear dissipative dispersive wave equations, nonlinear reaction diffusion equations, nonlinear hyperbolic equations and other kinds of nonlinear evolution equations). We have coupled together the method of reduction of order, the method of undetermined coefficients, some nonlinear transformations, particular solutions of Burnoulli equations and other ideas in differential equations to establish the bounded explicit particular solutions of several second order nonlinear ordinary differential equations. These particular solutions of the second order nonlinear ordinary differential equations may be applied to solve nonlinear evolution equations for representations of bounded explicit traveling wave solutions, as indicated below.

3.1 Project 9: Traveling Wave Problems of Some Nonlinear Dispersive Wave Equations

The next theorem is developed and motivated by the traveling wave problems of several nonlinear dispersive wave equations, such as the nonlinear Korteweg-de Vries equation

$$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} + \frac{\partial}{\partial x}(u^p) = 0,$$

the nonlinear Benjamin-Bona-Mahony equation

$$\frac{\partial u}{\partial t} - \frac{\partial^3 u}{\partial x^2 \partial t} + \frac{\partial}{\partial x}(\beta u + u^p) = 0,$$

the nonlinear Schrödinger equation

$$\mathrm{i}\frac{\partial u}{\partial t}+\alpha\frac{\partial^2 u}{\partial x^2}+\beta|u|^{2p}u=0,$$

the cubic nonlinear system of Schrödinger equations

$$\begin{aligned}\mathrm{i}\frac{\partial u}{\partial t}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial}{\partial x}[(|u|^2+|v|^2)u] &= 0,\\ \mathrm{i}\frac{\partial v}{\partial t}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial}{\partial x}[(|u|^2+|v|^2)v] &= 0,\end{aligned}$$

the general nonlinear system of Schrödinger equations

$$\begin{aligned}\mathrm{i}\frac{\partial u}{\partial t}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial}{\partial x}[(|u|^{2p}+|v|^{2p})u] &= 0,\\ \mathrm{i}\frac{\partial v}{\partial t}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial}{\partial x}[(|u|^{2p}+|v|^{2p})v] &= 0,\end{aligned}$$

the three-dimensional Kadomtsev-Petviashvili equation

$$\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial t}+\frac{\partial^3 u}{\partial x^3}+\frac{\partial}{\partial x}(u^p)\right]+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0,$$

and some other nonlinear dispersive wave equations, where $\alpha>0$, $\beta>0$ and $p>0$ are positive constants.

Theorem . (I) Consider the following second order nonlinear differential equation

$$\phi''(z)=\phi(z)\{\alpha^2-\beta^2[\phi(z)]^{2m}\},$$

where $\alpha>0$, $\beta>0$ and $m>0$ are positive constants. There exists a bounded explicit solution

$$\phi(z)=\left\{\frac{(m+1)\alpha^2}{\beta^2}[\mathrm{sech}(\alpha m z)]^2\right\}^{1/(2m)}.$$

In particular, the differential equation $\phi''(z) = \phi(z)\{\alpha^2 - \beta^2[\phi(z)]^2\}$ has the following bounded explicit solution

$$\phi(z) = \pm\left\{\frac{2\alpha^2}{\beta^2}[\text{sech}(\alpha z)]^2\right\}^{1/2}.$$

(II) Consider the second order nonlinear differential equation

$$\phi''(z) = \phi(z)[\alpha^2 - \beta^2\phi(z)] - \alpha^2\omega + \beta^2\omega^2,$$

where $\alpha > 0$ and $\beta > 0$ are positive constants, ω is a real constant, such that $\alpha^2 - 2\beta^2\omega > 0$. There exists a bounded explicit solution

$$\phi(z) = \frac{3(\alpha^2 - 2\beta^2\omega)}{2\beta^2}\left[\text{sech}\left(\frac{1}{2}\sqrt{\alpha^2 - 2\beta^2\omega z}\right)\right]^2 + \omega.$$

The next theorem is developed and motivated by the traveling wave problem of the Korteweg-de Vries equation with nonlinear dispersion

$$\frac{\partial u}{\partial t} + \frac{\partial^3}{\partial x^3}(u^2) + \frac{\partial}{\partial x}(u^2) = 0.$$

Theorem . Consider the second order nonlinear differential equation

$$\alpha\{[\phi(z)]^2\}'' + \beta[\phi(z)]^2 + \gamma\phi(z) = 0,$$

where $\alpha \neq 0$, $\beta \neq 0$ and $\gamma \neq 0$ are real nonzero constants, such that $\alpha\beta > 0$. There exist two bounded explicit periodic solutions

$$\phi(z) = -\frac{2\gamma}{3\beta} \pm \frac{2\gamma}{3\beta}\sin\left(\sqrt{\frac{\beta}{4\alpha}}z\right).$$

3.2 Project 10: Traveling Wave Problems of Some Nonlinear Dissipative Dispersive Wave Equations

The next theorem is developed and motivated by the traveling wave problems of several nonlinear dissipative dispersive wave equations, such as the nonlinear Korteweg-de Vries-Burgers equation

$$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} - \alpha \frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x}(u^2) = 0,$$

the general nonlinear Korteweg-de Vries-Burgers equation

$$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} - \alpha \frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x}(Au + Bu^2) = 0,$$

the nonlinear Benjamin-Bona-Mahony-Burgers equation

$$\frac{\partial u}{\partial t} - \frac{\partial^3 u}{\partial x^2 \partial t} - \alpha \frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x}(u + u^2) = 0,$$

the general Benjamin-Bona-Mahony-Burgers equation

$$\frac{\partial u}{\partial t} - \frac{\partial^3 u}{\partial x^2 \partial t} - \alpha \frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x}(Au + Bu^2) = 0,$$

the general n-dimensional Benjamin-Bona-Mahony-Burgers equation

$$\frac{\partial u}{\partial t} - \frac{\partial}{\partial t}\triangle u - \alpha \triangle u + \beta \cdot \nabla(Au + Bu^2) = 0,$$

and some other nonlinear dissipative dispersive wave equations, where $\alpha > 0$ is a positive constant, $A \neq 0$ and $B \neq 0$ are real nonzero constants, $\beta = (\beta_1, \beta_2, \cdots, \beta_n)$ is a real nonzero constant vector.

Theorem . (I) Consider the following second order nonlinear differential equation

$$\phi''(z) + A\phi(z) + B[\phi(z)]^2 = \alpha \phi'(z),$$

where $\alpha > 0$ is a positive constant, $A \neq 0$ and $B \neq 0$ are real nonzero constants, such that $A = \frac{6}{25}\alpha^2$. There exists a bounded explicit solution

$$\phi(z) = -\frac{3\alpha^2}{50B}\left\{1 + \tanh\left[\frac{\alpha}{10}z\right]\right\}^2.$$

(II) Consider the second order nonlinear differential equation

$$\phi''(z) + A\phi(z) + B[\phi(z)]^2 + C = \alpha\phi'(z),$$

where $\alpha > 0$ is a positive constant, A, B and C are real constants, such that

$$A + 2B\omega = \frac{6}{25}\alpha^2, \qquad A\omega + B\omega^2 + C = 0,$$

for some real constant ω. There exists a bounded explicit solution

$$\phi(z) = -\frac{3\alpha^2}{50B}\left\{1 + \tanh\left[\frac{\alpha}{10}z\right]\right\}^2 + \omega.$$

3.3 Project 11: Traveling Wave Problems of Several Nonlinear Reaction Diffusion Equations

The next theorem is developed and motivated by the traveling wave problems of several nonlinear reaction diffusion equations, such as the Fisher's equation

$$\frac{\partial u}{\partial t} - \alpha\frac{\partial^2 u}{\partial x^2} = \beta u(\gamma^2 - \delta^2 u),$$

the general Fisher's equation

$$\frac{\partial u}{\partial t} - \alpha\frac{\partial^2 u}{\partial x^2} = \beta u(\gamma^2 - \delta^2 u^m),$$

the nonlinear system of reaction diffusion equations (that is, the 2×2 Belousov-Zhabotinskii equations)

$$\begin{aligned}\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2} &= \alpha u(1-u)+\beta uv,\\ \frac{\partial v}{\partial t}-\frac{\partial^2 v}{\partial x^2} &= \gamma v(1-v)+\delta uv,\end{aligned}$$

the nonlinear system of reaction diffusion equations (that is, the 3×3 Belousov-Zhabotinskii equations)

$$\begin{aligned}\frac{\partial u}{\partial t}-D\frac{\partial^2 u}{\partial x^2} &= 2\alpha uw-\beta uv,\\ \frac{\partial v}{\partial t}-D\frac{\partial^2 v}{\partial x^2} &= -\beta uv+2\gamma vw,\\ \frac{\partial w}{\partial t}-D\frac{\partial^2 w}{\partial x^2} &= \beta uv-\alpha uw-\gamma vw,\end{aligned}$$

and some other nonlinear reaction diffusion equations, where $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\delta > 0$ and $m > 0$ are positive constants.

Theorem . Consider the following second order nonlinear differential equation

$$\nu\phi'(z)-\alpha\phi''(z)=\beta\phi(z)\{\gamma^2-\delta^2[\phi(z)]^m\},$$

where $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\delta > 0$ and $m > 0$ are positive constants, ν is a real parameter. Let

$$\nu=\pm(m+4)\gamma\sqrt{\frac{\alpha\beta}{2(m+2)}}.$$

There exist two bounded explicit solutions to the differential equation, given by

$$\phi(z)=\left\{\frac{\gamma}{2\delta}\left[1\pm\tanh\left(\frac{1}{2}\sqrt{\frac{\beta}{2\alpha(m+2)}}m\gamma z\right)\right]\right\}^{2/m}.$$

3.4 Project 12: Traveling Wave Problems of Some Nonlinear Hyperbolic Equations

The next theorem is developed and motivated by the traveling wave problem of the well known nonlinear Sine-Gordon equation

$$\frac{\partial^2 u}{\partial t^2} - \alpha^2 \frac{\partial^2 u}{\partial x^2} + 2\beta\gamma^2 \sin(2\beta u) = 0,$$

where $\alpha > 0$, $\beta > 0$ and $\gamma > 0$ are positive constants.

Theorem. (I) Consider the second order nonlinear differential equation

$$\phi''(z) = 2\beta\gamma^2 \sin(2\beta\phi(z)),$$

where $\beta > 0$ and $\gamma > 0$ are positive constants. There exists a bounded explicit solution

$$\phi(z) = \frac{1}{\beta} \cos^{-1}(\tanh(2\beta\gamma z)).$$

(II) Consider the second order nonlinear differential equation

$$\phi''(z) = -2\beta\gamma^2 \sin(2\beta\phi(z)),$$

where $\beta > 0$ and $\gamma > 0$ are positive constants. There exist two bounded explicit solutions

$$\phi(z) = \pm\frac{1}{\beta} \sin^{-1}(\tanh(2\beta\gamma z)).$$

The next theorem is developed and motivated by the Sine-Gordon equation with dissipation

$$\frac{\partial^2 u}{\partial t^2} - \alpha^2 \frac{\partial^2 u}{\partial x^2} + \beta\frac{\partial u}{\partial t} + \gamma \sin(\lambda u) + \delta \sin(2\lambda u) = 0,$$

where $\alpha > 0$, $\beta \neq 0$, $\gamma \neq 0$, $\delta \neq 0$ and $\lambda \neq 0$ are real nonzero constants.

Theorem . Consider the second order nonlinear differential equation

$$\alpha\phi''(z) + \beta\phi'(z) + \gamma\sin(\lambda\phi(z)) + \delta\sin(2\lambda\phi(z)) = 0,$$

where $\alpha \neq 0$, $\beta \neq 0$, $\gamma \neq 0$, $\delta \neq 0$ and $\lambda \neq 0$ are real nonzero constants. If the constants satisfy the condition $\alpha\gamma^2\lambda + 2\beta^2\delta = 0$, then there exists a bounded explicit solution

$$\phi(z) = \frac{1}{\lambda}\cos^{-1}\left(\tanh\left(\frac{\gamma\lambda}{\beta}z\right)\right).$$

3.5 Project 13: Traveling Wave Problems of Some Nonlinear Evolution Equations Involving Cubic Polynomials

The next theorem is developed and motivated by the traveling wave problems of nonlinear evolution equations involving cubic polynomial functions, such as the nonlinear scalar bistable equation

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = u(1-u)(u-a);$$

the nonlinear scalar reaction diffusion equation

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = u^2(1-u);$$

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = -u(1-u)^2;$$

the Gray-Scott reaction diffusion equations

$$\begin{aligned} \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} &= \beta\frac{\partial u}{\partial x} + \alpha(1-u) - uv^2, \\ \frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} &= \beta\frac{\partial v}{\partial x} + uv^2 - \alpha v; \end{aligned}$$

the reaction diffusion equations

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} + \beta\frac{\partial u}{\partial x} + 2\alpha^2(R^2 - u^2 - v^2)u,\\ \frac{\partial v}{\partial t} &= \frac{\partial^2 v}{\partial x^2} + \beta\frac{\partial v}{\partial x} + 2\alpha^2(R^2 - u^2 - v^2)v;\end{aligned}$$

the Selkov equations

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} - \alpha u + \beta v + u^2 v + \rho,\\ \frac{\partial v}{\partial t} &= \frac{\partial^2 v}{\partial x^2} - \gamma v - u^2 v + \sigma;\end{aligned}$$

the nonlinear Korteweg-de Vries-Burgers equation

$$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} - \alpha\frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x}(Au + Bu^2 + Cu^3) = 0;$$

the nonlinear Benjamin-Bona-Mahony-Burgers equation

$$\frac{\partial u}{\partial t} - \frac{\partial^3 u}{\partial x^2 \partial t} - \alpha\frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x}(Au + Bu^2 + Cu^3) = 0;$$

the nonlinear Klein-Gordon equation

$$\frac{\partial^2 u}{\partial t^2} - \alpha^2\frac{\partial^2 u}{\partial x^2} + \alpha^2 u - \beta^2 u^3 = 0;$$

and some other nonlinear evolution equations, where α, β, γ, σ, ρ, A, B, C and a are real constants.

Theorem . Consider the following second order nonlinear differential equation

$$\phi''(z) + \delta\phi'(z) = \alpha\phi(z)[\phi(z) - \beta][\phi(z) - \gamma],$$

where $\alpha > 0$ and $\beta > 0$ are positive constants, γ and δ are real constants. If the constants α, β, γ, δ satisfy

$$\delta = (2\gamma - \beta)\sqrt{\frac{\alpha}{2}},$$

then there exists a bounded explicit solution to the differential equation, given by

$$\phi(z) = \frac{\beta}{2}\left[1 + \tanh\left(\frac{\beta}{2}\sqrt{\frac{\alpha}{2}}z\right)\right].$$

If the constants α, β, γ, δ satisfy

$$\delta = -(2\gamma - \beta)\sqrt{\frac{\alpha}{2}},$$

then there exists a bounded explicit solution

$$\phi(z) = \frac{\beta}{2}\left[1 - \tanh\left(\frac{\beta}{2}\sqrt{\frac{\alpha}{2}}z\right)\right].$$

3.6 Project 14: Traveling Wave Problems of Nonlinear Eevolution Equations Involving Special Functions

The next theorem is developed and motivated by the traveling wave problems of nonlinear systems of differential equations involving $(R^2 - u^2 - v^2)u$ and $(R^2 - u^2 - v^2)v$, such as the nonlinear system of hyperbolic equations

$$\begin{aligned}
\frac{\partial^2 u}{\partial t^2} &= \frac{\partial^2 u}{\partial x^2} + (R^2 - u^2 - v^2)u, \\
\frac{\partial^2 v}{\partial t^2} &= \frac{\partial^2 v}{\partial x^2} + (R^2 - u^2 - v^2)v;
\end{aligned}$$

and some other nonlinear systems of differential equations, where α, β and R are real constants.

Theorem . Consider the following second order nonlinear system of differential equations

$$\begin{aligned}
\alpha\phi''(z) + \beta\{R^2 - [\phi(z)]^2 - [\psi(z)]^2\}\phi(z) = 0, \\
\alpha\psi''(z) + \beta\{R^2 - [\phi(z)]^2 - [\psi(z)]^2\}\psi(z) = 0,
\end{aligned}$$

where $\alpha \neq 0$, $\beta \neq 0$ and $R \neq 0$ are real nonzero constants. (I) Suppose that the real constants α and β satisfy the condition $\alpha\beta < 0$. There exist two families of bounded explicit solutions

$$\phi(z) = \pm\left\{\frac{2R^2}{1+r^2}\left[\operatorname{sech}\left(\sqrt{-\frac{\beta}{\alpha}}Rz\right)\right]^2\right\}^{1/2},$$

$$\psi(z) = \pm r\left\{\frac{2R^2}{1+r^2}\left[\operatorname{sech}\left(\sqrt{-\frac{\beta}{\alpha}}Rz\right)\right]^2\right\}^{1/2},$$

where r is a real parameter.

