Lehigh University Sample Calculus Diagnostic August 2009 version 1

Name_____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Simplify the exponential expression.

1) $(5x^{-5}y^{9}z^{-2})^{-3}$ 1) _____

A) $\frac{y^{12}}{-15x^{8}z^{5}}$ B) $\frac{x^{15}z^{6}}{125y^{27}}$ C) $\frac{x^{15}z^{6}}{-15y^{-27}}$ D) $\frac{y^{12}}{125x^{8}z^{5}}$

Rationalize the denominator.

2) $\frac{4}{\sqrt{10}+\sqrt{14}}$ 2) _____

A) $\sqrt{10}-\sqrt{14}$ B) $\sqrt{4}$ C) $\sqrt{14}+\sqrt{10}$ D) $\sqrt{14}-\sqrt{10}$

Simplify the expression.

3) $\frac{\frac{x^2}{\sqrt{x^2+3}}-\sqrt{x^2+3}}{x^2}$ 3) _____

A) $\frac{2x^2+3}{x^2\sqrt{x^2+3}}$ B) $\frac{3}{x^2\sqrt{x^2+3}}$

C) $\frac{-3}{x^2\sqrt{x^2+3}}$ D) $\frac{\frac{x^2}{\sqrt{x^2+3}}-\sqrt{x^2+3}}{x^2}$

Solve the linear equation.

4) $\frac{2x}{5}-x=\frac{x}{40}-\frac{9}{8}$ 4) _____

A) $\left\{\frac{45}{23}\right\}$ B) $\left\{-\frac{9}{5}\right\}$ C) $\left\{\frac{9}{5}\right\}$ D) $\left\{-\frac{45}{23}\right\}$

Solve the quadratic equation using the quadratic formula.

5) $x^2+8x=3$ 5) _____

A) $\{4+\sqrt{19}\}$ B) $\{-4-\sqrt{19}, -4+\sqrt{19}\}$

C) $\{-1-\sqrt{19}, -1+\sqrt{19}\}$ D) $\{-4-2\sqrt{19}, -4+2\sqrt{19}\}$

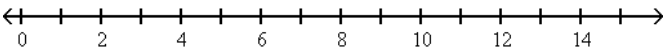
Solve the problem.

6) The formula C = 0.5x + 20 represents the estimated future cost of yearly attendance at State University, where C is the cost in thousands of dollars x years after 2002. Use a compound inequality to determine when the attendance costs will range from 24 to 26 thousand dollars. 6) _____

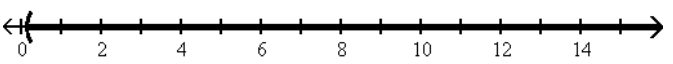
A) From 2010 to 2014
B) From 2009 to 2013
C) From 2011 to 2013
D) From 2011 to 2015

Solve the absolute value inequality. Other than ∅, use interval notation to express the solution set and graph the solution set on a number line.

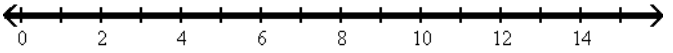
7) $|7x - 9| - 2 > -10$ 7) _____



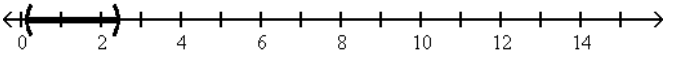
A) $\left(\frac{1}{7}, \infty\right)$



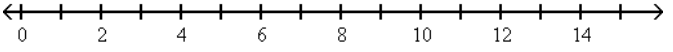
B) $(-\infty, \infty)$



C) $\left(\frac{1}{7}, \frac{17}{7}\right)$

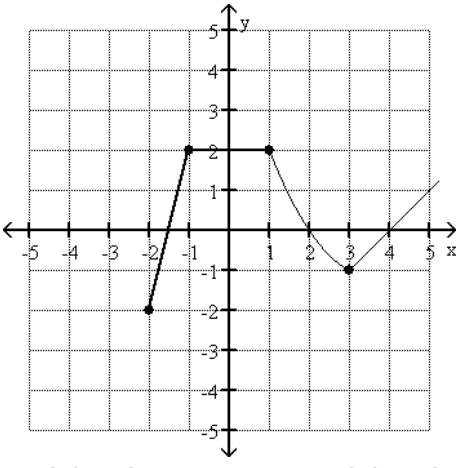


D) ∅



Identify the intervals where the function is changing as requested.

