Math 163 Exam 3 Lehigh University 4-4-2008
This is closed book, closed notes etc. You have 50 minutes to take this exam.
Points for each problem are indicated as $[\cdot]$.
Be sure to do the problems that you know well first.
There are 6 problems plus 2 bonus problems
When you are asked to pick a part or parts of a problem to answer clearly indicate your selection. If you attempt more than is asked those with lowest scores will be used.

1: [30] Consider the variants for strong duality listed below.
(a) Pick *one* of A' implies C' or C' implies A' and prove it.
(b) Pick *one* other implication and prove it. State clearly which implication you are proving.

Strong Duality for Linear Programming variants:
A': If both problems are feasible then :
$\max\{\boldsymbol{cx} | A\boldsymbol{x} \le \boldsymbol{b}\} = \min\{\boldsymbol{yb} | \boldsymbol{y}A = \boldsymbol{c}, \boldsymbol{y} \ge \boldsymbol{0}\}$
B': If both problems are feasible then :
$\max\{\boldsymbol{cx} | A\boldsymbol{x} \le \boldsymbol{b}, \boldsymbol{x} \ge \boldsymbol{0}\} = \min\{\boldsymbol{yb} | \boldsymbol{y}A \ge \boldsymbol{c}, \boldsymbol{y} \ge \boldsymbol{0}\}$
C': If both problems are feasible then :
$\max\{\boldsymbol{cx} | A\boldsymbol{x} = \boldsymbol{b}, \boldsymbol{x} \ge \boldsymbol{0}\} = \min\{\boldsymbol{yb} | \boldsymbol{y}A \ge \boldsymbol{c}\}$

2: [15] Consider the system of equations (in one variable) $x = a_1, x = a_2, \ldots, x = a_n$ for given numbers $a_1 \le a_2 \le \cdots \le a_n$. The best L_1 approximation is any value of x that minimizes $\sum_{i=1}^{n} |a_i - x|$. Prove that the median is such a value. For simplicity assume that $n = 2m + 1$ is odd and hence the median is a_{m+1}

3: [15] Consider the system of equations (in one variable) $x = a_1, x = a_2, \ldots, x = a_n$ for given numbers $a_1 \le a_2 \le \cdots \le a_n$. The best L_2 approximation is any value of x that minimizes $\sum_{i=1}^{n} (a_i - x)^2$. Prove that this is the arithmetic mean $\frac{1}{n} \sum_{i=1}^{n} a_i$.

4: [10] Use Lagrange interpolation to determine the unique parabola through the points $(-1, 5), (3, 4), (6, 7)$. You do not need to simplify the expressions that you get using the interpolation polynomials.

5: [15] For the points $(-1, 5), (3, 4), (6, 7)$ write down a linear programming problem whose solution will give the slope m and intercept b for
(a) The best L_1 line
(b) The best L_∞ line.

6: [15] Consider a system of m equations in the n variables $x_1, x_2, \ldots, x_n$ which we can write as $\sum_{j=1}^{n} a_{ij} x_j = b_i$ for $i = 1, 2, \ldots, m$ or alternatively in matrix notation $A\boldsymbol{x} = \boldsymbol{b}$. Write down (using $\sum$ notation) a linear programming problem whose solution gives
(a) the best L_1 approximation
(b) the best L_∞ approximation.

7: [5] We could use Lagrange interpolation to find the unique parabola through the points $(-1, 5), (3, 4), (6, 7)$. Instead we can determine the coefficients a_0, a_1, a_2 in the polynomial $a_0 + a_1 x + a_2 x^2$ by solving a system of equations. Write down that system.

8: [10] For problem 6 write down the answers to (a) and (b) using matrix notation.