(II) There exists a family of bounded explicit periodic solutions

$$\phi(z) = r\left\{a\cos\left[\sqrt{\frac{\beta}{\alpha}(R^2-r^2)}z\right] + b\sin\left[\sqrt{\frac{\beta}{\alpha}(R^2-r^2)}z\right]\right\},$$

$$\psi(z) = r\left\{b\cos\left[\sqrt{\frac{\beta}{\alpha}(R^2-r^2)}z\right] - a\sin\left[\sqrt{\frac{\beta}{\alpha}(R^2-r^2)}z\right]\right\},$$

where a, b and r are real parameters, such that $a^2+b^2=1$ and $\alpha\beta(R^2-r^2) > 0$.

The results in this lemma may be used to study the bifurcation of nonlinear waves of some differential equations.

3.7 Summary

I have appropriately coupled together the method of reduction of order, the method of undetermined coefficients, some nonlinear transformations and particular solutions of Bernoulli equations to generate a comprehensive and systematic method to accomplish the representations of bounded explicit particular solutions of several nonlinear second order ordinary differential equations. These results may be used to solve several kinds of nonlinear evolution equations (including nonlinear dispersive wave equations, nonlinear dissipative dispersive wave equations, nonlinear reaction diffusion equations, nonlinear hyperbolic equations and other kinds of nonlinear evolution equations) for bounded explicit traveling wave solutions, which are of special values in industry, engineering and applied mathematics. These nonlinear evolution equations have strong backgrounds in physics, chemistry, and biology.

4 THE RESEARCH STATEMENTS - Future Research Plans

There are still many very important and interesting open problems in applied mathematics. I have been very careful in selecting what mathematical model equations to work on. This is how I choose them: they must have strong backgrounds in physics or mathematical neuroscience by taking main physical points or main neurobiological points into account, the nonlinear functions and nonlocal interactions must be physically or biologically realistic and the model equations must be of public interests in the mathematical society. The model equations may describe the motion of incompressible fluids, or they may describe the propagation of nerve impulses, neurological disorders (for example, cortical epilepsy and migraine), and traveling waves in cardiac tissues. On the other hand, they cannot be too complicated so that no one can solve them either analytically or numerically.

More specifically, in fluid dynamics, I am very interested in the special structures of the n-dimensional incompressible Navier-Stokes equations. By making complete use of the special structures, it is possible to establish the uniform energy estimates for the first order derivatives and then for the higher order derivatives of the global weak solution. Therefore, I can accomplish the existence and uniqueness of the global smooth solution.

In mathematical neuroscience, I am very interested in the stability of the multiple traveling pulse solutions of two kinds of nonlinear singularly perturbed systems of differential equations, both have strong backgrounds: nonlinear singularly perturbed systems of reaction diffusion equations arising from mathematical neuroscience and nonlinear singularly perturbed system of integral differential equations arising from synaptically coupled neuronal networks.

In ordinary differential equations, I am very interested in developing more technical lemmas to find bounded explicit solutions whose graphs are like perfect multiple pulses. Then I can apply the lemmas to solve nonlinear evolution equations for bounded explicit multiple pulse solutions.

Needless to say, they are my research projects in the near future.

4.1 Future Project 1: An Open Problem in n-Dimensional Incompressible Navier-Stokes Equations

Consider the Cauchy problem for the n-dimensional incompressible Navier-Stokes equations

$$\frac{\partial}{\partial t}\mathbf{u} - \alpha\triangle\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f}(\mathbf{x},t), \quad \nabla\cdot\mathbf{u} = 0, \nabla\cdot\mathbf{f} = 0,$$
$$\mathbf{u}(\mathbf{x},0) = \mathbf{u}_0(\mathbf{x}), \quad \nabla\cdot\mathbf{u}_0 = 0.$$

There exist two hidden nonlinear diffusions in the n-dimensional incompressible Navier-Stokes equations. Moreover, they help each other when making energy estimates. I will make complete use of the special structures and couple together an unusual energy estimate method to establish the uniform energy estimates for both lower order derivatives and higher order derivatives of the global weak solutions. I will also make use of many traditional ideas, methods, techniques and results to achieve the main goal.

4.2 Future Project 2: Stability of Multiple Traveling Pulse Solutions of a Nonlinear Singularly Perturbed System of Reaction Diffusion Equations Arising from Mathematical Neuroscience

Consider the following nonlinear singularly perturbed system of reaction diffusion equations

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} + \alpha[\beta H(u-\theta) - u] - w, \\ \frac{\partial w}{\partial t} &= \varepsilon(u - \gamma w). \end{aligned}$$

There are several serious mathematical mistakes in other people's previous analysis about the stability of the multiple traveling pulse solutions. Therefore, the stability has been open. I will use linearization idea, in particular, I will construct and apply Evans functions to study the eigenvalues of a few eigenvalue problems to rigorously accomplish the stability of the multiple pulse solutions of the nonlinear singularly perturbed system of reaction diffusion equations.

4.3 Future Project 3: The Existence and Stability of Multiple Traveling Pulse Solutions of a Nonlinear Singularly Perturbed System of Integral Differential Equations Arising from Synaptically Coupled Neuronal Networks

Consider the following nonlinear singularly perturbed system of integral differential equations

$$\begin{aligned}
\frac{\partial u}{\partial t} + u + w &= \alpha \int_0^\infty \xi(c) \left[\int_{\mathbb{R}} K(x-y) H\left(u\left(y, t - \frac{1}{c}|x-y| \right) - \theta \right) \mathrm{d}y \right] \\
&+ \beta \int_0^\infty \eta(\tau) \left[\int_{\mathbb{R}} W(x-y) H(u(y, t-\tau) - \Theta) \mathrm{d}y \right] \mathrm{d}\tau, \\
\frac{\partial w}{\partial t} &= \varepsilon(u - \gamma w).
\end{aligned}$$

In this system, $(u, w) = (u(x,t), w(x,t))$ is a real vector-valued function of x and $t > 0$. I will study the existence and stability of multiple traveling pulse solutions. I will use ideas from my previous research and other people's ideas.

4.4 Future Project 4: The Representations of Bounded Explicit Traveling Wave Solutions of Some Nonlinear Evolution Equations

I will develop more technical lemmas about second order nonlinear ordinary differential equations for the representations of bounded explicit solutions which look perfectly like multiple pulse. Then I will apply them to solve nonlinear evolution equations, such as the nonlinear Korterweg-de Vries equation, the nonlinear Schrödinger equation and the Sine-Gordon equation

$$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} + \frac{\partial}{\partial x}(u^p) = 0,$$

$$\mathrm{i}\frac{\partial u}{\partial t} + \alpha\frac{\partial^2 u}{\partial x^2} + \beta|u|^{2p}u = 0,$$

$$\frac{\partial^2 u}{\partial t^2} - \alpha^2\frac{\partial^2 u}{\partial x^2} + 2\beta\gamma^2\sin(2\beta u) = 0,$$

where $\alpha > 0$, $\beta > 0$, $\gamma > 0$ and $p > 0$ are positive constants, for multiple pulse solutions.

Summary :

Overall, what I would like to do in the near future is to study interdisciplinary problems arising from mathematical neuroscience and in applied mathematics. I would like to have publications in high quality journals where rigorous mathematical analysis is appreciated and also in journals which highlight interdisciplinary topics.

I have thought about these problems for a long time and discussed them with many other experts in my research areas. I am confident that I have the right ideas to work them out. I hope to have more important results related to mathematical neuroscience.

My long term, main research goal is to generate various ideas and methods to solve important open problems in applied mathematics and mathematical neuroscience.

List of Citations by Other Mathematicians

Listed below are the citations of my papers by other mathematicians. No self citations are included.

Citation of Book Chapter [1]: This chapter has been cited twice by others:

1 Yan Jia, Xingwei Zhang and Boqing Dong, The asymptotic behavior of solutions to three-dimensional Navier-Stokes equations with nonlinear damping. *Nonlinear Analysis, Real World Applications*, **12**(2011), 1736-1747.

2 Jerry L. Bona and Laihan Luo, Large-time asymptotics of the generalized Benjamin-Ono-Burgers equation. *Discrete and Continuous Dynamical Systems, Series S*, **4**(2011), 15-50.

Citations of [1]: This paper has been cited twice by others:

1 A. S. Fokas and Laihan Luo, Global solutions and their asymptotic behavior for Benjamin-Ono-Burgers type equations. *Differential and Integral Equations*, **13**(2000), 115-124.

2 Jerry L. Bona and Laihan Luo, Large-time asymptotics of the generalized Benjamin-Ono-Burgers equation. *Discrete and Continuous Dynamical Systems, Series S*, **4**(2011), 15-50.

]

Citations of [3]: This paper has been cited five times by others:

1 Huijiang Zhao and Shaoqiang Tang, Nonlinear stability and optimal decay rate for a multidimensional generalized Kuramoto-Sivashinsky system. *Journal of Mathematical Analysis and Applications*, **246**(2000), 423-445.

2 Weijiu Liu and Miroslav Krstic, Stability enhancement by boundary control in the Kuramoto-Sivashinsky equation. *Nonlinear Analysis, Series A: Theory and Methods*, **43**(2001), 485-507.

3 Shangbin Cui and Cuihua Guo, Global existence and exponential decay of solutions of generalized Kuramoto-Sivashinsky equations. *Journal of Partial Differential Equations*, **18**(2005), 167-184.

4 C P. Massarolo, G P. Menzala and A F. Pazoto, A coupled system of Korteweg-de Vries equations as singular limit of the Kuramoto-Sivashinsky equations. *Advances in Differential Equations*, **12**(2007), 541-572.

5 C P. Massarolo and A F. Pazoto, Uniform stabilization of a nonlinear coupled system of Korteweg-de Vries equations as a singular limit of the Kuramoto-Sivashinsky system. *Differential and Integral Equations*, **22**(2009), 53-68.

Citations of [4]: This paper has been cited three time by others:

1 Boling Guo, Congratulation on Professor Zhou Yulin 80th birthday. Dedicated to the 80th birthday of Professor Zhou Yulin. *Journal of Partial Differential Equations*, **16**(2003), 1-7.

2 Shijin Ding, Changxing Miao, Shaobin Tan and Baoxiang Wang, On the occasion of the seventieth birthday of Professor Guo Boling. *Journal of Partial Differential Equations*, **20**(2007), 1-10.

3 Xueke Pu and Boling Guo, Existence and decay of solutions to the two-dimensional fractional quasigeostropic equation. *Journal of Mathematical Physics*, **51**(2010), 083101, 1-15.

]

]

Citations of [7]: This paper has been cited four time by others:

1 Huijiang Zhao, Decay estimates for the solutions of some multi-dimensional nonlinear evolution equations. *Communications in Partial Differential Equations*, **25**(2000), 377-422.

2 Maria E. Schonbek and Shubha V. Rajopadhye, Asymptotic behavior of solutions to the Korteweg-de Vries-Burgers system. *Ann. Inst. Henry Poincare, Analyse non lineaire*, **12**(1995), 425-457.

3 Mohammed Aassila, Stabilization of the Korteweg-de Vries-Burgers equation with non-periodic boundary feedbacks. *Journal of Applied Mathematics and Computations*, **11**(2003), 81-108.

4 Tomasz Dlotko, Maria B. Kania and Shan Ma, Korteweg-de Vries-Burgers system in $\mathbb{R}^n$. *Journal of Mathematical Analysis and Applications*, **411**(2014), 853-872.

Citations of [8]: This paper has been cited thirteen time by others:

1 Ming Mei, L^q-decay rates of solutions for Benjamin-Bona-Mahony-Burgers equations. *Journal of Differential Equations*, **158**(1999), 314-340.

2 Ming Mei and Christian Schmeiser, Asymptotic profiles of solutions for the BBM-Burgers equation. *Funkcial. Ekvac.* **44**(2001), 151-170.

3 Huijiang Zhao, Decay estimates for the solutions of some multi-dimensional nonlinear evolution equations. *Communications in Partial Differential Equations*, **25**(2000), 377-422.

4 Huijiang Zhao, Optimal temporal decay estimates for the solution to the multidimensional generalized BBM-Burgers equations with dissipative term. *Applicable Analysis*, **75**(2000), 85-105.

5 Shin-ichi Kinami, Ming Mei and Seiro Omata, Convergence to diffusion waves of the solutions for Benjamin-Bona-Mahony-Burgers equations. *Applicable Analysis*, **75**(2000), 317–340.

6 Elena I. Kaikina, Pavel I. Naumkin and I. A. Shishmarev, The Cauchy problem for a Sobolev-type equation with a power non-linearity. *Izv. Ross. Akad. Nauk Ser. Mat.* **69**(2005), 61-114.

7 Elena I. Kaikina, Initial-boundary value problems for nonlinear pseudoparabolic equations in a critical case. *Electronic Journal of Differential Equations*, (2007), article ID: 109, 25 pages.

8 Elena I. Kaikina, Nonlinear pseudoparabolic type equations on a half-line with large initial data. *Nonlinear Analysis*, **67**(2007), 2839-2858.

9 Guowang Chen and Hongxia Xue, Global existence of solution of Cauchy problem for nonlinear pseudo-parabolic equation. *Journal of Differential Equations*, **245**(2008), 2705-2722.

10 Elena I. Kaikina, Pavel I. Naumkin and I. A. Shishmarev, Long-time asymptotics of solutions of Sobolev-type nonlinear equations. *Uspekhi Mat. Nauk*, **64**(2009), 3-72.

11 Changhong Guo and Shaomei Fang, Optimal decay rates of solutions for a multidimensional generalized Benjamin-Bona-Mahony equation. *Nonlinear Analysis*, **75**(2012), 3385-3392.

12 Elena I. Kaikina, Pavel I. Naumkin and I. A. Shishmarev, The far-field asymptotics of solutions of a nonlinear equation with a fractional derivative. *Izv. Ross. Akad. Nauk Ser. Mat.*, **76**(2012), 37-66.

13 Weike Wang and Dandan Zhang, Large-time behavior for the solution to the generalized Benjamin-Bona-Mahony-Burgers equation with large initial data in the whole space. *Journal of Mathematical Analysis and Applications*, **411**(2014), 144-165.

Citations of [9]: This paper has been cited twice by others:

1 Daniel B. Dix, Large-time behaviour of solutions of Burgers' equation. *Proceedings of the Royal Society of Edinburgh, Section A*, **132**(2002), 843-878.

2 A Rashid, Convergence analysis of three-level Fourier pseudospectral method for Korteweg-de Vries-Burgers equation. *Comput Mathematics and Applications*, **52**(2006), 769-778.

Citations of [14]: This paper has been cited five time by others:

1 Corinne Laurey, On a nonlinear dispersive equation with time-dependent coefficients. *Advances in Differential Equations*, **6**(2001), 577-612.

2 Shaobin Tan, Blow-up solutions for mixed nonlinear Schrödinger equations. *Acta Mathematica Sinica, English Series*, **20**(2004), 115-124.

3 Baoxiang Wang and Chunyan Huang, Frequency-uniform decomposition method for the generalized BO, KdV and NLS equations. *Journal of Differential Equations*, **239**(2007), 213-250.

4 Kuoya, Imamura, Stability and bifurcation of periodic travelling waves in a derivative non-linear Schrödinger equation. *Hiroshima Mathematics Journal*, **40**(2010), 185-203.

5 Xiao Liu, Gideon Simpson and Catherine Sulem, Stability of solitary waves for a generalized derivative nonlinear Schrödinger equation. *Journal of Nonlinear Sciences*, **23**(2013), 557-583.

Citations of [15]: This paper has been cited ten times by others:

1 Daniel B. Dix, The dissipation of nonlinear dispersive waves: the case of asymptotically weak nonlinearity. *Communications in Partial Differential Equations*, **17**(1992), 1665-1693.

2 Huijiang Zhao, Decay estimates for the solutions of some multidimensional nonlinear evolution equations. *Communications in Partial Differential Equations*, **25**(2000), 377-422.

3 Nakao Hayashi, Elena I. Kaikina and Pavel I. Naumkin, Large-time behaviour of solutions to the dissipative nonlinear Schrödinger equation. *Proceedings of the Royal Society of Edinburgh, Section A*, **130**(2000), 1029-1043.

4 Jardel Morais Pereira, Global existence and decay of solutions of a coupled system of BBM-Burgers equations. *Rev Mat Complut*, **13**(2000), 423-443.

5 Jerry L. Bona and Laihan Luo, Asymptotic decomposition of nonlinear, dispersive wave equations with dissipation. *Physica D*, **152**(2001), 363-383.

6 Daniel B. Dix, Large-time behaviour of solutions of Burgers' equation. *Proceedings of the Royal Society of Edinburgh, Section A*, **132**(2002), 843-878.

7 Boqing Dong, Asymptotic behavior of the nonlinear parabolic equations. *Journal of Partial Differential Equations*, **17**(2004), 255-263.

8 Nakao Hayashi, Elena I. Kaikina and Pavel I. Naumkin, Asymptotics in the critical case for Whitham type equations. *Nonlinear Analysis*, **67**(2007), 2914-2933.

9 Celene Buriol, Eleni Bisognin, Vanilde Bisognin and Ruy Coimbra Charao, Asymptotic behavior for the Benjamin-Bona-Mahony equation in an exterior domain. *Differential and Integral Equations*, **25**(2012), 31-52.

