Math 163 Introductory Seminar - Lehigh University - Spring 2008 - Assignment 1
Due Wednesday January 23

1. Let W be a set of men and M a set of women (with the same number of men and women, $|W| = |M| = n$) and E a set of pairs (w, m) with $w \in W$ and $m \in M$.

If there are subsets $R \subseteq W$ and $S \subseteq M$ such that $|R| + |S| < n$ and every pair in E contains at least one member of $R \cup S$ (that is, for each $(w, m) \in E$ either $w \in R$ or $m \in S$ or both), then there is no matching of the men and women with each pair from E. The marriage theorem shows that the converse also holds: if there is no matching of the men and women then there are R and S as described in the previous sentence.

Another condition is as follows: If there is a set T of women who 'like' strictly less than $|T|$ men then there is no matching of the men and women. More formally, if there is $T \subseteq W$ such that $|\{m|(w, m) \in E \text{ for some } w \in T\}| < |T|$ then there is no matching of the men and women. Use the marriage theorem to prove that the converse also holds: if there is no matching of men and women then there is a set T as described in the previous sentence.

2. Prove by induction that the Fibonacci numbers satisfy the following formula:
$F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{-1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$.

3. Prove by induction that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.