8) Increasing 8) _____



A) (-2, 1)
B) (-1, ∞)
C) (-2, -1) or (3, ∞)
D) (-1, 3)

Find and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$ for the given function.

9) $f(x) = 5x^2$ 9) _____

A) $5(2x+h)$

B) $\frac{5(2x^2 + 2xh + h^2)}{h}$

C) $\frac{10}{h} + x + 5h$

D) 5

Use the given conditions to write an equation for the line in point-slope form.

10) Passing through (4, 2) and (3, 7) 10) _____

A) $y - 2 = 4(x + 4)$ or $y - 7 = 3(x - 2)$

B) $y + 2 = -5(x + 4)$ or $y + 7 = -5(x + 3)$

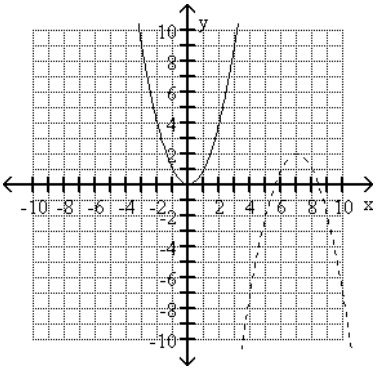
C) $y - 2 = -5(x - 4)$ or $y - 7 = -5(x - 3)$

D) $y - 2 = -5(x - 3)$ or $y - 7 = -5(x - 4)$

Begin by graphing the standard quadratic function $f(x) = x^2$. Then use transformations of this graph to graph the given function.

11) $h(x) = -(x + 7)^2 + 2$ 11) _____

A)



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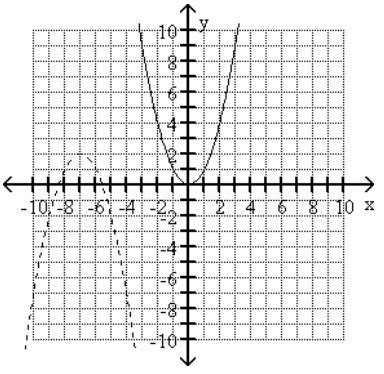

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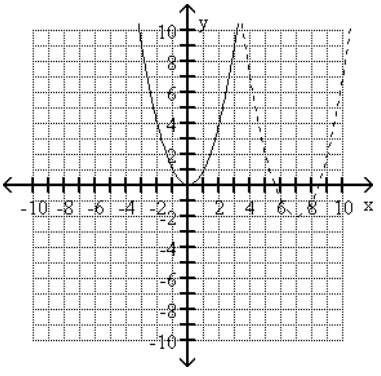
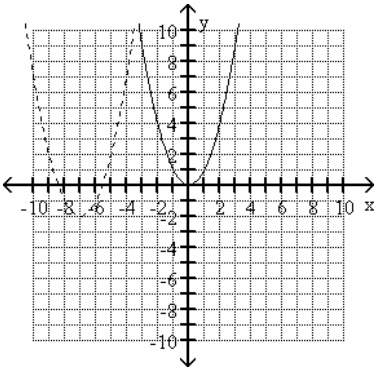


D)



Find the domain of the function.

12) $h(x) = \frac{x - 4}{x^3 - 16x}$ 12) _____

A) $(-\infty, 0) \cup (0, \infty)$

B) $(-\infty, 4) \cup (4, \infty)$

C) $(-\infty, -4) \cup (-4, 0) \cup (0, 4) \cup (4, \infty)$

D) $(-\infty, \infty)$

For the given functions f and g , find the indicated composition.

13) $f(x) = \frac{4}{x + 6}$, $g(x) = \frac{7}{8x}$ 13) _____

$(f \circ g)(x)$

A) $\frac{32x}{7 + 48x}$ B) $\frac{4x}{7 + 48x}$ C) $\frac{32x}{7 - 48x}$ D) $\frac{7x + 42}{32x}$

Solve the problem.