10 Yuexing Yang, Algebraic L^2 decay for weak solutions of the nonlinear heat equations in whole space $\mathbb{R}^3$. *Abstract and Applied Analysis*, (2014), article ID 329474, 1-4.

Citations of [16]: This paper has been cited twenty-four times by others:

1 Maria E. Schonbek, Tomas P. Schonbek and Endre Suli, Large-time behavior of solutions to the magneto-hydrodynamics equations. *Mathematische Annalen*, **304**(1996), 717-756.

2 Takayoshi Ogawa, Shubha V. Rajopadhye and Maria E. Schonbek, Energy decay for a weak solution of the Navier-Stokes equation with slowly varying external forces. *Journal of Functional Analysis*, **144**(1997), 325-358.

3 Maria E. Schonbek, The Fourier splitting method. Advances in Geometric Analysis and Continuum Mechanics, (1995), 269-274. International Press. Cambridge Massachusetts.

4 Shubha V. Rajopadhye, Maria E. Schonbek and Michael Wiegner, Asymptotic decay to a generalized Boussinesq system.

Journal of Dynamics and Differential Equations, **11**(1999), 595-623.

5 Maria E. Schonbek and Tomas P Schonbek, On the boundedness and decay of moments of solutions to the Navier-Stokes equations. *Advances in Differential Equations*, **5**(2000), 861-898.

6 Cherif Amrouche, Vivette Girault, Maria E. Schonbek and Tomas P. Schonbek, Pointwise decay of solutions and of higher derivatives to Navier-Stokes equations. *SIAM Journal on Mathematical Analysis*, **31**(2000), 740-753.

7 Maria E. Schonbek, On decay of solutions to the Navier-Stokes equations. Applied Nonlinear Analysis, Editors: A Sequeira, H. Beirao da Vega, J H. Videman, Kluwer Academic/Plenum Publisher. 1999.

8 Songmu Zheng, Asymptotic behavior for strong solutions of the Navier-Stokes equations with external forces. *Nonlinear Analysis, Series A: Theory and Methods*, **45**(2001), 435-446.

9 Andras Balogh, Weijiu Liu and Miroslav Krstic, Stability enhancement by boundary control in 2-D channel flow. *IEEE Trans. Automat. Control*, **46**(2001), 1696-1711.

10 Boqing Dong, Asymptotic behavior of the nonlinear parabolic equations. *Journal of Partial Differential Equations*, **17**(2004), 255-263.

11 Boqing Dong and Yongsheng Li, Large time behavior to the system of incompressible non-Newtonian fluids in $\mathbb{R}^2$. *Journal of Mathematical Analysis and Applications*, **298**(2004), 667-676.

12 Boqing Dong, Decay of solutions to equations modelling incompressible bipolar non-Newtonian fluids. *Electronic Journal of Differential Equations*, (2005), article ID: 125, 13 pages.

13 Boqing Dong and Yongsheng Li, Sharp rate of decay for solutions to non-Newtonian fluids in $\mathbb{R}^2$. *Acta Mathematica Sinica*, **48**(2005), 1065-1070.

14 Maria E. Schonbek, Asymptotic behavior of solutions to viscous conservation laws with slowly varying external forces. *Mathematischen Annalen*, **336**(2006), 505-538.

15 Cesar J. Niche and Maria E. Schonbek, Decay of weak solutions to the 2D dissipative quasi-geostrophic equation. *Communications in Mathematical Physics*, **276**(2007), 93-115.

16 Clayton Bjorland and Maria E. Schonbek, On questions of decay and existence for the viscous Camassa-Holm equations. *Ann. Inst. H. Poincare Anal. Non Lineaire*, **25**(2008), 907-936.

17 Clayton Bjorland and Cesar J. Niche, On the decay of infinite energy solutions to the Navier-Stokes equations in the plane. *Physica D*, **240**(2011), 670-674.

18 Cesar J. Niche and Gabriela Planas, Existence and decay of solutions in full space to Navier-Stokes equations with delays. *Nonlinear Analysis*, **74**(2011), 244-256.

19 Yan Jia, Xingwei Zhang and Boqing Dong, The asymptotic behavior of solutions to three-dimensional Navier-Stokes equations with nonlinear damping. *Nonlinear Analysis, Real World Applications*, **12**(2011), 1736-1747.

20 Boqing Dong and Juan Song, Global regularity and asymptotic behavior of modified Navier-Stokes equations with fractional

dissipation. *Discrete and Continuous Dynamical Systems*, **32**(2012), 57-79.

21 Cesar J. Niche and Gabriela Planas, Existence and decay of solutions to the dissipative quasi-geostrophic equation with delays. *Nonlinear Analysis*, **75**(2012), 3936-3950.

22 Caidi Zhao, Yunyun Liang and Min Zhao, Upper and lower bounds of time decay rate of solutions to a class of incompressible third grade fluid equations. *Nonlinear Analysis, Real World Applications*, **15**(2014), 229-238.

23 Xiaomei Hu, Error estimates for solutions of the semilinear parabolic equation in whole space. *Abstract and Applied Analysis*, (2014), article ID 501280, 1-5.

24 Cesar J. Niche and Rafael Orive-Illera, Decay of solutions to a porous media equation with fractional diffusion. *Advances in Differential Equations*, **19**(2014), 133-160.

Citations of [17]: This paper has been cited many times by others:

1 Ming Mei, L^q-decay rates of solutions for Benjamin-Bona-Mahony-Burgers equations. *Journal of Differential Equations*, **158**(1999), 314-340.

2 Ming Mei and Christian Schmeiser, Asymptotic profiles of solutions for the BBM-Burgers equation. *Funkcial. Ekvac.* **44**(2001), 151-170.

3 Huijiang Zhao, Decay estimates for the solutions of some multidimensional nonlinear evolution equations. *Communications in Partial Differential Equations*, **25**(2000), 377-422.

4 Huijiang Zhao, Optimal temporal decay estimates for the solution to the multidimensional generalized BBM-Burgers equations with dissipative term. *Applicable Analysis*, **75**(2000), 85-105.

5 Raul Prado and Enrique Zuazua, Asymptotic expansion for the generalized Benjamin-Bona-Mahony-Burgers equation. *Differential and Integral Equations*, **15**(2002), 1409-1434.

6 Elena I. Kaikina, Pavel I. Naumkin and I. A. Shishmarev, The Cauchy problem for a Sobolev-type equation with a power nonlinearity. *Izv. Ross. Akad. Nauk Ser. Mat.*, **69**(2005), 61-114.

7 Maxim Olegovich Korpusov and A. G. Sveshnikov, On the blow-up of a solution of a Sobolev-type equation with a nonlocal source. *Sibirsk. Mat. Zh.* **46**(2005), 567-578.

8 Maxim Olegovich Korpusov and A. G. Sveshnikov, On the blow-up of solutions of a class of strongly nonlinear dissipative wave equations of Sobolev type with sources. *Izv. Ross. Akad. Nauk Ser. Mat.* **69**(2005), 89-128.

9 Milena Stanislavova, Atanas Stefanov and Bixiang Wang, Asymptotic smoothing and attractors for the generalized Benjamin-Bona-Mahony equation on $\mathbb{R}^3$. *Journal of Differential Equations*, **219**(2005), 451-483.

10 Maxim Olegovich Korpusov and A. G. Sveshnikov, On the "blow-up" of solutions of nonlinear wave equations of Sobolev type with cubic sources. *Mat. Zametki*, **78**(2005), 559-578.

11 Jerry L. Bona and Laihan Luo, Asymptotic decomposition of nonlinear, dispersive wave equations with dissipation. *Physica D*, **152**(2001), 363-383.

12 Maxim Olegovich Korpusov and A. G. Sveshnikov, Blow-up of solutions of Sobolev-type nonlinear equations with cubic sources. *Differ. Uravn*, **42**(2006), 404-415.

13 Elena I. Kaikina, Pavel I. Naumkin and I. A. Shishmarev, Asymptotics for a Sobolev-type equation with a critical nonlinearity. *Differ. Uravn*, **43**(2007), 655-668.

14 Hui Yin, Huijiang Zhao and Jongsung Kim, Convergence rates of solutions toward boundary layer solutions for generalized Benjamin-Bona-Mahony-Burgers equations in the half-space. *Journal of Differential Equations*, **245**(2008), 3144-3216.

15 Guowang Chen and Hongxia Xue, Global existence of solution of Cauchy problem for nonlinear pseudo-paraboloic equation. *Journal of Differential Equations*, **245**(2008), 2705-2722.

16 Shaomei Fang and Boling Guo, The decay rates of solutions of generalized Benjamin-Bona-Mahony-Burgers equations in multidimensions. *Nonlinear Analysis*, **69**(2008), 2230-2235.

17 Maxim Olegovich Korpusov and A G. Sveshnikov, Blow-up of solutions of strongly nonlinear equations of pseudoparabolic type. *Journal of Mathematical Science*, **148**(2008), 1-142.

18 Hui Yin, Shuyue Chen and Jing Jin, Convergence rate to traveling waves for generalized Benjamin-Bona-Mahony-Burgers equations. *Zeitschrift fur angewandte Mathematik und Physik (ZAMP)*, **59**(2008), 969-1001.

19 M Meyvaci, Blow up of solutions of pseudoparabolic equations. *Journal of Mathematical Analysis and Applications*, **352**(2009), 629-633.

20 Hui Yin and Jiayi Hu, Exponential decay rate of solutions toward traveling waves for the Cauchy problem of generalized Benjamin-Bona-Mahony-Burgers equations. *Nonlinear Analysis*, **73**(2010), 1729-1738.

21 Jerry L. Bona and Laihan Luo, Large-time asymptotics of the generalized Benjamin-Ono-Burgers equation. *Discrete and Continuous Dynamical Systems, Series S*, **4**(2011), 15-50.

22 Celene Buriol, Eleni Bisognin, Vanilde Bisognin and Ruy Coimbra Charao, Asymptotic behavior for the Benjamin-Bona-Mahony equation in an exterior domain. *Differential and Integral Equations*, **25**(2012), 31-52.

23 Tinggang Zhao, Xiaoxian Zhang, Jinxia Huo, Wanghui Su, Yongli Liu and Yujiang Wu, Optimal error estimate of Chebyshev-Legendre spectral method for the generalized Benjamin-Bona-Mahony-Burgers equations. *Abstract and Applied Analysis*, (2012), article ID: 106343, 22 pages.

24 Yan Liu, Weisheng Jiang and Falun Huang, Asymptotic behaviour of solutions to some pseudoparabolic equations. *Applied Mathematics Letter*, **25**(2012), 111-114.

25 Qinghua Xiao and Huijiang Zhao, Nonlinear stability of generalized Benjamin-Bona-Mahony-Burgers shock profiles in several dimensions. *Journal of Mathematical Analysis and Applications*, **406**(2013), 165-187.

26 Qinghua Xiao and Huijiang Zhao, Nonlinear stability of planar stationary waves for generalized Benjamin-Bona-Mahony-Burgers equations in the half space. *Journal of Hyperbolic Differential Equations*, **10**(2013), 283-333.

27 Hongmei Xu and Yan Liang, Global existence and pointwise estimates of solutions to generalized Benjamin-Bona-Mahony equations in multi-dimensions. *Chinese Annals of Mathematics*, **35B**(2014), 659-668.

28 Hongmei Xu and Guyu Xin, Global existence and decay rates of solutions of generalized Benjamin-Bona-Mahony equations in multiple dimensions. *Acta Math Vietnam*, **39**(2014), 121-131.

29 Yuzhu Wang and Keyan Wang, Large time behavior of solutions to the nonlinear pseudo-parabolic equation. *Journal of Mathematical Analysis and Applications*, **417**(2014), 272-292.

30 Belkacem Said-Houari, Improved decay rates for solutions for a multi-dimensional generalized Benjamin-Bona-Mahony equation. *Asymptotic Analysis*, **86**(2014), 155-164.

31 Yang Cao and Yuanyuan Nie, Blow-up of solutions of the nonlinear Sobolev equation. *Applied Mathematics Letter*, **28**(2014), 1-6.

32

Citations of [18]: This paper has been cited three time by others:

1 Shijin Ding, Changxing Miao, Shaobin Tan and Baoxiang Wang, On the occasion of the seventieth birthday of Professor Guo Boling. *Journal of Partial Differential Equations*, **20**(2007), 1-10.

2 Xueke Pu and Boling Guo, Existence and decay of solutions to the two-dimensional fractional quasigeostropic equation. *Journal of Mathematical Physics*, **51**(2010), article ID: 083101, 15 pages.

3 Pigong Han and Cheng He, Decay properties of solutions to the incompressible magnetohydrodynamics equations in a half space. *Mathematical Methods in Applied Sciences*, **35**(2012), 1472-1488.

Citations of [19]: This paper has been cited once by others:

1 Huijiang Zhao, Decay estimates for the solutions of some multidimensional nonlinear evolution equations. *Communications in Partial Differential Equations*, **25**(2000), 377-422.

Citations of [21]: This paper has been cited once by others:

1 Yinghua Li and Yang Cao, A pseudo-parabolic type equation with weakly nonlinear sources. *Journal of Partial Differential Equations*, **26**(2013), 363-372.

Citations of [32]: This paper has been cited four time by others:

1 Radu C. Cascaval, Local and global well-posedness for a class of nonlinear dispersive equations. *Advances in Differential Equations*, **9**(2004), 85-132.

2 Masanori Otani, Bilinear estimates with applications to the generalized Benjamin-Ono-Burgers equations. *Differential and Integral Equations*, **18**(2005), 1397-1426.

3 Stephane Vento, Well-posedness and ill-posedness results for dissipative Benjamin-Ono equations. *Osaka Journal of Mathematics*, **48**(2011), 933-958.

4 Zihua Guo, Lizhong Peng, Baoxiang Wang and Yuzhao Wang, Uniform well-posedness and inviscid limit for the Benjamin-Ono-Burgers equation. *Advances of Mathematics*, **228**(2011), 647-677.

Citations of [33]: This paper has been cited once by others:

1 Todd Kapitula, Nathan Kutz and Björn Sandstede, The Evans function for nonlocal equations. *Indiana University Mathematics Journal*, **53**(2004), 1095-1126.

Citations of [35]

1 Yuzo Hosono, Traveling waves for a diffusive Lotka-Volterra competition model. I. Singular perturbations. *Discrete and Continuous Dynamical Systems, Series B*, **3**(2003), 79-95.

2 Shengfan Zhou and Linshan Wang, Kernel sections for damped non-autonomous wave equations with critical exponents. *Discrete and Continuous Dynamical Systems*, **9**(2003), 399-412.

3 Björn Schmalfuss, Attractors for nonautonomous and random dynamical systems perturbed by impulses. *Discrete and Continuous Dynamical Systems*, **9**(2003), 727-744.

4 Shyan-Shiou Chen and Chih-Wen Shih, Asymptotic behaviors in a transiently chaotic neural network. *Discrete and Continuous Dynamical Systems*, **10**(2004), 805-826.

5 Suyun Wang, Desheng Li and Chengkui Zhong, On the dynamics of a class of nonclassical parabolic equations. *Journal of Mathematical Analysis and Applications*, **317**(2006), 565-582.

Citations of [37]: This paper has been cited time by twenty-eight times by others:

1 Remus Osan, Jonathan Rubin and G. Bard Ermentrout, Regular traveling waves in a one-dimensional network of theta neurons. *SIAM Journal on Applied Mathematics*, **62**(2002), 1197-1221.

2 Axel Hutt, Effects of nonlocal feedback on traveling fronts in neural fields subject to transmission delay. *Physical Review E*, **70**(2004), 052902.

3 Paul C. Bressloff and Stefanos E. Folias, Front bifurcations in an excitatory neural network. *SIAM Journal on Applied Mathematics*, **65**(2004), 131-151.

4 Stephen Coombes and M. R. Owen, Evans functions for integral neural field equations with Heaviside firing rate function. *SIAM Journal on Applied Dynamical Systems*, **3**(2004), 574-600.

5 Todd Kapitula, Nathan Kutz and Björn Sandstede, The Evans function for nonlocal equations, *Indiana University Mathematics Journal*, **53**(2004), 1095-1126.

6 Jonathan E. Rubin, A nonlocal eigenvalue problem for the stability of a traveling wave in a neuronal medium. *Discrete and Continuous Dynamical Systems, Series A,* **10**(2004), 925-940.

7 Jonathan E. Rubin and William C. Troy, Sustained spatial patterns of activity in neuronal populations without recurrent excitation. *SIAM Journal on Applied Mathematics*, **64**(2004), 1609-1635.

8 Yixin Guo and Carson C. Chow, Existence and stability of standing pulses in neural networks: I. Existence. II Stability. *SIAM Journal on Applied Dynamical Systems*, **4**(2005), I: 217-248, II 249-281.

9 Paul C. Bressloff, Weakly interacting pulses in synaptically coupled neural media. *SIAM Journal on Applied Mathematics*,

66(2005), 57-81.

