Math 163 Introductory Seminar - Lehigh University - Spring 2008 - Assignment 6
Due Wednesday February 27

18. Apply Fourier-Motzkin elimination to the following systems. Using this method, either determine one solution or give a certificate showing the original system is inconsistent. Use Fourier-Motzkin elimination to get your answers, showing how it could work on much larger systems.

$$\textbf{(a)}\quad \begin{array}{rcrcr} x_1 & + & x_2 & \le & 3 \\ 2x_1 & - & 8x_2 & \le & 8 \\ -3x_1 & + & 9x_2 & \le & -15 \end{array} \qquad \textbf{(b)}\quad \begin{array}{rcrcr} x_1 & + & x_2 & \le & 3 \\ 2x_1 & - & 8x_2 & \le & 4 \\ -3x_1 & + & 9x_2 & \le & -15 \end{array}$$

19. Prove the following version of weak duality:
If both problems are feasible then $\max\{\boldsymbol{cx} | A\boldsymbol{x} \le \boldsymbol{b}\} \le \min\{\boldsymbol{yb} | \boldsymbol{y}A = \boldsymbol{c}, \boldsymbol{y} \ge \boldsymbol{0}\}$.
Give *two* proofs. One using matrix notation and one using $\sum$ notation.

20. Consider the following statement: If the dual is unbounded then the primal is infeasible. Prove this for the versions of linear programming problems in problem 19 by first stating the contrapositive and then proving that. You may use the result of problem 19.

21. Consider the linear programming problem:

$$\begin{array}{rrcrcrcr} \max & x_1 & + & 2x_2 & + & 3x_3 & & \\ \text{s.t.} & 4x_1 & - & 5x_2 & + & 6x_3 & = & 7 \\ & 8x_1 & & & + & 9x_3 & = & 10 \\ & x_1 & + & x_2 & + & x_3 & \le & 0 \\ & x_1 & - & x_2 & + & 13x_3 & \le & 1 \\ & x_1 & , & x_2 & & & \ge & 0 \end{array}$$

(a) Write down an equivalent problem that is in the form $\max\{\boldsymbol{cx} | A\boldsymbol{x} = \boldsymbol{b}, \boldsymbol{x} \ge \boldsymbol{0}\}$. Write out the equations as above. Also give the matrix $A$ and vectors $\boldsymbol{b}, \boldsymbol{c}$ for this.
(b) Write down the duals to both the original problem and the problem in part (a).

22. Prove the equivalence of B and C:

B: Exactly one of the following holds:
(I) $A\boldsymbol{x} \le \boldsymbol{b}, \boldsymbol{x} \ge \boldsymbol{0}$ has a solution $\boldsymbol{x}$ *or* (II) $\boldsymbol{y}A \ge \boldsymbol{0}, \boldsymbol{y} \ge \boldsymbol{0}, \boldsymbol{yb} < 0$ has a solution $\boldsymbol{y}$

C: Exactly one of the following holds:
(I) $A\boldsymbol{x} = \boldsymbol{b}, \boldsymbol{x} \ge \boldsymbol{0}$ has a solution $\boldsymbol{x}$ *or* (II) $\boldsymbol{y}A \ge \boldsymbol{0}, \boldsymbol{yb} < 0$ has a solution $\boldsymbol{y}$

23. Prove the equivalence of A' and B':

A': If both problems are feasible then : $\max\{\boldsymbol{cx} | A\boldsymbol{x} \le \boldsymbol{b}\} = \min\{\boldsymbol{yb} | \boldsymbol{y}A = \boldsymbol{c}, \boldsymbol{y} \ge \boldsymbol{0}\}$

B': If both problems are feasible then : $\max\{\boldsymbol{cx} | A\boldsymbol{x} \le \boldsymbol{b}, \boldsymbol{x} \ge \boldsymbol{0}\} = \min\{\boldsymbol{yb} | \boldsymbol{y}A \ge \boldsymbol{c}, \boldsymbol{y} \ge \boldsymbol{0}\}$