14) A person invested $20,000, part of the money, x, was placed in a stock that paid 15% annual interest. The rest of the money suffered a 4% loss. Express the total annual income from both investments, I, as a function of x. 14) _____

A) I(x) = 0.04x - 0.15(20,000 - x)
B) I(x) = 0.15x - 0.04(20,000 - x)
C) I(x) = 0.15x + 0.04(20,000 - x)
D) I(x) = -0.15x + 0.04(20,000 - x)

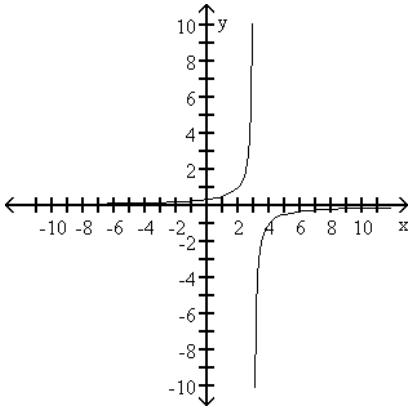
Solve the polynomial equation. In order to obtain the first root, test the possible rational roots.

15) $x^3 - 6x^2 + 7x + 2 = 0$ 15) _____

A) {1, -1, -2}
B) $\{2, 2 + \sqrt{5}, 2 - \sqrt{5}\}$
C) $\{2, 4 + \sqrt{2}, 4 - \sqrt{2}\}$
D) $\{-2, 4 + \sqrt{5}, 4 - \sqrt{5}\}$

Use the graph of the rational function shown to complete the statement.

16) 16) _____



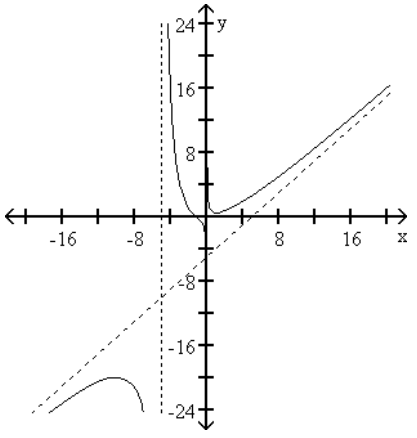
As $x \to 3^+$, $f(x) \to$?

A) 0 B) $+\infty$ C) 3 D) $-\infty$

Graph the function.

17) $f(x) = \frac{x^3 + 2}{x^2 + 5x}$ 17) _____

A)



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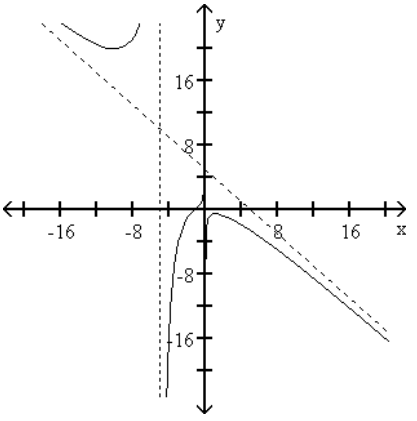

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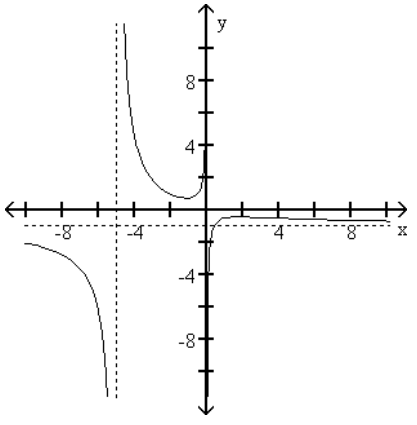

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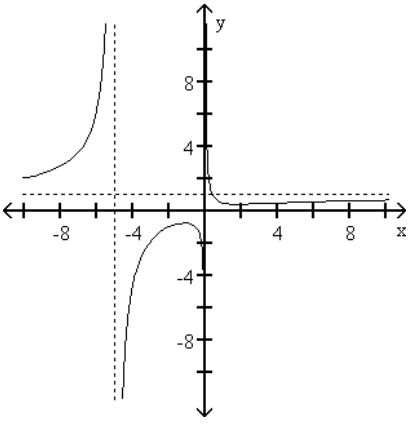


Solve the problem.