10 Stefanos E. Folias and Paul C. Bressloff, Stimulus-locked traveling waves and breathers in an excitatory neural network. *SIAM Journal on Applied Mathematics*, **65**(2005), 2067-2092.

11 Stephen Coombes, Waves, bumps, and patterns in neural field theories. *Biological Cybernetics*, **93**(2005), 91-108.

12 David J. Pinto, Russell K. Jackson and C. Eugene Wayne, Existence and stability of traveling pulses in a continuous neuronal network. *SIAM Journal on Applied Dynamical Systems*, **4**(2005), 954-984.

13 A Tonnelier, Stabilization of pulse waves through inhibition in a feedforward neural network. *Physica D*, **210**(2005), 118-137.

14 Fatihcan M. Atay and Axel Hutt, Neural fields with distributed transmission speeds and long-range feedback delays. *SIAM Journal on Applied Dynamical Systems*, **5**(2006), 670-698.

15 Sittipong Ruktamatakul, Jonathan Bell and Yongwimon Lenbury, Wave-front solution behaviour for continuous neutral networks with lateral inhibition. *IMA Journal of Applied Mathematics*, **71**(2006), 544-564.

16 William C. Troy and Vladimir Shusterman, Patterns and features of families of traveling waves in large-scale neuronal networks. *SIAM Journal on Applied Dynamical Systems*, **6**(2007), 263-292.

17 Björn Sandstede, Evans functions and nonlinear stability of travelling waves in neuronal network models. *International*

Journal of Bifurcation and Chaos in Applied Sciences and Engineering, **17**(2007), 2693-2704.

18 William C. Troy, Traveling waves and synchrony in an excitable large-scale neuronal network with asymmetric connections. *SIAM Journal on Applied Dynamical Systems*, **7**(2008), 1247-1282.

19 Zachary P. Kilpatrick, Stefanos E. Folias and Paul C. Bressloff, Traveling pulses and wave propagation failure in inhomogeneous neural media. *SIAM Journal on Applied Dynamical Systems*, **7**(2008), 161-185.

20 Zachary P. Kilpatrick, Stefanos E. Folias and Paul C. Bressloff, Traveling pulses and wave propagation failure in inhomogeneous neural media. *SIAM Journal on Applied Dynamical Systems*, **7**(2008), 161-185.

21 H. Schmidt-Martens, Axel Hutt and L Schimansky-Geier, Wave fronts in inhomogeneous neural field models. *Physica D*, **238**(2009), 1101-1112.

22 David J. Pinto, William C. Troy and Timothy Kneezel, Asymmetric activity waves in synaptic cortical systems. *SIAM Journal on Applied Dynamical Systems*, **8**(2009), 1218-1233.

23 Paul C. Bressloff, Stochastic neural field theory and the system-size expansion. *SIAM Journal on Applied Mathematics*, **70**(2009), 1488-1521.

24 Stefanos E. Folias, Nonlinear analysis of breathing pulses in a synaptically coupled neural network. *SIAM Journal on Applied Dynamical Systems*, **10**(2011), 744-787.

25 P Grindrod and D A. Pinotsis, On the spectra of certain integro-differential-delay problems with applications in neurodynamics. *Physica D*, **240**(2011), 13-20.

26 Yixin Guo, Existence and stability of traveling fronts in a lateral inhibition neural network. *SIAM Journal on Applied Dynamical Systems*, **11**(2012), 1543-1582.

27 Paul C. Bressloff and Matthew Webber, Front propagation in stochastic neural fields. *SIAM Journal on Applied Dynamical Systems*, **11**(2012), 708-740.

28 Gregory Faye, Existence and stability of traveling pulses in a neural field equation with synaptic depression. *SIAM Journal on Applied Dynamical Systems*, **12**(2013), 2032-2067.

29 Samuel Carroll and Paul C. Bressloff, Binocular rivalry waves in a directionally selective neural field model. *Physica D*, **285**(2014), 8-17.

Citations of [38]: This paper has been cited many times by others:

1 Stephen Coombes, Gabriel J. Lord, and M. R. Owen, Waves and bumps in neuronal networks with axo-dendritic synaptic interactions. *Physica D*, **178**(2003), 219-241.

2 Stefanos E. Folias and Paul C. Bressloff, Stimulus-locked traveling waves and breathers in an excitatory neural network. *SIAM Journal on Applied Mathematics*, **65**(2005), 2067-2092.

3 Sittipong Ruktamatakul, Jonathan Bell and Yongwimon Lenbury, Wave-front solution behaviour for continuous neutral networks with lateral inhibition. *IMA Journal of Applied Mathematics*, **71**(2006), 544-564.

4 Emmanuel Chasseigne, Manuela Chaves and Julio D. Rossi, Asymptotic behavior for nonlocal diffusion equations. *J. Math. Pures Appl.* **86**(2006), 271-291.

5 Ademir F. Pazoto and Julio D. Rossi, Asymptotic behaviour for a semilinear nonlocal equation. *Asymptotic Analysis*, **52**(2007), 143-155.

6 Liviu I. Ignat and Julio D. Rossi, A nonlocal convection-diffusion equation. *Journal of Functional Analysis*, **251**(2007), 399-437.

7 Carmen Cortazar, Manuel Elgueta, Julio D. Rossi and Noemi Wolanski, How to approximate the heat equation with Neumann boundary conditions by nonlocal diffusion problems. *Archive for Rational Mechanics and Analysis*, **187**(2008), 137-156.

8 Fuensanta Andreu, Jose M. Mazon, Julio D. Rossi and Julian Toledo, The Neumann problem for nonlocal nonlinear diffusion equations. *Journal of Evolution Equations*, **8**(2008), 189-215.

9 Zachary P. Kilpatrick, Stefanos E. Folias, and Paul C. Bressloff, Traveling pulses and wave propagation failure in inhomogeneous neural media. *SIAM Journal on Applied Dynamical Systems*, **7**(2008), 161-185.

11 F Andreu, J M. Mazon, J D. Rossi and J A. Toledo, A nonlocal p-Laplacian evolution equation with Neumann boundary conditions. *J Math Pures Appl*, **90**(2008), 201-227.

12 Diabate Nabongo, Theodore K. Boni, Blow-up time for a nonlocal diffusion problem with Dirichlet boundary conditions. *Communications in Analysis and Geometry*, **16**(2008), 865-882.

13 Liviu I. Ignat and Julio D. Rossi, Refined asymptotic expansions for nonlocal diffusion equations. *Journal of Evolution Equations*, **8**(2008), 617-629.

14 Jorge Garcia-Melian and Julio D. Rossi, On the principal eigenvalue of some nonlocal diffusion problems. *Journal of Differential Equations*, **246**(2009), 21-38.

15 Carmen Cortazar, Manuel Elgueta, Jorge Garcia-Melian and Salome Martinez, Existence and asymptotic behavior of solutions to some inhomogeneous nonlocal diffusion problems. *SIAM Journal on Mathematical Analysis*, **41**(2009), 2136-2164.

16 Jorge Garcia-Melian and Julio D. Rossi, A logistic equation with refuge and nonlocal diffusion. *Communications on Pure and Applied Analysis*, **8**(2009), 2037-2053.

17 Carmen Cortazar, Manuel Elgueta and Julio D. Rossi, Nonlocal diffusion problems that approximate the heat equation with Dirichlet boundary conditions. *Israel Journal of Mathematics*, **170**(2009), 53-60.

18 Julio D. Rossi and Carola-Bibiane Schonlieb, Nonlocal higher order evolution equations. *Applicable Analysis*, **89**(2010), 949-960.

19 Shuxia Pan, Wantong Li and Li Guo, Existence and stability of traveling wavefronts in a nonlocal diffusion equation with delay. *Nonlinear Analysis*, **72**(2010), 3150-3158.

20 Liviu I. Ignat and Julio D. Rossi, Asymptotic expansions for nonlocal diffusion equations in L^q-norms for $1 \leq q \leq 2$. *Jour-*

nal of Mathematical Analysis and Applications, **262**(2010), 190-199.

21 Carmen Cortazar, Manuel Elgueta, Jorge Garcia-Melian and Salome Martinez, Stationary sign changing solutions for an inhomogeneous nonlocal problem. *Indiana University Mathematics Journal*, **60**(2011), 209-232.

22 Joana Terra and Noemi Wolanski, Large time behavior for a nonlocal diffusion equation with absorption and bounded initial data. *Discrete and Continuous Dynamical Systems*, **31**(2011), 581-605.

23 Joana Terra and Noemi Wolanski, Asymptotic behavior for a nonlocal diffusion equation with absorption and nonintegrable initial data. The supercritical case. *Proceedings of the American Mathematical Society*, **139**(2011), 1421-1432.

24 F. Andreu, J M. Mazon, J D. Rossi and J Toledo, Local and nonlocal weighted p-Laplacian evolution equations with Neumann boundary conditions. *Publ Mat*, **55**(2011), 27-66.

25 Mayte Perez-Llanos and Julio D. Rossi, Numerical approximations for a nonlocal evolution equation. *SIAM Journal on Numerical Analysis*, **49**(2011), 2103-2123.

26 Fu Zheng, Guangtian Zhu and Chao Gao, Well-posedness and stability of the repairable system with N failure modes and one standby unit. *Journal of Mathematical Analysis and Applications*, **375**(2011), 174-184.

27 Yixin Guo, Existence and stability of traveling fronts in a lateral inhibition neural network. *SIAM Journal on Applied Dynamical Systems*, **11**(2012), 1543-1582.

28 Feiying Yang, Yan Li, Wantong Li and Zhicheng Wang, Traveling waves in a nonlocal dispersal Kermack-McKendrick epidemic model. *Discrete and Continuous Dynamical Systems, Series B*, **18**(2013), 1969-1993.

29 Xiaojie Hou, Biao Wang and and Zhengce Zhang, Mutual inclusion in a nonlocal competitive Lotka Volterra system. *Japan Journal of Industrial and Applied Mathematics*, **31**(2014), 87-110.

30 Jiebao Sun, Jing Li and Qiang Liu, Cauchy problem of a nonlocal p-Laplacian evolution equation with nonlocal convection. *Nonlinear Analysis*, **95**(2014), 691-702.

31 Wenlong Wang and Guanghua He, The spectral analysis of k out of N, G redundant system. *Journal of Difference Equations and Applications*, **19**(2013), 1580-1600.

32 Feiying Yang, Runlong Qiao, Wantong Li, Traveling waves for a nonlocal dispersal epidemic model. *Annals of Differential Equations*, **30**(2014), 222-235.

33 Wantong Li and Feiying Yang, Traveling waves for a nonlocal dispersal SIR model with standard incidence. *Journal of Integral Equations and Application*, **26**(2014), 243-273. J. Zhou, J Xu, J Wei, H Xu, Existence and non-existence of traveling wave solutions for a nonlocal dispersal SIR epidemic model with nonlinear incidence rate. *Nonlinear Analysis: Real World Applications*, **41**(2018), 204-231.

Citations of [41]: This paper has been cited two time by others:

1 Boqing Dong, Asymptotic behavior of the nonlinear parabolic equations. *Journal of Partial Differential Equations*, **17**(2004), 255-263.

2 Yuexing Yang, Algebraic L^2 decay for weak solutions of the nonlinear heat equation in whole space $\mathbb{R}^3$. *Abstract and Applied Analysis*, (2014), article ID 329474, 1-4.

Citations of [42]: This paper has been cited two time by others:

1 Yan Jia, Xingwei Zhang and Boqing Dong, The asymptotic behavior of solutions to three-dimensional Navier-Stokes equations with nonlinear damping. *Nonlinear Analysis, Real World Applications*, **12**(2011), 1736-1747.

2 Caidi Zhao, Yunyun Liang and Min Zhao, Upper and lower bounds of time decay rate of solutions to a class of incompressible third grade fluid equations. *Nonlinear Analysis, Real World Applications*, **15**(2014), 229-238.

Citations of [44]: This paper has been cited twice by others:

1 Manuel Delgado and Antonio Suarez, Study of an elliptic system arising from angiogenesis with chemotaxis and flux at the boundary. *Journal of Differential Equations*, **244**(2008), 3119-3150.

2 Tianran Zhang and Qingming Gou, Minimal wave speed of bacterial colony model with saturated functional response. *Abstract and Applied Analysis*, (2014), article ID 510671, 1-9.

Citations of [45]: This paper has been cited twice by others:

1 Paul C. Bressloff and Zachary P. Kilpatrick, Two-dimensional bumps in piecewise smooth neuroal fields with synaptic depression. *SIAM Journal on Applied Mathematics*, **71**(2011), 379-408.

2 Gregory Faye, Existence and stability of traveling pulses in a neural field equation with synaptic depression. *SIAM Journal on Applied Dynamical Systems*, **12**(2013), 2032-2067.

Citations of [47]: This paper has been cited two time by others:

1 Guangying Lu and Mingxin Wang, Traveling waves of some integral differential equations arising from neuronal networks with oscillatory kernels. *Journal of Mathematical Analysis and Applications*, **370**(2010), 82-100.

2 Lijun Zhang, Existence and unique of wave fronts in neuronal network with nonlocal post-synaptic axonal and delayed nonlocal feedback connections. *Advances in Differential Equations*, **243**(2013), 1-15.

Citations of [48]: This paper has been cited seven time by others:

1 H. Schmidt-Martens, Axel Hutt and L Schimansky-Geier, Wave fronts in inhomogeneous neural field models. *Physica D*, **238**(2009), 1101-1112.

2 David J. Pinto, William Troy and Timothy Kneezel, Asymmetric activity waves in synaptic cortical systems. *SIAM Journal on Applied Dynamical Systems*, **8**(2009), 1218-1233.

3 Felicia Maria G. Magpantay and Xingfu Zou, Wave fronts in neuronal fields with nonlocal post-synaptic axonal connections and delayed nonlocal feedback connections. *Mathematical Biosciences and Engineering*, **7**(2010), 421-442.

4 Guangying Lu and Mingxin Wang, Traveling waves of some integral differential equations arising from neuronal networks with oscillatory kernels. *Journal of Mathematical Analysis and Applications*, **370**(2010), 82-100.

5 Lianzhong Li, Maoan Han and Yuanyuan Liu, Existence and uniqueness of traveling wave front of a nonlinear singularly perturbed system of reaction-diffusion equations with a Heaviside step function. *Journal of Mathematical Analysis and Applications*, **410**(2014), 202-212.

6 Gregory Faye, Existence and stability of traveling pulses in a neural field equation with synaptic depression. *SIAM Journal on Applied Dynamical Systems*, **12**(2013), 2032-2067.

7 Lijun Zhang, Existence and unique of wave fronts in neuronal network with nonlocal post-synaptic axonal and delayed nonlocal feedback connections. *Advances in Differential Equations*, **243**(2013), 1-15.

Citations of [50]: This paper has been cited three times by others:

1 Yan Jia, Xingwei Zhang and Boqing Dong, The asymptotic behavior of solutions to three-dimensional Navier-Stokes equations with nonlinear damping. *Nonlinear Analysis, Real World Applications*, **12**(2011), 1736-1747.

2 Boqing Dong and Juan Song, Global regularity and asymptotic behavior of modified Navier-Stokes equations with fractional dissipation. *Discrete and Continuous Dynamical Systems*, **32**(2012), 57-79.

3 Junbai Ren, Large time behavior for weak solutions of the 3D globally modified Navier-Stokes equations. *Abstract and Applied Analysis*, (2014), article ID 879780, 1-5.

Citations of [52]: This paper has been cited once by others:

1 Shuxia Pan and Guo Lin, Invasion traveling wave solutions of a competitive system with dispersal. *Boundary Value Problems*, **2012**(2012), article ID , pages.

Citations of [53]: This paper has been cited three time by others:

1 G. G. Rigatos, Estimation of wave-type dynamics in neurons' membrane with the use of the Derivative-free nonlinear Kalman Filter, *Neurocomputing*, **131**(2014), 286-299.

2 Lianzhong Li, Maoan Han and Yuanyuan Liu, Existence and uniqueness of traveling wave front of a nonlinear singularly perturbed system of reaction-diffusion equations with a Heaviside step function. *Journal of Mathematical Analysis and Applications*, **410**(2014), 202-212.

3 Lijun Zhang, Existence and unique of wave fronts in neuronal network with nonlocal post-synaptic axonal and delayed nonlocal feedback connections. *Advances in Differential Equations*, **243**(2013), 1-15.

Citations of [54]: This paper has been cited once by others:

1 Lianzhong Li, Maoan Han and Yuanyuan Liu, Existence and uniqueness of traveling wave front of a nonlinear singularly perturbed system of reaction-diffusion equations with a Heaviside step function. *Journal of Mathematical Analysis and Applications*, **410**(2014), 202-212.

List of Impact Factors of Some Mathematics Journals

The impact factors for some mathematics journals can not be found on-line. The impact factors for each journal may be different from different sources. For example, the Introduction of the SIAM journals says that the impact factor of SIAM Journal on Applied Dynamical Systems is 2.159, but the Web of Sciences http://isiknowledge.com/wos says it is 1.703. Please note that, compared with biology or other natural sciences, the overall impact factors for mathematics journals are very low.

