11. We can consider perfect matchings in graphs in general, not just bipartite graphs. For this
we use Tutte’s condition: $\text{odd}(G - S) \leq |S|$ for all $S \subseteq V$. Do not worry what $\text{odd}(G - S)$
means or what the condition is about for now. It is known that Tutte’s condition is necessary
and sufficient for a graph $G$ to have a perfect matching.

(a) State a condition equivalent to: Tutte’s condition is a necessary condition for $G$ to have a
perfect matching.
(b) State the contrapositive of the statement in (a).
(c) State a condition equivalent to: Tutte’s condition is a sufficient condition for $G$ to have a
perfect matching.
(d) State the contrapositive of the statement in (a).

For (a) and (c) one of your statements should be of the form ‘If $G$ has a perfect matching then ...
and the other should be of the form ‘If ... then $G$ has a perfect matching.’ The ‘...’ should
be statements using terms like ‘for some $S \subseteq V$, ‘for all $S \subseteq V$, $|S|$, $\text{odd}(G - S)$, $\leq$ and $>$. For (b) and (c) you should give ‘if ... then’ statements using terms like those in the previous
sentence (and you should not use the word ‘not’) and ‘$G$ does not have a perfect matching’.

(a) If $G$ has a perfect matching then $\text{odd}(G - S) \leq |S|$ for all $S \subseteq V$.
(b) If $\text{odd}(G - S) > |S|$ for some $S \subseteq V$ then $G$ does not have a perfect matching.
(c) If $\text{odd}(G - S) \leq |S|$ for all $S \subseteq V$ then $G$ has a perfect matching.
(d) If $G$ does not have a perfect matching then $\text{odd}(G - S) > |S|$ for some $S \subseteq V$. 