18) The average cost per unit, y, of producing x units of a product is modeled by $y = \frac{300{,}000 + 0.25x}{x}$. 18) _____

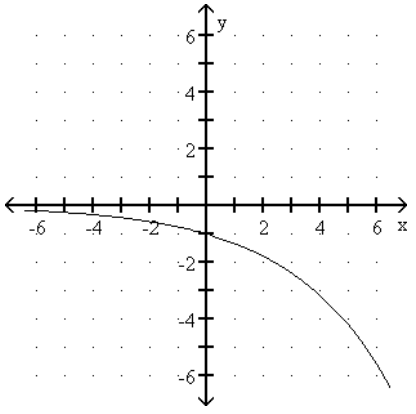
Describe the company's production level so that the average cost of producing each unit does not exceed $1.75.

A) At least 300,000 units
B) At least 200,000 units
C) Not more than 300,000 units
D) Not more than 200,000 units

Graph the function.

19) $f(x) = \left(\frac{3}{4}\right)^x$ 19) _____

A)



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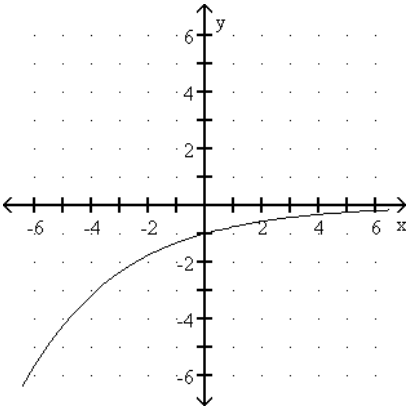

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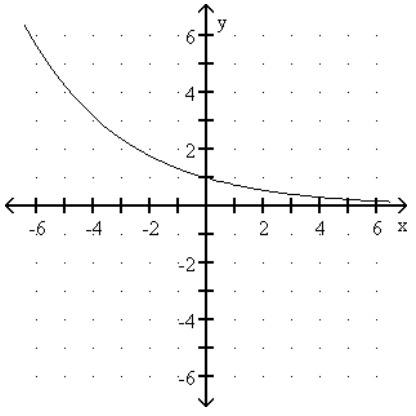

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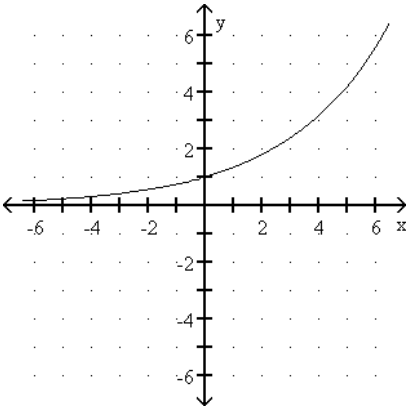


Use properties of logarithms to condense the logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions.

20) $5 \log_b q - \log_b r$ 20) _____

A) $\log_b (q^5 - r)$

B) $\log_b \left(\frac{5q}{r}\right)$

C) $\log_b \left(\frac{q^5}{r}\right)$

D) $\log_b q^5 \div \log_b r$

Solve the equation by expressing each side as a power of the same base and then equating exponents.

21) $2^{(7 - 3x)} = \frac{1}{4}$ 21) _____

A) $\left\{\frac{1}{2}\right\}$ B) {-3} C) {1} D) {3}

Convert the angle in degrees to radians. Express answer as a multiple of π.

22) 144° 22) _____

A) $\frac{4\pi}{5}$ radians B) $\frac{3}{5}\pi$ radians C) $\frac{3\pi}{4}$ radians D) $\frac{5\pi}{6}$ radians

Use the unit circle to find the value of the trigonometric function.