1	Acta Mathematicae Applicatae Sinica, English Series:	0.534
2	Calculus of Variation and Partial Differential Equations:	0.992
3	Communications on Pure and Applied Mathematics	2.031
4	Communications in Partial Differential Equations	1.094
5	Discrete and Continuous Dynamical Systems, Series A:	1.087
6	Differential and Integral Equations:	0.641
7	Journal of American Mathematical Society	2.552
8	Journal of Differential Equations:	1.166
9	Journal of Functional Analysis	0.866
10	Journal of Partial Differential Equations:	0.539
11	Journal of Dynamics and Differential equations:	0.435
12	Journal of Mathematical Analysis and Applications	0.758
13	Mathematische Zeitschrift:	0.570

14 SIAM Journal on Applied Dynamical Systems: 1.703

15 Nonlinear Analysis, Theory and Methods: 0.677

16 Nonlinear Analysis, Real World Applications: 1.194

17 Chinese Annals of Mathematics 0.470

Overall, I have many significant papers published in first-class, worldwidely well known, high rank mathematics journals. I also have published a lot of papers in other slightly less significant but popular journals (e.g. Acta Mathematicae Applicatae Sinica, Journal of Partial Differential Equations). This undoubtedly enhance the reputation of Lehigh University.

Brief background information on the journals where my papers have been published

Acta Mathematicae Applicatae Sinica publishes high quality research papers from all branches of applied mathematics, and particularly welcomes those from partial differential equations, computational mathematics, applied probability, mathematical finance, statistics, dynamical systems, optimization and management science.

Bulletin of the Institute of Mathematics publishes original papers in all areas of mathematics.

Chinese Annals of Mathematics is probably the best mathematics journal in China. It publishes research papers in all areas of mathematics. Every year four issues are published.

Differential and Integral Equations will publish carefully selected research papers on mathematical aspects of differential and integral equations and on applications of the mathematical theory to issues arising in the sciences and in engineering.

Discrete and Continuous Dynamical Systems publishes peer-reviewed high quality original papers and invited expository papers on the theory and methods of analysis, differential equations and dynamical systems. This journal is committed to being the record for important new results in its field, and will maintain the highest standards of innovation and quality. To be published in this journal, an original paper must be correct, new, nontrivial and of interest to a substantial number of readers.

Dynamics in Partial Differential Equations publishes novel results in the areas of partial differential equations and dynamical systems in general, and priority will be given to dynamical system theory or dynamical aspects of partial differential equations.

Journal of Differential Equations is concerned with the theory and the application of differential equations. The articles published are addressed not only to mathematicians but also to those engineers, physicists, and other scientists for whom differential equations are valuable research tools.

Journal of Dynamics and Differential Equations serves as an international forum for the publication of high-quality, peer-reviewed original papers in the field of mathematics, biology, engineering, physics, and other areas of science. The dynamical issues treated in the journal cover all the classical topics, including attractors, bifurcation theory, connection theory, dichotomies, stability theory, and transversality, as well as topics in new and emerging areas of the field.

Journal of Partial Differential Equations publishes research papers and short communications of high quality in theory and applications including numerical analysis of partial differential equations. The main interest of the Journal is to encourage the research of partial differential equations, and to promote national and international exchange between partial differential equations, engineering, physics and different fields of mathematics.

Mathematische Zeitschrift is devoted to pure and applied mathematics; papers on theoretical physics and astronomy may be accepted if they present interesting mathematical results.

Nonlinear Analysis is concerned, as the title stresses, with three major activities. It is a multidisciplinary journal, which has applications in main academic subjects as well as in industry and government. The journal publishes important research and expository papers and preliminary communications devoted to solving nonlinear problems in all areas of theory, methods and applications of nonlinear analysis. Clearly, papers that tend to integrate and interrelate theory, methods and applications within the scope of the journal will be particularly welcomed.

Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences: The criteria for selection are scientific excellence, originality and interest across disciplines within mathematical, physical and engineering sciences.

SIAM Journal on Applied Dynamical Systems publishes research articles on the mathematical analysis and modeling of dynamical systems and its application to the physical, engineering, and life sciences. It is published in electronic format only.

The Teaching Statements

First of all, let me provide the information from Course Evaluation Summary Report in the last years. I will only highlight the first two items. In each block below, the first line is the year and the semester, the second line and the third line are the courses I taught and the scores from (1) Overall, the instructor's teaching was effective) and (2) Overall the quality of the course was good. The total score for each item is 5.00.

$$\begin{pmatrix} 2009 & Sp \\ & \\ & \end{pmatrix} \quad \begin{pmatrix} 2009 & Su \\ & \\ & \end{pmatrix} \quad \begin{pmatrix} 2009 & Au \\ & \\ & \end{pmatrix}$$

$$\begin{pmatrix} 2010 & Sp \\ & \\ & \end{pmatrix} \quad \begin{pmatrix} 2010 Su \\ \\ \end{pmatrix} \quad \begin{pmatrix} 2010 & Au & \\ M205 & 4.21 & 4.19 \\ M21 & 4.21 & 4.11 \end{pmatrix}$$

$$\begin{pmatrix} 2011 & Sp & \\ M205 & 4.49 & 4.41 \\ M320 & 4.17 & 4.17 \end{pmatrix} \quad \begin{pmatrix} 2011 & Su & \\ M22 & 4.87 & 4.87 \\ M52 & 4.67 & 4.67 \end{pmatrix} \quad \begin{pmatrix} 2011 & Au & \\ M301 & 4.33 & 3.93 \\ M205 & 3.73 & 3.63 \end{pmatrix}$$

$$\begin{pmatrix} 2012 & Sp & \\ M23 & 4.19 & 4.34 \\ M320 & 4.27 & 4.27 \end{pmatrix} \quad \begin{pmatrix} 2012 & Su & \\ M22 & 4.38 & 4.54 \\ M52 & 4.71 & 4.71 \end{pmatrix} \quad \begin{pmatrix} 2012 & Au & \\ M405 & 4.60 & 4.40 \\ M21 & 4.15 & 3.66 \end{pmatrix}$$

$$\begin{pmatrix} 2013 & Sp \\ Sabbatical & leave \\ Sabbatical & leave \end{pmatrix} \quad \begin{pmatrix} 2013 & Su & \\ M22 & 4.10 & 4.14 \\ M52 & 4.00 & 4.29 \end{pmatrix} \quad \begin{pmatrix} 2013 & Au & \\ M23 & 4.25 & 4.24 \\ M23 & 4.10 & 4.20 \end{pmatrix}$$

$$\begin{pmatrix} 2014 & Sp & \\ M21 & 4.35 & 4.43 \\ M320 & 4.33 & 4.25 \end{pmatrix} \quad \begin{pmatrix} 2014 & Su & \\ M22 & 4.43 & 4.55 \\ M52 & 4.60 & 4.80 \end{pmatrix} \quad \begin{pmatrix} 2014 & Au \\ M205 & \\ M205 & \end{pmatrix}$$

(A) Chronological list of courses taught with number

of credits per course, and the number of grades assigned in each course.

2002: Autumn Semester. Mathematics 75 (two credits) and Mathematics 205 (three credits).
Six sections in Math 75, 91 grades assigned, the average grade is “B+” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.
One section in Math 205, 35 grades assigned, the average grade is “B” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.

2003: Spring Semester. Mathematics 76 (two credits) and Mathematics 406 (three credits).
Six sections in Math 76, 87 grades assigned, the average grade is “B+” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.
One section in Math 406, 4 grades assigned, the average grade is “A” and the range of grades is “A” and “A-”.

2003: Autumn Semester. Mathematics 21 (four credits) and Mathematics 205 (three credits).
Four sections in Math 21, 69 grades assigned, the average grade is “B” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.
One section in Math 205, 45 grades assigned, the average grade is “B” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.

2004: Spring Semester. Mathematics 22 (four credits) and Mathematics 450 (three credits).
Four sections in Math 22, 57 grades assigned, the average grade is “B” and the range of grades is: “A”, “A-”, “B+”,

"B", "B-", "C+", "C", "C-", "D" and "F".

One section in Math 450, 1 grade assigned, the average grade is "A" and the range of grade is "A".

2004: Autumn Semester. Mathematics 75 (two credits), Mathematics 405 (three credits) and Mathematics 450 (three credits).

Four sections in Math 75, 71 grades assigned, the average grade is "B+" and the range of grades is: "A", "A-", "B+", "B", "B-", "C+", "C", "C-", "D" and "F".

One section in Math 405, 3 grades assigned, the average grade is "A" and the range of grades is "A".

One section in Math 450, 2 grades assigned, the average grade is "A" and the range of grades is also "A".

2005: Spring Semester. Mathematics 205 (three credits) and Mathematics 320 (three credits).

One section in Math 205, 39 grades assigned, the average grade is "B" and the range of grades is: "A", "A-", "B+", "B", "B-", "C+", "C", "C-", "D" and "F".

Two sections in Math 320, 8 grades assigned, the average grade is "B+" and the range of grades is "A", "A-", "B", "C-", "F" and "W".

2005: Autumn Semester. Mathematics 205 (three credits), Mathematics 435 (three credits) and Choices and Decisions (one credit).

One section in 205, 49 grades assigned the average grade is "B" and the range of grades is: "A", "A-", "B+", "B", "B-", "C+", "C", "C-", "D" and "F".

One section in 435, 3 grades assigned, the average grade is "A-" and the range of grades is "A-". One section in Choices and Decisions, 17 grades assigned, the average grade is "Pass" and the range of grades is also "Pass".

2006: Spring Semester. Mathematics 23 (four credits), Mathematics 320 (three credits) and Mathematics 450 (three credits).

Four sections in Math 23, 80 grades assigned, the average grade is "B" and the range of grades is: "A", "A-", "B+", "B", "B-", "C+", "C", "C-", "D" and "F".

Two sections in 320, 4 grades assigned, the average grade is "A-" and the range of grades is "A", "A-" and "B+".

One section in Math 450, 3 grades assigned, the average grade is "A-" and the range of grades is "A" and "B+".

2006: Summer Semester. Mathematics 21 (four credits) and Mathematics 205 (three credits).

One section in Math 21, 10 grades assigned, the average grade is "B+" and the range of grades is "A", "A-", "B+", "B", "'B-", "C+", "C", "C-", "D" and "F".

One section in Math 205, 4 grades assigned, the average grade is "A-" and the range of grades is "A", "A-", "B+".

2006: Autumn Semester. Sabbatical leave, visiting the Mathematical Biosciences Institute of The Ohio State University. 231 West 18th Avenue, Columbus, Ohio 43210 USA. September-December, 2006.

2007: Spring Semester. Mathematics 205 (three credits), Mathematics 341 (three credits) and Mathematics 450 (three credits).

One section in Math 205, 40 grades assigned, the average grade is "B" and the range of grades is: "A", "A-", "B+", "B", "B-", "C+", "C", "C-", "D" and "F".

One section in Math 341, 11 grades assigned, the average grade is “B+” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.

One section in Math 450, 1 grade assigned, the average grade is “A” and the range of grade is also “A”.

2007: Summer Semester. Mathematics 21 (four credits).

One section in Math 21, 12 grades assigned, the average grade is “B+” and the range of grades is “A”, “A-”, “B+”, “B”, “‘B-”, “C+”, “C”, “C-”, “D” and “F”.

2007: Autumn Semester. Mathematics 205 (three credits), Mathematics 295 (three credits), Mathematics 450 (three credits), and Choices and Decisions (one credit).

One section in Math 205, 40 grades assigned the average grade is “B+” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.

One section in Math 295, 10 grades assigned, the average grade is “B+’, the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C” and “C-”.

One section in Math 450, 2 grades assigned, the average grade is “A” and the range of grade is also “A”.

One section in Choices and Decisions, 13 grades assigned, the average grade is “Pass” and the range of grades is also “Pass”.

2008: Spring Semester. Mathematics 205 (three credits), Mathematics 320 (three credits) and Mathematics 450 (three credits).

One section in Math 205, 57 grades assigned, the average grade is “B+” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.

Two sections in Math 320, 20 grades assigned, the average grade is "B+" and the range of grades is "A", "A-", "B+", "B", "B-", "C+", "C" and "C-".

One section in Math 450, 2 grades assigned, the average grade is "A" and the range of grade is also "A".

2008: Summer Semester. Mathematics 21 (four credits) and Mathematics 76 (two credits).

One section in Math 21, 8 grades assigned, the average grade is "B" and the range of grades is: "A", "A-", "B+", "B", "B-", "C+", "C" and "C-".

One section in Math 76, 5 grades assigned, the average grade is "B" and the range of grades is "A", "B+", "B" and "C+".

2008: Autumn Semester. Mathematics 75 (two credits), Mathematics 405 (three credits) and Mathematics 450 (three credits).

Four sections in Math 75, 63 grades assigned, the average grade is "B" and the range of grades is: "A", "A-", "B+", "B", "B-", "C+", "C", "C-", "D" and "F".

One section in Math 405, 4 grades assigned, the average grade is "A" and the range of grades is "A".

One section in Math 450, 1 grade assigned, the average grade is "A" and the range of grades is "A".

2009: Spring Semester. Mathematics 76 (two credits), Mathematics 320 (three credits).

Four sections in Math 76, 66 grades assigned, the average grade is "B+" and the range of grades is: "A", "A-", "B+", "B", "B-", "C+", "C", "C-", "D" and "F".

Two sections in Math 320, 11 grades assigned, the average grade is “B+” and the range of grades is “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C” and “C-”.

2009: Summer Semester. Mathematics 22 (four credits) and Mathematics 76 (two credits).
One section in Math 22, 18 grades assigned, the average grade is “B+” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C” and “F”.
One section in Math 76, 6 grades assigned, the average grade is “B” and the range of grades is “A”, “B+”, “B” and “C+”.

2009: Autumn Semester. Mathematics 205 (three credits), Mathematics 435 (three credits).
One section in Math 205, grades assigned, the average grade is “B” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.
One section in Math 435, 4 grades assigned, the average grade is “A-” and the range of grades is “A-”.

2010: Spring Semester. Mathematics 22 (four credits), Mathematics 320 (three credits).
Four sections in Math 22, 58 grades assigned, the average grade is “B-” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.
Two sections in Math 320, 9 grades assigned, the average grade is “B+” and the range of grades is “A”, “A-”, “B+”, “B”, “B-”, “C” and “C-”.

2010: Summer Semester. Mathematics 22 (four credits).

One section in Math 22, 23 grades assigned, the average grade is “B+” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-” and “F”.

2010: Autumn Semester. Mathematics 21 (four credits), Mathematics 205 (three credits).
Four sections in Math 21, grades assigned, the average grade is “B” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.

2011: Spring Semester. Mathematics 205 (three credits), Mathematics 320 (three credits), Mathematics 450 (three credits).
One sections in Math 205, 51 grades assigned, the average grade is “B+” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.
Two sections in Math 320, 6 grades assigned, the average grade is “B+” and the range of grades is “A”, “B” and “C-”.
One section in Math 450, 1 grade assigned, the average grade is “A” and the range of grades is “A”.

2011: Summer Semester. Mathematics 22 (four credits) and Mathematics 52 (three credits).
One section in Math 22, 16 grades assigned, the average grade is “B+” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”.
One section in Math 52, 8 grades assigned, the average grade is “B+” and the range of grades is: “A”, “B+”, “C+”, “C” and “F”.

2011: Autumn Semester. Mathematics 205 (three credits) and Mathematics 301 (three credits).

One section in Math 205, 44 grades assigned, the average grade is “B+” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.

Two sections in Math 301, 28 grades assigned, the average grade is “B+” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.

One section in Math 450, 1 grades assigned, the average grade is “A” and the range of grades is “A”.

2012: Spring Semester. Mathematics 23 (four credits), Mathematics 320 (three credits).

Four sections in Math 23, grades assigned, the average grade is “B” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.

Two sections in Math 320, grades assigned, the average grade is “B+” and the range of grades is “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C” and “C-”.

2012: Summer Semester. Mathematics 22 (four credits) and Mathematics 52 (three credits).

One section in Math 22, grades assigned, the average grade is “B+” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-” and “F”.

One section in Math 52, 8 grades assigned, the average grade is “B+” and the range of grades is “A”, “A-”, “B+”, “B”, “B-”, “C”.

2012: Autumn Semester. Mathematics 21 (four credits), Mathematics 405 (three credits).
Four sections in Math 21, grades assigned, the average grade is “B” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.
One section in Math 405, 5 grades assigned, the average grade is “A” and the range of grades is “A”.

2013: Spring Semester. Sabbatical leave - visiting the Institute for Mathematics and its Applications at the University of Minnesota.

2013: Summer Semester. Mathematics 22 (four credits) and Mathematics 52 (three credits).
One section in Math 22, 22 grades assigned, the average grade is “B+” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-” and “D”.
One section in Math 52, 7 grades assigned, the average grade is “B+” and the range of grades is “A”, “A-”, “B+”, “B”, “B-”, “C”.

2013: Autumn Semester. Two Sections of Mathematics 23 (four credits).
Eight sections in Math 23, 133 grades assigned, the average grade is “B” and the range of grades is: “A”, “A-”, “B+”, “B”, “B-”, “C+”, “C”, “C-”, “D” and “F”.

2014: Spring Semester. Mathematics 21 (four credits), Math-

ematics 320 (three credits).
Four sections in Math 21, 80 grades assigned, the average grade is "B" and the range of grades is: "A", "A-", "B+", "B", "B-", "C+", "C", "C-", "D" and "F".
Two sections in Math 320, 13 grades assigned, the average grade is "B+" and the range of grades is "A", "B" and "C-".

2014: Summer Semester. Mathematics 22 (four credits) and Mathematics 52 (three credits).
One section in Math 22, 28 grades assigned, the average grade is "B+" and the range of grades is: "A", "A-", "B+", "B", "B-", "C+", "C", "C-".
One section in Math 52, 5 grades assigned, the average grade is "B" and the range of grades is: "A", "B+", "C+", "C" and "F".

2014: Autumn Semester. Mathematics 205 (three credits).
Two sections in Math 205, 105 grades assigned, the average grade is "B" and the range of grades is: "A", "A-", "B+", "B", "B-", "C+", "C", "C-", "D" and "F".

2015: Spring Semester. Mathematics 205 (three credits), Mathematics 320 (three credits).
Four sections in Math 205, 58 grades assigned, the average grade is "B" and the range of grades is: "A", "A-", "B+", "B", "B-", "C+", "C", "C-", "D" and "F".
Two sections in Math 320, 8 grades assigned, the average grade is "B+" and the range of grades is "A", "A-", "B+", "B", "B-", "C+", "C" and "C-".

Recent Innovations in Teaching

Two New Courses: I have prepared two new courses: Mathematics 450-A (Introduction to Dynamical Systems in Mathematical Biology, for both undergraduate students and graduate students) and Mathematics 450-B (Topics in Mathematical Neuroscience, for students in the Ph.D program). Why do I prepare these new courses? Interdisciplinary research activities are becoming more and more important worldwide. As well known, it is not easy to obtain very interesting results in a single research area. It is desirable to couple together mathematics and other natural sciences (such as physics, chemistry, biology, astronomy) to discover new phenomena. These two courses are designed in such a way that students may have very good opportunities to learn very interesting topics in mathematical neuroscience. I use my research results in mathematical neuroscience published in very good journals and book chapters in biophysics (see a list of references later) as teaching materials in Mathematics 450-A and in Mathematics 450-B. The main reason is that the model equations have been used worldwide and the results are standard. I developed several topics in modern applied mathematics, including traveling waves, existence analysis, stability analysis, speed analysis, bifurcation analysis, etc. Of special biological importance in these two courses are nonlinear scalar integral differential equations, nonlinear singularly perturbed systems of integral differential equations, nonlinear scalar reaction diffusion equations and nonlinear singularly perturbed systems of reaction diffusion equations. In the long run, these courses may be very important for Lehigh University's interdisciplinary programs.

1 On stability of traveling wave solutions in synaptically coupled neuronal networks. *Differential and Integral Equations*, **16**(2003), 513-536.

2 Traveling waves of a singularly perturbed system of integral-differential equations arising from neuronal networks. *Journal of Dynamics and Differential Equations*, **17**(2005), 489-522.

3 How do synaptic coupling and spatial temporal delay influence traveling waves in nonlinear nonlocal neuronal networks? *SIAM Journal on Applied Dynamical Systems*, **6**(2007), 597-644.

4 Evans functions and bifurcations of nonlinear waves of some reaction diffusion equations. *Journal of Differential Equations*, **263**(2017), 3627-3686.

5 Traveling Waves Arising from Synaptically Coupled Neuronal Networks. *Advances in Mathematics Research*, Volume **10**. Editor-in-Chief: Albert R. Baswell. Nova Science Publishers Inc. New York. ISBN: 978-1-60876-265-1. 2010. Pages 53-204.

Innovations of Existing Courses: Very often I teach Math 320: Ordinary Differential Equations and Math 405: Partial Differential Equations. I have developed new materials in Mathematics 320 to reflect most recent advances. Since several topics in the old Math 320 have been moved to Math 319, now I have time to cover several new topics. What new materials have I developed recently? First of all, by coupling together the method of reduction of order, the method of undetermined coefficients, some nonlinear transformations and particular solutions of Bernoulli equations, I have established many technical lemmas to find the bounded, smooth, explicit solutions of more than ten second order, nonlinear ordinary differential equations. Secondly, by applying these lemmas, I have solved five classes of nonlinear evolution equations for bounced, smooth, explicit traveling wave solutions. These equations have strong backgrounds in physics, chemistry, biology, biophysics, biochemistry, astronomy, or other interdisciplinary. As well known, bounded explicit traveling wave solutions of nonlinear evolution equations play very important roles in industry, engineering and applied mathematics. The engineering college at Lehigh University is very famous nationwide. It is my strong desire to use my results to support the engineering college. These bounded explicit solutions will help the faculty, graduate students, post-doctoral, and visitors in the engineering college to better understand many important and interesting phenomena. Moreover, the explicit solutions may help to discover new phenomena in engineering. That is why I spend a lot of time to write two chapters of new materials and teach them in Math 320 (Ordinary Differential Equations).

In Mathematics 405 (Partial Differential Equations), I also updated some of the materials. For example, whenever time is allowed, I would teach the sharp rate of decay of the global weak solutions of the n-dimensional incompressible Navier-Stokes equations. I understand that many faculty, post-doctoral and visitors are solving the Navier-Stokes equations in one way or another. The study of the existence and the long time asymptotic behaviour of the global weak solutions will help experts to have a better understanding of the Navier-Stokes equations. The sharp rate of decay comes from the application of the Fourier transformation, the Parseval's identity and some elementary inequalities, including the Cauchy-Schwartz's inequality, the Hölder's inequality and the Young's inequality. This motivates graduate students from the Department of Mathematics and other disciplinary at Lehigh University to use fundamental concepts and techniques to do research.

I keep updating materials from year to year in Math 450 so that the graduate students can learn the most advanced ideas, methods, results and research directions in applied mathematics.

Overall, I have developed many new technical lemmas and let students apply these lemmas to solve nonlinear partial differential equations with strong physical backgrounds for bounded explicit traveling wave solutions. In this way, many students at Lehigh have applied ODE techniques to solve many real world model equations (partial differential equations).

3. On top of regular teaching loads, very often I taught Math 450 as extra courses. Here are the most recent extra courses.

Fall of 2015, Math 450 - Traveling Pulse Solutions.

Fall of 2016, Math 450 - Partial Differential Equations; Math 450 - Traveling Pulses.

Spring of 2016, Math 450 - Special topics in Evans functions and stability

Fall of 2014, Math 450 - Partial Differential Equations.

(B) Selected teaching materials (Note: This part is only for reviewers inside Lehigh University. Reviewers outside Lehigh University will not receive these materials.) Please see the attachment for the syllabus and examinations for each course.

(C) Significance and impact of mathematics courses: Studying mathematics courses very well is extremely important in one's career. Every mathematics course may be viewed as a corner stone in laying one's career foundation. As it is well known, mathematics is the base of learning many other natural sciences well. Nowadays, mathematics is playing more and more important roles than ever before. Not only in natural sciences, but also in economics, business, psychology, etc, researchers need to build mathematical models to accurately describe what are going on in their fields. Without understanding various concepts, ideas and methods in mathematics, it is almost impossible to master other subjects and it is impossible to do any serious research in sciences. For examples, I have seen many differential equations and integrals appearing in course materials of electrical engineering, chemical engineering and industrial engineering. Overall, it is hard to over emphasize the impact of doing well in mathematics. Many Nobel Prize winners in biology, chemistry, physics, economics know how to set up mathematical relationships (including many differential equations) between various quantities. This is really a time to couple together mathematics and other disciplines to obtain more and more interesting and important results in science. That is why I love mathematics and I love to teach it very well. After all, tomorrow's Nobel Prize winners come from today's students. Therefore, all of us will have to try our best to teach very well so that our students can achieve their goals. I stress the importance and impact to my students, and ask them to have correct and positive attitudes to learn it.

When I teach courses, I would normally check attendance, announce homework to be due the next time, announce important information (including date, time and classroom of a review session, an exam or a competition), announce other information, remind students to turn in homework on time and prepare for quizzes, send emails to students who did not do well in recent midterm exams to invite them to come to see me for extra help, have interactions with students, encourage them to think about something new and important, give them a partly correct answer and let them correct the mistakes, and then provide the completely correct answer, arrange extra office hours and review sessions before exams. I have taught calculus, linear methods, etc, for many times and I think I know how to teach them very well. Sometimes I reverse the order of some sections in a chapter. In this way I think the students would understand the materials better. I do make and distribute some of the most important materials, for example, solutions to very important or difficult problems for my students. In the last five years, many colleagues have observed my classes. All of them have made positive comments about my teaching.

(D) Teaching Philosophy: First of all, I love to teach and I was one of the top 2 percent (50 out of 2200) teaching assistants in the Graduate School at The Ohio State University.

My goal is to provide a convenient, comfortable environment such that every student in my class has the best chance to learn well. I present the materials in an easy-to-understand way, to let the students understand the main points to solve problems and to get ready to answer all questions. I always prepare very well before going to teach classes.

I always try my best to make abstract, difficult theorems very interesting to learn. I would motivate the backgrounds of the main theorems, let them know the key points (assumptions and results) and how to apply them. This teaching skill has been highly appreciated by my students. Many of my students no longer find mathematics courses frustrating. When I teach something extremely important and difficult, I will emphasis or paraphrase them. I also talk to a student individually to see how he or she feels in the class. I often give my students twenty or thirty seconds to think about problems to be solved. I also teach them how to be careful. For example, I would give a partially correct answer, and ask them to think about the correct one. I also present mistakes made by previous students and ask them to correct it. After I solve long and difficult problems, I usually summarize the main points or steps to complete the problems, pointing out what kind of theorem was applied in each step. They like this method of teaching very much and they have learned a lot.

I care about all students progress in my class and I usually arrange office hours immediately after my lecture, so that they can ask me for help if they have questions. I always encourage them to attend every lecture and recitation. I often go over the homework problems before I assign them and I strongly recommend that they solve the homework problems as soon as possible after the material has been covered in class - preferably on the same day. They will benefit far more from the lectures if they familiarize themselves with the material to be covered before class and come to prepared to ask questions.

(E) Teaching Interests: I love to teach both undergraduate level and graduate level courses. In particular, I love to teach

Calculus sequences, Ordinary Differential Equations, Partial Differential Equations, Real Analysis, Complex Analysis, Functional Analysis, and Linear Algebra. I am also very interested in developing and teaching new courses closely related to mathematical neuroscience and topics in fluid dynamics (such as the n-dimensional incompressible Navier-Stokes equations and n-dimensional magnetohydrodynamics equations).

(F) Comments from former undergraduate students, mostly from calculus classes and Math 205 I am including several comments from my students.

"Professor Zhang gave very good examples and was especially helpful before tests, when students need the most help. Prof Zhang taught the material very well."

"Teacher was very devoted to our learning, this was truly an excellent course."

"Thank you. I was taught more, and better than any other mathematics class. Your funny sense of humor makes the class much better."

"Prof. Zhang is very nice, fair and intelligent."

"Your teaching style really helped me understand the material. I like the 'many examples' approach."

"Zhang is very enthusiastic."

"Professor Zhang is an effective teacher and his examples are clear and valuable."

"Professor Zhang is great. All I have to say."

"Professor Zhang did a great job of making confusing topics simple."

"The professor was very clear when he was giving explanations. The professor was very knowledgeable in the subject. I especially appreciate his short-cut methods"

"This student is friend with this professor."

"Overall, Zhang did a very good job."

"Professor Zhang made Math 205 very exciting. I really enjoyed his enthusiasm. He also was always open for office hours and genuinely wanted us to learn the material."

"Thank you for your care. Sometimes in college it seems that teachers don't really care about how their students perform, and it is nice that teachers such as yourself go out of your way."

"Professor Zhang teaches in a very candid, fun manner, I enjoyed class with him and would take another from him. $\cdots$"

"He (Linghai Zhang) is an excellent professor and he is always available to help students even when students come at odd times for help."

"Mr. Zhang was an excellent teacher. Funny and keeps the class alive."

"Professor Zhang is an excellent mathematics professor who is very devoted to all his students not only getting good grades, but also truly learning the subject matters to further their understanding of mathematics for their future benefits."

"Linghai was very effective."

"Professor Zhang was very enthusiastic and he really cared about what the students learned. I really enjoyed his class. I would recommend Professor Zhang's courses because of this."

"Professor Zhang did a good job of emphasizing the important parts."

"Professor Linghai Zhang is undoubtedly the best mathematics prof I've ever had. Always available and very helpful. Give him a raise."

"Professor Zhang was very helpful in office hours. He made class enjoyable and taught the subject well."

"Calculus III is pretty hard, but Zhang was great in breaking it down."

"Zhang is a great Professor. He makes class fun and gives relevant examples."

"Professor Zhang was very engaged in class. He always make an effort to learn students names which I really appreciated."

"Zhang is one of the best Math professors I have had. Let him teach Math 205."

"The Professor is good and well prepared."

"Best teacher I have had in college!"

"I love Zhang!"

"Great class - I really enjoyed having time to practice with presentation time."

"Really cares about his students and gives great examples to keep class interested."

"Professor Zhang made the class interesting and truly cared about the success of each student."

(G) Impacts of teaching

The majority of my students have good understanding of mathematical concepts, ideas, methods. This will enhance their future mathematics study and other natural sciences study. By correcting mistakes made by current students and by previous students, I trained the students how to become very careful and how to avoid many major, common mistakes in mathematics and other applied sciences. By carefully examining conditions and results of many theorems, students learned rigorous reasoning. Their overall ability in mathematics are becoming stronger and stronger.

(H) Ph.D students Under my guidance, Melissa Anne Stoner had received her Ph.D in May 2011. Currently I am supervising Alan Dyson in the mathematics Ph.D program. Very likely Alan will graduate in May, 2019.

The Service Statements

Linghai Zhang has been performing many services for Lehigh University, the College of Arts and Sciences, the Department of Mathematics, and the mathematical society.

I: Service to Lehigh University

Educational Policy Committee (CAS Representative) Linghai Zhang was elected to be a representative of the College of Arts and Sciences on the Educational Policy Committee for 2016 - 2019. He is very grateful to his colleague Professor David Johnson for substituting him on the committee (fall of 2016) due to class conflict with the meeting schedule of the committee. Often he offers constructive comments or suggestions on things being discussed, such as

Schedules of Summer Classes

Dates of Make Up Final Exams

Polices on Winter Sessions

Many Other Policies

So far he has never been absent from any meeting in the Educational Policy. He would like to make more substantial contributions to these committees. Attending commencements.

II: Service to the College of Arts and Sciences

College of Arts and Sciences Policy Committee Linghai Zhang was elected to be a member of the College of Arts and Sciences Policy Committee (CASPC) for three academic years

2016 - 2019. During 2016 - 2017, he was also on the Policy Committee's Course and Curriculum (C & C) sub-Committee. His main duty is to approve or disapprove course and program changes (to find out the main reasons to add or drop classes, to make substantial or non-substantial changes of classes). If there is not enough strong reason, then he would roll it back to the corresponding department chair for more reasons for the course or program change.

To help all instructors to teach well and to help all students to learn well in all classes, to minimize unnecessary distractions and noises during classes, Linghai initiated a very important and interesting policy on electronic devises. This policy has been approved during a College of Arts and Sciences meeting. Here is the policy: No cell phones or any electronic devices are allowed in CAS classrooms, unless the instructor permits.

During fall of 2017, he was the secretary to record the minutes of the CASPC meetings.

It is his desire to continue to make more positive contributions to CASPC.

III: Service to the Department of Mathematics

1 Hiring search committee member for many times.

2 Calculus committee member for six years (spring 2007 to autumn 2012).

3 Graduate committee member for three years (autumn 2003 to spring 2006).

4 Organizer of Seminar in Applied Mathematics for three years (autumn 2003 to spring 2006).

5 Course leader of Math 205 and Math 22 for many times.

6 Making, proctoring and grading comprehensive examinations and qualifying examinations for many times.

7 Advisor of undergraduate applied mathematics majors since autumn 2013.

8 Advisor of undergraduate applied mathematics minors since autumn 2013.

9 Advisor of undergraduate mathematics majors for two years (autumn 2011 to autumn 2013).

10 Advisor of undergraduate non-mathematics majors, since autumn 2005.

11 Advisor of undergraduate pure mathematics minors for two years (autumn 2011 to autumn 2013).

12 Library representative for two years (autumn 2004 to spring 2006).

13 Overall coordinator for the conference for undergraduates considering graduate school in mathematics. Title: *Making the Most of Mathematics Graduate School*. April 22, 2006. Department of Mathematics, Lehigh University.