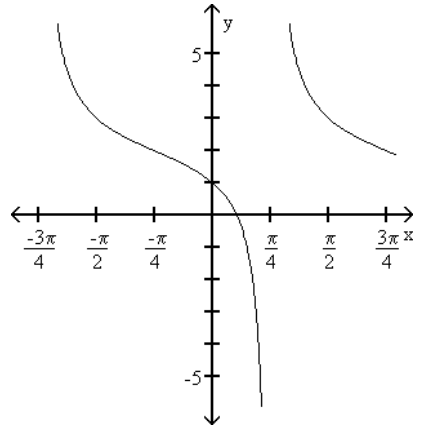
23) $\sec\frac{7\pi}{6}$ 23) _____

A) $\frac{2\sqrt{3}}{3}$ B) $-\frac{1}{2}$ C) $-\frac{\sqrt{3}}{2}$ D) $-\frac{2\sqrt{3}}{3}$

Graph the function.

24) $y = 2 - \tan\left(x + \frac{\pi}{4}\right)$ 24) _____

A)



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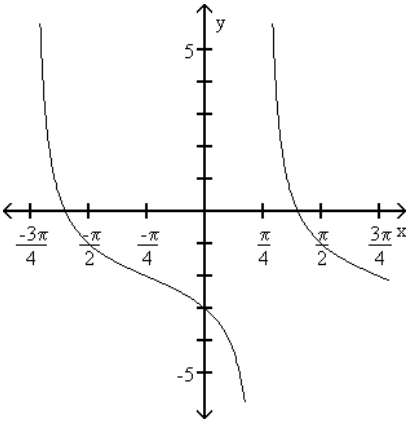

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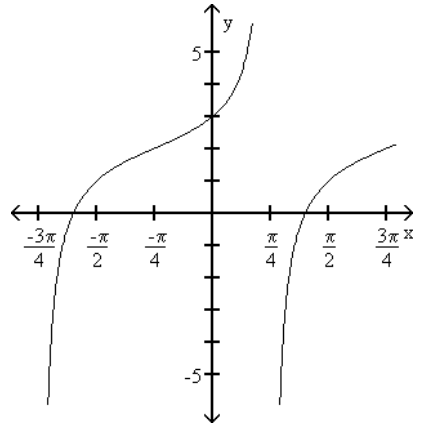
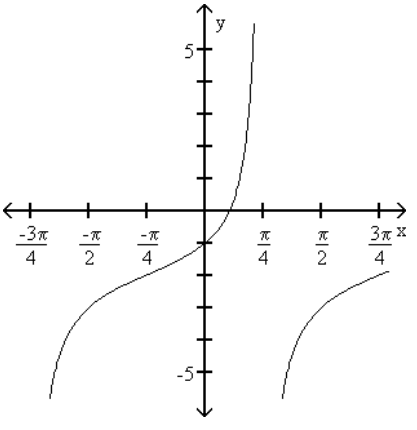


D)



Find the exact value of the expression.

25) $\sin^{-1}\frac{\sqrt{3}}{2}$ 25) _____

A) $\frac{3\pi}{4}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{4}$ D) $\frac{2\pi}{3}$

Use a sketch to find the exact value of the expression.

26) $\cos\left(\tan^{-1}\frac{2}{3}\right)$ 26) _____

A) $\frac{3\sqrt{13}}{13}$ B) $\frac{3}{13}$ C) $\frac{\sqrt{13}}{3}$ D) $\frac{2}{3}$

Solve the problem.

27) A surveyor is measuring the distance across a small lake. He has set up his transit on one side of the lake 140 feet from a piling that is directly across from a pier on the other side of the lake. From his transit, the angle between the piling and the pier is 30°. What is the distance between the piling and the pier to the nearest foot? 27) _____

A) 70 feet B) 121 feet C) 81 feet D) 242 feet

Complete the identity.

28) $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = ?$ 28) _____

A) $-2\tan^2 x$ B) $\sin x \tan x$ C) $1 + \cot x$ D) $\sec x \csc x$