IV: Service to the Mathematical Society

1 Main organizer of the international conference: Nonlinear Systems of Fluid Dynamics Equations and Applications, in Sanya, China. December 19-22, 2015. For information of the conference, please visit: http://msc.tsinghua.edu.cn/sanya/upcoming.aspx. The co-organizer is Professor Shijin Ding, dingsj@scnu.edu.cn. South China Normal University.

2 Organizer of the special session: Nonlinear Waves of Differential Equations, November 14 - 15, 2015, at the American Mathematical Society Sectional Meeting at Rutgers University in New Brunswick, New Jersey. For information of the special session[1], please visit: http://www.ams.org/meetings/sectional/2227_special.html.

3 Co-organizer of the mini-symposium on "Neuronal and Biological Dynamical Systems" at the Fifth International Congress on Industrial and Applied Mathematics. Sydney, Australia. July 7 - 11, 2003. (another organizer: Dr. Jianzhong Su, su@uta.edu, University of Texas at Arlington).

4 Associate Editor of International Journal of Mathematical Physics. July 2018 - now.

5 Guest co-editor of the Special Issue "Nonlinear Partial Differential Equations in Mathematics and Physics" for Abstract and Applied Analysis in 2014. For information of the special issue, please visit http://www.hindawi.com/journals/aaa/si/489087/cfp/.

6 Refereeing papers for academic journals, about three to five papers each year.

[1] Whenever the AMS conference site is reasonably close to Lehigh University in the future, I would like to organize more special sessions in my main research fields, such as "The Existence of Global Smooth Solutions of n-Dimensional Nonlinear Evolution Equations", or "Evans Functions and Stability of Traveling Wave Solutions of Nonlinear Integral Differential Equations", etc. On one hand, it is the organizer who benefit the most in applied mathematics. On the other hand, it is a very good opportunity for experts in these fields to get together to exchange ideas, methods, techniques and mutually improve each other's ideas and results.

7 Reviewing papers for the American Mathematical Review, about five papers each year.

V: Service to Interdisciplinary Programs

1 In the Summer of 2006, Linghai Zhang supervised three undergraduate students to do research in mathematical neuroscience - speed analysis of traveling waves arising from synaptically coupled neuronal networks.

2 In the Summer of 2007, Michael Burger and Linghai Zhang supervised a team of students to conduct research in mathematical neuroscience.

3 In the Summer of 2008, Michael Burger, Ping-Shi Wu and Linghai Zhang supervised a team of students to conduct research in mathematical neuroscience.

4 In the Summer of 2010, Linghai Zhang supervised Neil Whitman Dexter to conduct research in mathematical neuroscience.

5 In the Spring of 2018, Linghai Zahng supervised a team of students (Wang Shuai, Xu Duo, Liu Yanxi) to participate in the Mathematics Contest in Modelling.

VI: Refereeing Papers for Academic Journals

I have refereed papers for the following journals in the last twenty years or so:

Acta Mathematica Scientia

Acta Mathematica Sinica

Acta Mathematicae Applicatae Sinica

Annales de L'Institut Henri Poincare, Analyse Non Lineaire

Archive for Rational Mechanics and Analysis

Boundary Value Problems

Calculus of Variations and Partial Differential Equations

Chinese Advances in Mathematics

Chinese Annals of Mathematics

Communications in Geometry and Analysis

Communications in Partial Differential Equations

Discrete and Continuous Dynamical Systems

Dynamics in Partial Differential Equations

Journal of the American Mathematical Society

Journal of Differential Equations

Journal of Functional Analysis

Journal of Mathematical Analysis and Applications

Journal of Partial Differential Equations

Mathematical Biosciences - an international journal

Mathematical and Computer Modeling

Physica D

Proceedings of the American Mathematical Society

Proceedings of the Royal Society of Edinburgh, Section A: Mathematics

Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences

SIAM Journal on Applied Dynamical Systems

SIAM Journal on Applied Mathematics

SIAM Journal on Mathematical Analysis

Transactions of the American Mathematical Society

I am trying my best to take responsibility and to have a positive attitude in performing these services.

Impact of the Seminar in Applied Mathematics: It fosters collaborations and interchanging research ideas among scholars. The faculty, graduate students and undergraduate students have more opportunity to interact with experts when attending the seminar. I would like to chair the colloquium lecture at the Department of Mathematics in the future and have a focus each semester.

Impact as advisor of undergraduate students: students received good advice and they know how to learn well, how to manage their time and how to maintain self-control. The students have became much more potential to succeed.

As a library representative, I have a good opportunity to serve the faculty in the Department of Mathematics. I hope to have some feedbacks from the faculty and students so that I can make good progress.

By refereeing and reviewing papers, I can learn some of the best-known results in the mathematical society. I hope to have more chance to organize mini-symposium during international conferences. This will accelerate interaction and collaboration between experts in many different areas.

From 2018 CV: Research Statement **The main difficulty:** Previously, many mathematicians have tried to apply the Cauchy-Schwartz's inequality, the Hölder's inequality, the Gagliardo-Nirenberg's interpolation inequality and the Gronwall's inequality to establish uniform energy estimates of the derivatives of the global weak solutions to accomplish the existence and uniqueness of the global smooth solutions of the Cauchy problems for nonlinear systems of fluid dynamics equations. This method works perfectly for equations in lower-dimensional spaces and for equations with special structure (such as the Hopf-Cole transformation) or special feature (such as a maximum principle). However, for equations in three-dimensional and higher-dimensional spaces without the special structure or the special feature, such as the n-dimensional magnetohydrodynamics equations and the n-dimensional incompressible Navier-Stokes equations, it is extremely difficult to establish the uniform energy estimates even for the first order derivatives of the global weak solution. Mathematicians cannot use the Gagliardo-Nirenberg's interpolation inequalities because some crucial conditions are not satisfied. This is the main barrier to establish the uniform energy estimates of any order derivatives of the global weak solution and this is why the existence of the global smooth solutions of the Cauchy problems for the n-dimensional magnetohydrodynamics equations and the n-dimensional incompressible Navier-Stokes equations have been open for more than a hundred years.

The main strategy: Here, I accomplish the existence of the global smooth solution of the Cauchy problem for the n-dimensional nonlinear system of fluid dynamics equations in a very different method. Different from before, instead of directly working on the uniform energy estimates of the derivatives of the global weak solution, we work on the Fourier transformation $\widehat{\mathbf{u}}(\xi, t)$ of the global weak solution, that is, to establish an exponential decay estimate of the Fourier transformation $\widehat{\mathbf{u}}(\xi, t)$ of the global weak solution.

The details: First of all, I establish some elementary uniform energy estimates. The existence of the global weak solution of the nonlinear system of fluid dynamics equations can be established by coupling together Lax-Milgram's theorem, representation of continuous linear functionals in Hilbert spaces, Leray-Schauder's fixed point principle and the elementary uniform energy estimates. For example, the existence of the global weak solution of the n-dimensional incompressible Navier-Stokes equations may be established in this way. Second, we couple together the representation of the Fourier transformation of the global weak solution, the Fourier splitting method and the elementary uniform energy estimates to establish elementary decay estimates with sharp rates for the global weak solution. Third, by making use of the representation of the Fourier transformation of the global weak solution, an appropriate assumption on the Fourier transformation of the nonlinear function and by making use of Gronwall's inequality, I establish an exponential decay estimate of the Fourier transformation $\widehat{\mathbf{u}}(\xi,t)$. The assumption is that the Fourier transformation of the nonlinear function is controlled by a nonlinear function of the Fourier transformation $\widehat{\mathbf{u}}(\xi,t)$ and (ξ,t), which is motivated by the elementary uniform energy estimates and the elementary decay estimates of the global weak solution. Fourth, by coupling together Plancherel's identity and the exponential decay estimate of $\widehat{\mathbf{u}}(\xi,t)$, establish uniform energy estimates of any order derivatives and decay estimates with sharp rates of any order derivatives of the global weak solution. if, by coupling together the representation of the global weak solution and the uniform energy estimates, we accomplish the existence of the global smooth solution of the Cauchy problem for the very general nonlinear system of fluid dynamics equations.

Apparently, this is very different method to establish $n$fluid dynamics. The main results have number of applications in physics, engineering and industry. Needless to say, this new method to accomplish the existence of the global smooth solution of the nonlinear fluid dynamics equations is novel. Additional to nonlinear systems of fluid dynamics equations, this method may be applied to accomplish the existence of other equations, such as functional and integra, etc. **The main results:** Isults thissearch:

(1) the existence smooth ,

(2) of any order derivatives,

(3) the stability estimates of any order derivatives,

(4) smooth of the very general nonlinear system of fluid dynamics equations.

That is $\int_{\mathbb{R}^2} \mathrm{d}x\mathrm{d}y \ \int_{\mathbb{R}^2} \mathrm{d}x\mathrm{d}y \int_{\mathbb{R}^2} \mathrm{d}x\mathrm{d}y \ x, y \ \mathcal{A}$, I proved the following results. $C, , m \geq v0$ **Theorem 3. (The existence of**

the global smooth solution) Suppose that the initial function $\mathbf{u}_0 n L^1(\mathbb{R}^n) L^2(n)$ and that the external force $\mathbf{f} L^1$. There exists a unique global smooth solution 1 to the Cauchy problem for the nonlinear system of fluid dynamics equations: **Theorem 4. (The uniform energy estimates)** T the following uniform energy estimates for all integers, where is a positive constant, independent of and, it depends on. **Theorem 5. (The decay estimates with sharp rates)** There hold the following decay estimates with sharp rates for all integers, where is a positive constant, independent of and, it depends on and. **Theorem 6. (The stability estimates)** Let with, let be the global smooth solution with and. the following stability estimates for all integers, where is a positive constant, independent of and, it depends on and.$====, ()$ **Theorem 7. (The exact limits)** Suppose that the initial functionand the external force. Then the global smooth solution of the Cauchy problem enjoys The next goal in the near future is to justify that the assumption on the Fourier transformation of the nonlinear function is valid, that is, the Fourier transformation of the nonlinear function is controlled by a nonlinear function of the Fourier transformation $\widehat{(}\xi, t)$ and (ξ, t), which is motivated by the elementary uniform energy estimates and the elementary decay estimates of. I - 4: The Global Smooth Solution of A Two-Dimensional Nonlinear System of Differential Equations Arising from Geostrophics. Consider the following two-dimensional nonlinear singular system of differential equations arising from geostrophics and $> 0 \neq 0 \frac{1}{2} D D_1 y x \sum_0^n + - 0 -_0 = ++++++ \mathbf{x},, -. \cdot + n/2p, q C p, q = \quad . J = u_k u_k -,, t === C2$ This is a singular, highly nonlinear, strongly coupled system of differential equations. The existence and uniqueness of are very difficult problems and have been open for a long time. The main purpose of this project is to accomplish the existence and uniqueness of the global smooth solution of the Cauchy problems for the two-dimensional nonlinear system of differential equations arising from geostrophics. We are able to overcome the main difficulty by completely making use of the special structure of the nonlinear system and by coupling together many inequalities (including Cauchy-Schwartz's inequality, Hölder's inequality, Gagliardo-Nirenberg's interpolation inequalities for fractional order derivatives and classical derivatives) to establish the uniform energy estimates of any order derivatives. The uniform energy estimates obtained in this project distinguish from those in previous related papers where the authors did not make use of the special structure of the nonlinear system to overcome the main difficulty. The main ingredients in the rigorous mathematical analysis include Leray-Schauder's fixed point principle and uniform energy estimates. **Theorem 8.** For all initial functions and, for all there exists a global weak solution, such that for all time, where is positive constant, independent of, and.0 **Theorem 9.** Let. Suppose that the initial functionsand. Then there exists a unique global smooth solution to the Cauchy problems for the nonlinear system of differential equations. There holds the following uniform energy estimate for all nonnegative integersand for all time, where is a positive constant, independent of and. **Theorem 1.** Let. Suppose that, for the Fourier transformation of the global smooth solution of the Cauchy problems, there holds the following estimate for all and for some positive constants where, independent of and. Then the global smooth solution enjoys the following decay estimates with sharp rates for all positive integers $1\partial\partial \sum_{k=1}^{n} u_k t > 0 H^6 > 3/t >> 30(x,y), 0(,)a0 << mH^6,,..-,,$ where, and are positive constants, independent of and. This general system includes, the n-dimensional nonlinear system of Newton-Boussinesq equations as examples. In this system, $\alpha > 0$ is a positive constant. The unknown function $=, , s$, is a real vector valued function of $= ()s, n$ and $t > 0 =,$. The nonlinear function $=,$ is sufficiently smooth. The function $, , t$ represents the initial function and the nonhomogeneous function f represents an external force.

the n-dimensional nonlinear system of Newton-Boussinesq equations for Some n-Dimensional nonlinear system of Korteweg-de Vries-Burgers $= gt = 0 = 0, (u)p - (u = x, t)$ nonlinear system of Benjamin-Ono-Burgers equations $Hx- = | \leq, -u,$. s equations my b contained general nonlinear system of fluid equations Consider gwsp nonlinear system of fluid dynamics equations. (I) Let the function n and the external force $fn,), f(x, t$en holde for all me $t > 0$, where the positive constant > 0 independent$\mathbf{u}$ and $--,$ but d, , and the () ofitial function u_0. (II) Let thecap and external force cap, fes, uch,but depends on the ()1 the () of . **Remark.** The of do not tisfy the first decay but they satisfy the second decay estimate, simply because existence, D EsSRs, stability estimates and el global smooth solutions of nonlinear systems of fluid dynamics equations e of fundamental importance in applied mathematics. Let me list the specific problems I have solved in nonlinear systems of uid dynamics equations. The papers [8], [15], [16], [17] in this area have been highly recognized and cited by many other mathematicians.

Summary: A general direction of my research has been to accomplish the existence and stability of traveling wave fronts of, the existence and stability of traveling pulse solutions of arising from synaptically coupled neuronal networks, to study the influence of the mechanisms in mathematical neuroscience on wave speeds and the bifurcations of standing wave solutions, to have some broad impact in both improving my ability in modeling more realistic problems in mathematical neuroscience and in applied mathematics, and to create new and significant mathematics. It turns out that substantial progress has been made and the stage is set for even deeper mathematical understanding. The main objective has been not only to develop my investigation further, but also to synthesize these results and build a solid theory to give a systematic treatment of the existence and stability of traveling wave solutions.

Project . The existence and stability *multiple* traveling pulse solutions of singularly perturbed system of .
First of all, consider

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} + \alpha[\beta H(u-\theta) - u] - w,\\ \frac{\partial w}{\partial t} &= \varepsilon(u - \gamma w).\end{aligned}$$

I couple together implicit function theorem, intermediate value theorem, mean value theorem and any other important ideas, methods and techniques in dynamical systems to accomplish the existence of the traveling pulse solutions. I a global strong maximum principle for Evans functions and Hopf lemma to accomplish the stability of the fast pulse solutions and to establish the nonlinear instability of theow trling pulse solutions.

Theorem 15. (Existence) There exist two kinds of traveling pulse solutions $(U, W) \in C^1(\mathbb{R}) \cap C^2(\mathbb{R} - \{0\})$ t singularly perturbed: the fast traveling pulse solutions $(U, W) = (U(\varepsilon, x + \nu_{\text{fast}}(\varepsilon)t), W(\varepsilon, x + \nu_{\text{fast}}(\varepsilon)t))$ with the fast moving coordinates $z = x + \nu_{\text{fast}}(\varepsilon)t$ and the fast wave speeds $\nu_{\text{fast}}(\varepsilon)$; the slow traveling pulse solutions $(U, W) = (U(\varepsilon, x + \nu_{\text{slow}}(\varepsilon)t), W(\varepsilon, x + \nu_{\text{slow}}(\varepsilon)t))$ with the slow moving coordinates $z = x + \nu_{\text{slow}}(\varepsilon)t$ and the slow wave speeds $\nu_{\text{slow}}(\varepsilon)$.

For all positive integers $m \geq 1$, there exist $2m$ constants: $0 = Z_1(\varepsilon) < Z_2(\varepsilon) < Z_3(\varepsilon) < \cdots < Z_{2m-1}(\varepsilon) < Z_{2m}(\varepsilon)$ (for the fast pulses and the slow pulses, these constants are very different), such that the traveling pulse solutions satisfy the following conditions

$$U(\varepsilon, Z_{2k-1}(\varepsilon)) = \theta, U_z(\varepsilon, Z_{2k-1}(\varepsilon)) > 0, U(\varepsilon, Z_{2k}(\varepsilon)) = \theta, U_z(\varepsilon, Z_{2k}(\varepsilon)) <$$
$$U > \theta \text{ on } (Z_{2k-1}(\varepsilon), Z_{2k}(\varepsilon)), \qquad U < \theta \text{ on } (Z_{2k}(\varepsilon), Z_{2k+1}(\varepsilon)).$$

The traveling pulse solutions enjoy the following boundary conditions

$$\lim_{z\to\pm\infty}(U(\varepsilon, z), W(\varepsilon, z)) = (0, 0), \lim_{z\to\pm\infty}(U_z(\varepsilon, z), W_z(\varepsilon, z)) = (0, 0).$$

(Stability) Consider the following Cauchy problems

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} + \alpha[\beta H(u-\theta) - u] - w,\\ \frac{\partial w}{\partial t} &= \varepsilon(u - \gamma w).\end{aligned}$$

$+\nu_{\text{fast}}(\varepsilon)\frac{\partial P}{\partial z} + \nu_{\text{fast}}(\varepsilon)\frac{\partial Q}{\partial z}$,
$P(z, 0) = P_0(z), \qquad Q(z, 0) = Q_0(z)$. There exist three positive constants $C > 0$, $M > 0$ and $\rho > 0$, such that if the initial functions (P_0, Q_0) satisfy the condition

$$\sup_{z\in\mathbb{R}} |(P_0(z) - U_{\text{fast}}(\varepsilon, z), Q_0(z) - W_{\text{fast}}(\varepsilon, z))| \leq C,$$

then the global solution of the Cauchy problems enjoys the following decay estimate

$$\begin{aligned}&\sup_{z\in\mathbb{R}} |(P(z,t) - U_{\text{fast}}(\varepsilon, z+h), Q(z,t) - W_{\text{fast}}(\varepsilon, z+h))|\\ \leq\ & M \sup_{z\in\mathbb{R}} |(P_0(z) - U_{\text{fast}}(\varepsilon, z), Q_0(z) - W_{\text{fast}}(\varepsilon, z))| \exp(-\rho t),\end{aligned}$$

where h is a real nonzero time-independent constant, satisfying the estimate

$$|h| \le M \sup_z |(P_0(z) - U_{\text{fast}}(\varepsilon, z), Q_0(z) - W_{\text{fast}}(\varepsilon, z))|.$$

In another word, the fast traveling pulse solutions $(U, W) = (U_{\text{fast}}(\varepsilon, z), W_{\text{fast}}(\varepsilon$
are stable.
The slow traveling pulse solutions $(U, W) = (U_{\text{slow}}(\varepsilon, z), W_{\text{slow}}(\varepsilon, z))$
are unstable.

Theorem 16. Suppose that the positive constants $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\theta > 0$ and $\Theta > 0$ satisfy the following conditions

$$\gamma 1 + \gamma \neq, \qquad (\alpha + \beta)\gamma 1 + \gamma < \Theta.$$

The traveling pulse solutions satisfy the traveling wave equations

$$\mu(\varepsilon)U' + U + WK(z)U()y - \mu(\varepsilon)||zW(z)H()U()y - \mu() \int_{\mathbb{R}} K(x-y)H(u($$

$$\mu(\varepsilon)W$$

and the following boundary conditions

$$\begin{aligned} \lim_{z\to\pm\infty} (U_{\text{fast-pulse}}(\varepsilon, z), W_{\text{fast-pulse}}(\varepsilon, z)) &= (0, 0), \\ \lim_{z\to\pm\infty} (U_{\text{fast-pulse}}(\varepsilon, z), W_{\text{fast-pulse}}(\varepsilon, z)) &= (0, 0). \end{aligned}$$

The first traveling pulse solution is called a large pulse because it crosses both thresholds for $2m$ times, the next two traveling pulse solutions are called small pulses because they cross only one threshold. Let $m \ge 1$ be a positive integer. The fast traveling pulse solutions cross their thresholds in the following way, respectively.

(I) There exist real numbers $-\infty < Z_1 < Z_2 < Z_3 < \cdots < Z_{2m-1} < Z_{2m} < \infty$ and $-\infty < Z'_1 < Z'_2 < Z'_3 < \cdots < Z'_{2m-1} < Z'_{2m} < \infty$, (-au)(-bu) such that $(Z'_{2k-1}, Z'_{2k}) \subset$

(Z_{2k-1}, Z_{2k}), for all $k = 1, 2, 3, \cdots, m$. Define $Z_0 = -\infty$ and $Z_{2m+1} = \infty$. Then

$$U(\varepsilon, Z_{2k-1}) = \theta, U_z(\varepsilon, Z_{2k-1}) > 0, U(\varepsilon, Z_{2k}) = \theta, U_z(\varepsilon, Z_{2k}) < 0,$$
$$U(\varepsilon, z) > \theta, \text{ on } (Z_{2k-1}, Z_{2k}), U(\varepsilon, z) < \theta, \text{ on } (Z_{2k}, Z_{2k+1}),$$
$$U(\varepsilon, Z'_{2k-1}) = \Theta, U_z(\varepsilon, Z'_{2k-1}) > 0, U(\varepsilon, Z'_{2k}) = \Theta, U_z(\varepsilon, Z'_{2k}) < 0,$$
$$U(\varepsilon, z) > \Theta, \text{ on } (Z'_{2k-1}, Z'_{2k}), U(\varepsilon, z) < \Theta, \text{ on } (Z'_{2k}, Z'_{2k+1}).$$

Overall, the large traveling pulse solution $(U, W) = (U_{\text{pulse}-1}(\varepsilon, z), W_{\text{pulse}-}$ crosses the small threshold θ exactly $2m$ times and it crosses the large threshold Θ exactly $2m$ times.

(II) The second traveling pulse solution $(U, W) = (U_{\text{pulse}-2}(\varepsilon, z), W_{\text{pulse}-2}(\varepsilon, z$ crosses the small threshold θ exactly $2m$ times but it does not cross the large threshold Θ.

(III) The third traveling pulse solution $(U, W) = (U_{\text{pulse}-3}(\varepsilon, z), W_{\text{pulse}-3}(\varepsilon, z)$ crosses the large threshold Θ exactly $2m$ times but it does not cross the small threshold θ.

Appendix of THE RESEARCH STATEMENTS

Section One

I have coupled together the representation of the Fourier transformation of the global weak solution, the assumption on the Fourier transformation of the nonlinear function and Gronwall's inequality to obtain an exponential decay estimate of the global weak solution to accomplish the existence of the global smooth solution of the Cauchy problem, with any initial function and any external force. The assumption is that the Fourier transformation of the nonlinear function is controlled by a nonlinear function of the Fourier transformation, which is motivated by the elementary uniform energy estimates and the elementary decay estimates of the global weak solution. I am very interested in justifying the assumption on the Fourier transformation of the nonlinear function is valid.

Consider the Cauchy problems for the nonlinear system of Korteweg-de Vries-Burgers equations also consider the Cauchy problems for the nonlinear system of Benjamin-Bona-Mahony-Burgers equations where $\alpha > 0$ is a positive constant, for a positive constant $C > 0$, is a real nonzero constant vector. The nonlinear functions, are real vector-valued functions of, where are real scalar smooth functions of for a positive constant $C > 0$. , The following exact limits have been open. is the global smooth solution or the global strong solution The results may play very important roles in finding solutions to significant open problems in nonlinear systems of fluid dynamics equations.

$$\frac{\partial}{\partial t}(\cdot\nabla)(\cdot\nabla)(\cdot\nabla)(\cdot\nabla)(\cdot\nabla)(\cdot\nabla)(\cdot\nabla)()$$
$$\frac{\partial}{\partial t}(\cdot\nabla)(\cdot\nabla)(\cdot\nabla)(\cdot\nabla)(\cdot\nabla)(\cdot\nabla)(\cdot\nabla)()$$

subsectionthe exact limits of global weak solutions
subsectiontwo-dimensional nonlinear singular system
subsection$L^{\lambda} - L^{\mu}$ regularity of weak solutions
subsectiondecay estimates with sharp rates
Section Two
subsectionstability of traveling wave fronts
subsectionstability of traveling pulse solutions

1 Jian Young Wu, Li Guan and Yang Tsau, Propagating activation during oscillations and evoked responses in neocortical slices. *Journal of Neuroscience*, **19**(1999), 5005-5015.

2 Xiaoying Huang, William C. Troy, Qian Yang, Hongtao Ma, Carlo R. Laing, Steven J. Schiff and Jian-Young Wu, Spiral waves in disinhibited mammalian neocortex. *The Journal of Neuroscience*, **24**(2004), 9897-9902.

3 Carlo R. Laing, Spiral waves in nonlocal equations. *SIAM Journal on Applied Dynamical Systems*, **4**(2005), 588-606.

4 Weifeng Xu, Xiaoying Huang, Kentaroh Takagaki and Jian-young Wu, Compression and reflection of visually evoked cortical waves. *Neuron*, **55**(2007), 119-129.

5 Xiaoying Huang, Weifeng Xu, Jianmin Liang, Kentaroh Takagaki, Xin Gao and Jian-young Wu, Spiral wave dynamics in neocortex. *Neuron*, **68**(2010), 978-990.

mmm(u-n)(u-n)(u-n)(-au)(-au)(-bu)(-bu) subsectionmathematical analysis of wave speeds

Theorem . (I) Let μ_0 be the wave speed of the traveling wave front of the integral differential equation. Then

$$0 < \frac{m}{2}\frac{\alpha - a\theta - 2m\theta + 2mn}{(\alpha - a\theta)K(0)} < \mu_0 \le \frac{m}{\ln\frac{\alpha}{\alpha - 2m\theta + 2mn}} \int_{\mathbb{R}} |x|K(x)\mathrm{d}x,$$

(II) Let μ_0 be the wave speed of the traveling wave front of the integral differential equation. Then

$$\mu_0 < \frac{m}{\ln\frac{\beta}{\beta - 2m\theta + 2mn}} \int_{\mathbb{R}} |x|W(x)\mathrm{d}x,$$

$$\mu_0 > \frac{m}{2}(\beta - b\theta - 2m\theta + 2mn)/\left\{(\beta - b\theta)W(0)\left[\int_0^\infty \eta(\tau)\exp(m\tau)\mathrm{d}\tau\right]\right\} >$$

(III) Let μ_0 be the wave speed of the traveling wave front of the integral differential equation. Then

$$\mu_0 \le \frac{m}{(\alpha + \beta)\ln\frac{\alpha+\beta}{\alpha+\beta-2m\theta+2mn}} \int_{\mathbb{R}} [\alpha|x|K(x) + \beta|x|W(x)]\mathrm{d}x,$$

$$\mu_0 > \frac{m}{2}(\alpha+\beta-a\theta-b\theta-2m\theta+2mn)$$
$$/ \left\{(\alpha-a\theta)K(0)+(\beta-b\theta)W(0)\left[\int_0^\infty \eta(\tau)\exp(m\tau)\mathrm{d}\tau\right]\right\} > 0.$$

Theorem . Let $\mu_1=\mu_1(\alpha,\beta,\xi,\eta,K,W,\theta)$, $\mu_2=\mu_2(\alpha,\xi,K,\theta)$ and $\mu_3=\mu_3(\beta,\eta,W,\Theta)$ represent the three wave speeds of the three traveling wave fronts.

(I) There hold the following limits

$$\lim_{\theta/(\alpha+\beta)\to 1/2}\mu_1(\alpha,\beta,\xi,\eta,K,W,\theta)=0,$$
$$\lim_{\theta/(\alpha+\beta)\to 1/2}\left\{\mu_1(\alpha,\beta,\xi,\eta,K,W,\theta)/\left(\frac{1}{2}-\frac{\theta}{\alpha+\beta}\right)\right\}$$
$$= 1/\left\{\frac{\alpha}{\alpha+\beta}K(0)+\frac{\beta}{\alpha+\beta}\int_0^\infty(1+\tau)\eta(\tau)\mathrm{d}\tau W(0)\right\},$$
$$\lim_{\theta/(\alpha+\beta)\to 0}\mu_1(\alpha,\beta,\xi,\eta,K,W,\theta)=c_0,$$
$$\lim_{\theta/(\alpha+\beta)\to 0}\left\{\frac{\alpha+\beta}{\theta}[c_0-\mu_1(\alpha,\beta,\xi,\eta,K,W,\theta)]\right\}$$
$$= {c_0}^2/\left\{\frac{\alpha}{\alpha+\beta}\int_0^\infty\xi(c)\left[\int_{-\infty}^0|x|\exp\left(\frac{c-c_0}{cc_0}x\right)K(x)\mathrm{d}x\right]\mathrm{d}c\right.$$
$$\left.+\frac{\beta}{\alpha+\beta}\int_0^\infty\eta(\tau)e^\tau\left[\int_{-\infty}^{-c_0\tau}|x|\exp\left(\frac{x}{c_0}\right)W(x)\mathrm{d}x\right]\mathrm{d}\tau\right\}.$$

(II) Let $K(0) > 0$. There hold the following limits

$$\lim_{\theta/\alpha\to 1/2} \mu_2(\alpha,\xi,K,\theta) = 0,$$

$$\lim_{\theta/\alpha\to 1/2} \left\{\mu_2(\alpha,\xi,K,\theta)/\left(\frac{1}{2}-\frac{\theta}{\alpha}\right)\right\} = \frac{1}{K(0)} > 0,$$

$$\lim_{\theta/\alpha\to 0} \mu_2(\alpha,\xi,K,\theta) = c_0,$$

$$\lim_{\theta/\alpha\to 0} \left\{\frac{\alpha}{\theta}[c_0-\mu_2(\alpha,\xi,K,\theta)]\right\}$$
$$= {c_0}^2/\left\{\int_0^\infty \xi(c)\left[\int_{-\infty}^0 |x|\exp\left(\frac{c-c_0}{cc_0}x\right)K(x)\mathrm{d}x\right]\mathrm{d}c\right\}.$$

(III) Let $W(0) > 0$. There hold the following limits

$$\lim_{(\Theta-\alpha)/\beta\to 1/2} \mu_3(\beta,\eta,W,\Theta) = 0,$$

$$\lim_{(\Theta-\alpha)/\beta\to 1/2} \left\{\mu_3(\beta,\eta,W,\Theta)/\left(\frac{1}{2}-\frac{\Theta-\alpha}{\beta}\right)\right\}$$
$$= 1/\left\{\int_0^\infty (1+\tau)\eta(\tau)\mathrm{d}\tau W(0)\right\} > 0,$$

$$\lim_{(\Theta-\alpha)/\beta\to 0} \mu_3(\beta,\eta,W,\Theta) = \infty,$$

$$\lim_{(\Theta-\alpha)/\beta\to 0} \left\{\frac{\Theta-\alpha}{\beta}\mu_3(\beta,\eta,W,\Theta)\right\} = 0.$$

subsectionbifurcation of nonlinear waves

I am very interested in the existence and spiral waves s of integral differential equations study biologically relevant conditions for the existence, uniqueness and stability of the spiral waves. also study wave width and rotation rates. Additionally, I will study the convergence of a spiral wave to a periodic wave. A spiral wave is a rotating wave traveling outward from its center. Such spiral waves have been observed in many neurobiological systems, such as invertebrates, mammals, heart ventricular fibrillation, retinal spreading depression, fertilizing calcium waves, and glial calcium waves in of cortical tissue culture. Spiral waves are a basic feature of excitable systems. in Mathematical Neuroscience. Many previous work mathematical models of sal waves has voled w spatial interaction. Sri ied existence and siral waves y studying a crc region and moving in a coordinate frae that rotas with the spiral. Spiral waves n become time independent two-dimensional differential equation their stability can be solved by investigating the eigenvalues square matrix which results from the differential equations.of the differential equation and, such that $u_0(x)$. where where

References

x^3 α of (ξ, t) and (ξ, t) $a_0 L^1 \cap$, such that $\cdot_0 = 0$ $f \in \cap L^1$ Second, c $fu \leq C$; $=_0 /0_n =_u 02n == sstm)$g(x,t)s $sN_n N_{n,}\beta^2$, such that $eqC\{\}C\ \eta\eta == +v - v + u - 2\delta u,^2 v - v - uv + Aw, -w,^2 w ====$ $+uw$,
$(u), ub - u - uv, \& = \& + u - ()v - v^2$, $m \geq 0^3 x \lim_{t\to\infty} 11111 + t^{2m+n/2} \int^m \mathbf{x}, + - +[] =$, $(x) = 0$ represents nonnegative integers. Section Three subsectiondispersive wave equations

subsectiondissipative dispersive wave equations subsectionreaction diffusion equations 4.1. Consider the nonlinear system of Fitzhugh-Nagumo equations where $0 < a < \frac{1}{2}$, $0 < \gamma < \frac{4}{(1-a)^2}$ and $0 < \varepsilon \ll 1$ are positive constants. This is a nonlinear singularly perturbed system of reaction diffusion equations and it is a simplified version of the celebrated the Hodgkin-Huxley equations in mathematical neuroscience. There have been great progress in the existence and stability of traveling pulse solutions of this model since early 1970's. The representation of bounded traveling pulse solutions of the nonlinear singularly perturbed system of Fitzhugh-

Nagumo equations has been open. 4.2. Consider the nonlinear singularly perturbed system of reaction diffusion equations

This system is also called the diffusive predator-prey equations. The representation of bounded traveling wave solutions of the nonlinear system of reaction diffusion equations has been open. 4.3. Consider the nonlinear Belousov-Zhabotinskii system of reaction diffusion equations

where $\alpha > 0$, > 0, $\delta > 0$, $\varepsilon > 0$ and are positive constants. The representation of bounded traveling pulse solutions to the nonlinear system of reaction diffusion equations has been open.

subsectionnonlinear hyperbolic equations
subsectionother nonlinear evolution equations